METHODOLOGY FOR THE CONCEPTUAL DESIGN OF A ROBUST AND OPPORTUNISTIC SYSTEM-OF-SYSTEMS

A Dissertation
Presented to
The Academic Faculty

by

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In Partial Fulfillment
Of the Requirements for the Degree
Doctor of Philosophy in the
School of Aerospace Engineering

Georgia Institute of Technology

December 2008

METHODOLOGY FOR THE CONCEPTUAL DESIGN OF A ROBUST AND OPPORTUNISTIC SYSTEM-OF-SYSTEMS

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Date Approved: October 29, 2008

For two men who never cease to amaze and inspire me:

Stephen B. Talley and Matthew A. Rizzo

ACKNOWLEDGEMENTS

I would like to thank my family and friends for their unwavering support. I especially want to thank Matt Rizzo, Steve Talley, Penny Talley, and Susan Talley for their daily encouragement and selfless efforts in helping me complete this process. I truly cannot thank you all enough. Without a doubt each of you directly contributed to my success, and I would not be where I am today without your support. I also would like to thank both Patricia Talley and Mary Lou Noonan for their love and support.

Additionally I would like to thank my friends and coworkers in the Aerospace Systems Design Laboratory (ASDL) at Georgia Tech. I learned an enormous amount from all of you and in a very real sense this research is the culmination of all of our conversations and collaborations over the years. I would especially like to recognize and thank my office mates: Shuo-Ju Chou, Carl Johnson, and Daniel Cooksey as well as the following individuals: Kristin Kelly, Dan Crowley, William Engler, Steve Tangen, Patrick Biltgen, Ben Poole, Kelly Fry, Ryan Iandoli, Stephane DuFresne, Bjorn Cole, Santiago Balestrini, Hernando Jimenez, Jan Osburg, Rob McDonald, Blake Moffitt, Jeff Schutte, Brian Kestner, Jesse Hester, Ian Stults, Irian Ordaz, Tommer Ender, Elena Garcia, Simon Briceño, Holger Pfaender, Loretta Carroll, and countless other people in the lab. I consider myself privileged to have worked you all, and I am looking forward to seeing what all of you accomplish in the coming years.

I am also extremely fortunate to have had the support of a number of individuals outside of the lab. I would like to recognize and thank Patrick Engdahl, Michele Evans, Brenda Drake, Michelle Stanfield, Rebecca Hoover, and Lynn Rizzo. I would also like to thank the entire absolutely wonderful staff of the Georgia Tech Craft Center for providing hours of stress relief and for making my second job such a rewarding experience.

The NASA Graduate Student Research Program (GSRP) proved to be an invaluable opportunity for me. The work I did through the GSRP served as the main motivation for this body of research and helped me to develop many of the initial concepts. I would like to thank Craig Nickol, Bill Kimmell, and everyone in the Aeronautics Systems Analysis Branch at NASA Langley for their support during my fellowship.

I would also like to sincerely thank the members of my committee for their comments and guidance throughout this process: Dr. Daniel Schrage, whose guidance and encouragement was instrumental in the development of this research; Dr. Carlee Bishop, whose comments and questions throughout the process helped to refine the topic and identify key issues; Dr. Mark Costello, whose comments repeatedly helped in clarifying and expressing the concepts of my research; and Mr. Dave Brown, who not only provided part of the initial motivation for this project through the Air Force Research Laboratory (AFRL) Revolutionary Hunter/Killer UAV Project but who offered practical guidance throughout the process that helped me to explore and expand the potential of these research concepts.

And finally, and I would like to thank my advisor and committee chairman, Dr. Dimitri Mavris. Over the past five years I have learned an enormous amount from you, but most importantly I have learned how little I know in the grand scheme of things and the importance of continued learning and personal development. I have constantly been inspired by your ever evolving vision and am truly grateful for the many opportunities that I have had under your guidance. I am looking forward to seeing what you and the lab are able to achieve over the coming years, and I am very proud to be part of the ASDL family.

TABLE OF CONTENTS

TABLES

FIGURES

NOMENCLATURE

AAA	Anti-Aircraft Artillery
ACCost	Aircraft Acquisition Cost
AF	Air Force
AFRL	Air Force Reseaerch Lab
AMPR	Aeronautical Manufacturers Planning Report
ATMAS	Aircraft Time critical Target Mission Analysis Simulation
AVRU	Actual Value of Reducing Uncertainty
Bel	Belief Function
BPA	Basic Probability Assignment
BRAINN	Basic Regression Analysis for Integrated Neural Networks
C4ISR	Command, Control, Communications, Computers, Intelligence, Surveillance and Reconnaissance
CD0	Zero lift drag coefficient
CLMAX	Maximum Lift Coefficient
Con	Constraint
CONDOR-SS	CONceptual Design of Opportunistic and Robust System-of-Systems
CS	Combined set
CSSUA	Concurrent Subsystem Uncertainty Analysis
DCI	Design Capability Indices
DOE	Design of Experiements

E	Expectation, Subset of Set S
ECRU	Expected Cost to Reduce Uncertainty
ELR	Expected Loss Ratio
EMI	Error Margin Index
EOL	Expected Opportunity Loss
EPV	Empirical Performance Validity
ESV	Empirical Structural Validity
ET	Evidence Theory
EVI	Expected Value of Information
EVIU	Expected Value of Including Information
EVPI	Expected Value of Perfect Information
EVRU	Expected Value of Reducing Uncertainty
FLAMES	FLexible Analysis, Modeling, and Exercise System
FLTC	Focused Long Term Challenges
FORM	First Order Reliability Method
FOS	Family-of-Systems
FST	Fuzzy Set Theory
HUMM	Hybrid Uncertainty Modeling Method
IDEM	Inductive Design Exploration Method
IGT	Info-Gap Theory
INCOSE	International Council on Systems Engineering
Int	Intercept

JPDM	Joint Probabilistic Decision Making
K1	Coefficient in lift-drag polar equation
K2	Coefficient in lift-drag polar equation
kts	Knots
L	Loss
L	Low value of Interval
L/D	Lift over Drag
LANTIRN	Low Altitude Navigation and Targeting Infrared for Night
lbs	pounds
LT	Loiter Time
M	Mach Number, Monetary value
m	Möbius
M&S	Modeling and Simulation
MADM	Multi-Attribute Decision Making
Max	Maximum
MC	Monte Carlo
MCS	Monte Carlo Simulation
Min	Minimum
MODM	Multi-Objective Decision Making
MQ	Multi-role unmanned aircraft
NATS	National Air Transportation System
NFS	Number of runs from Full Factorial DOE for Fuzzy Set Theory

NI	Number of Intervals
NN	Neural Net
NO	The number of Options/ Fuzzy Sets in the analysis
NPSI	Number of product space intervals
NPV	Number of Probablity Theory Uncertainty Variables, Net Present Value
NR	Number of DOE Runs while using Probability Theory
NR	Number of runs from Full Factorial DOE for Probability Theory Uncertainty Variables
NS	Number of runs from Full Factorial DOE for Evidence Theory Uncertainty Variables
NS	Number of Sources
NSI	Number of sub-intervals
NumHK	Number of Hunter/Killer Aircraft
NUV	Number of uncertainty variable
OEC	Overall Evaluation Criterion
OES	Operational Environment and Scenario
OIF	Operation Iraqi Freedom
Opp	Opportunistic
OppOnly	Opportunistic Only
ORG	Original
P	Probability, Penalty Weight
pdf	Probability density function
PF	Penalty Factor

Pl	Plausibility Function
PS	Product space
PT	Probability Theory
q	Dynamic pressure
q_{MAX}	Maximum dynamic pressure
R	Range
R^2	Coefficent of determination
RADAR	Radio Detection And Ranging
RandO	Robust and Opportunistic
RCEM	Robust Concept Exploration Method
Ronly	Robust Only
RSE	Response Surface Equation
S	Wing Area
S&T	Science & Technology
SAM	Surface to Air Missiles
SEAS	System Effectiveness Analysis Simulation
SOS	System-of-Systems
SUA	System Uncertainty Analysis
T/W	Thrust to Weight
TOGW	Take-off Gross Weight
TOP	Take-off Performance
TOPSIS	Technique for Order Preference by Similarity to Ideal Solution

TP	Turboprop
TPV	Theoretical Performance Validity
TSFC	Thrust Specific Fuel Consumption
T_{SL}	Thrust at sea level
TSV	Theoretical Structural Validity
U	Upper value of Interval
U	Utility
UAS	Unmanned Aerial System
UAV	Unmanned Aerial Vehicle
V	Velocity
VRUM	Value of Reducing Uncertainty Method
W_{AMPR}	AMPR Weight
w_c	penalty weight
W_e	Empty Weight
W_{empty}	Empty Weight
W_f	Fuel Weight
W_{new}	New Weight
W_o	Take-off Gross Weight
W_{org}	Original Weight
$W_{payload}$	Payload Weight
W_{TO}	Take-off Weight
ZSU	Zenitnaya Samokhodnaya Ustanovka

$\hat{\alpha}$	Info-Gap Theory uncertainty parameter
$\hat{\beta}$	Info-Gap Theory uncertainty parameter
α	Robustness Function, beta distribution parameter 1, installed thrust lapse
β	Opportunity Function, beta distribution parameter 2, instantaneous weight fraction
δ_t	Discount rate for period t
λ	Multiplier
μ	Membership function
π	Probability, pi
ρ	Density, correlation coefficient
σ	Standard deviation

SUMMARY

Systems are becoming more complicated, complex, and interrelated. Designers have recognized the need to develop systems from a holistic perspective and design them as Systems-of-Systems (SoS). The design of the SoS, especially in the conceptual design phase, is generally characterized by significant uncertainty. As a result, it is possible for all three types of uncertainty (aleatory, epistemic, and error) and the associated factors of uncertainty (randomness, sampling, confusion, conflict, inaccuracy, ambiguity, vagueness, coarseness, and simplification) to affect the design process. While there are a number of existing SoS design methods, several gaps have been identified: the ability to modeling all of the factors of uncertainty at varying levels of knowledge; the ability to consider both the pernicious and propitious aspects of uncertainty; and, the ability to determine the value of reducing the uncertainty in the design process.

While there are numerous uncertainty modeling theories, no one theory can effectively model every kind of uncertainty. This research presents a Hybrid Uncertainty Modeling Method (HUMM) that integrates techniques from the following theories: Probability Theory, Evidence Theory, Fuzzy Set Theory, and Info-Gap theory. The HUMM is capable of modeling all of the different factors of uncertainty and can model the uncertainty for multiple levels of knowledge.

In the design process, there are both pernicious and propitious characteristics associated with the uncertainty. Existing design methods typically focus on developing robust designs that are insensitive to the associated uncertainty. These methods do not capitalize on the possibility of maximizing the potential benefit associated with the uncertainty. This research demonstrates how these deficiencies can be overcome by identifying the most robust and opportunistic design.

In a design process it is possible that the most robust and opportunistic design will not be selected from the set of potential design alternatives due to the related uncertainty. This research presents a process called the Value of Reducing Uncertainty Method (VRUM) that can determine the value associated with reducing the uncertainty in the design problem before a final decision is made by utilizing two concepts: the Expected Value of Reducing Uncertainty (EVRU) and the Expected Cost to Reducing Uncertainty (ECRU).

The techniques developed in this research, were combined into a robust and opportunistic design method specifically for the conceptual design of a SoS called CONceptual Design of Opportunistic and Robust System-of-Systems (CONDOR-SS). The utility of the individual techniques and the CONDOR-SS method as a whole are demonstrated through a number of example SoS design problems related to a persistent strike battlespace scenario.

CHAPTER 1: MOTIVATION

The world is made up of systems from roads and cities to the internet. The International Council on Systems Engineering (INCOSE) System Engineering Handbook defines systems as “a combination of interacting elements organized to achieve one or more stated purposes.” [79] Systems have allowed us to do everything from communicating with people around the globe, to mowing our lawn, to artificial organs. In many cases, these systems can be combined to form a System-of-Systems (SoS). Examples of SoSs include: automobiles, manufacturing lines, power plants, or even an item as small as a digital camera. [79]

As the complexity of these systems increases, so does the complexity in designing these systems. When designing a SoS there will be uncertainty. In many cases, the requirements of the overall system or subsystems will be unknown or ambiguous. An example many of us are familiar with pertains to city planning of highways. The designer was uncertain about what the future traffic demand would be and designed the roadways according to the estimated demand. In some areas these forecasts were adequate and in others much less so, resulting in expensive roadway expansion projects. Another source of uncertainty is in the interactions between the subsystems of the SoS. For instance, in the air transportation system, a variety of types of aircraft are used to transport people between different airports around the world. These aircraft must communicate with each other and the air traffic controllers in order to safely operate in the same area. Part of their interactions also deals with the air flow between the aircraft. Wingtip vortices from large aircraft can represent a serious hazard to small light aircraft. Generally the exact location and strength of a wingtip vortex is unknown. This is an example of a typically invisible interaction that must be considered by the air traffic controllers when regulating the traffic pattern.

These examples illustrate why it is important to consider uncertainty in the design of a System-of-Systems. This is especially critical in the aerospace field where system problems or failures can often have catastrophic results. While existing design methods can be used to design aerospace systems, they lack techniques for identifying, quantifying, propagating, and tracking all of the different factors of uncertainty that are in a SoS. This observation presents a need in the industry and leads to the question of how an aerospace SoS should be designed considering the inherent system complexity and associated uncertainty. The objective of this research is to systematically answer this question.

1.1. Evolution of Aerospace Design

Systems have been originally designed by optimizing for given requirements and with specified constraints. In the aerospace community, aircraft were typically designed to maximize the performance and minimize the weight. [107] Within the last couple of decades a paradigm shift occurred in the aerospace industry from design for performance to design for affordability and quality. [107] A paradigm shift occurs when the paradigm, or collection of assumptions, concepts, values, and practices that constitutes a common way of viewing reality for a particular group, changes fundamental concepts and inspires "new standards of evidence, new research techniques, and new pathways of theory and experimentation". [107,167] This paradigm of design for affordability continued to place significant emphasis on the vehicle itself but now also included life cycle considerations, breakthrough technologies, and new design methods. [107]

Additionally, the industry has come to realize that a system does not operate in isolation, and therefore the most effective and efficient systems are those designed as part of a larger system. Design requirements need to consider interactions and relationships or interfaces between systems. [79] The industry is starting to move beyond vehicle system design to consider the design of the entire System-of-Systems (SoS).

System-of-Systems will be defined in detail in the following chapter, but a general definition from the INCOSE System Engineering Handbook is that “a SoS applies to a system whose elements are themselves systems; typically entail large scale inter-disciplinary problems with multiple, heterogeneous, distributed systems.” [79] While several examples of SoS were provided in the previous section, examples of aerospace SoS include: jet engines, commercial airliners, air transportation systems, theater missile defense systems, and battlespace operations. However, a SoS does not have to be a large-scale system. Some small micro-air vehicles can also be considered SoS.

It is apparent that SoS and their related subsystems surround us in our professional and private lives. It is logical that in order to develop the best design for a system, its environment, interactions, and overall purpose must be considered. However, considering the complete system together brings an increasing amount of complexity and a number of challenges.

1.2. INCOSE Challenges

For the development of System-of-Systems, INCOSE has identified the following challenges:

System elements operate independently.

Many of the systems in a SoS are likely to be operated autonomously. An example of this is commercial aircraft fleets since each aircraft is piloted independently. This fact brings a fair amount of uncertainty into the problem. For instance, it is uncertain how these systems will interact with each other. Another issue is how these independent systems should be operated and managed. Each of these systems may have their own individual objectives which complicates the operation of the SoS and the design of the overall system.

System elements have different life cycles.

By definition, SoS are made up of multiple elements or systems. In many cases some of the systems have already been in operation while other components are being developed specifically for the system. The life cycles of these systems are different, but the systems still need to be able to interact. An example would be when an avionics system is upgraded in an existing aircraft platform. The aircraft platform needs to be capable of interfacing with both the old and the new avionics systems. The uncertainty affecting this aspect of the SoS design process is figuring out how the different lifecycles affect the operation of the system and how subsystems can be added, removed, or replaced.

The initial requirements are likely to be ambiguous.

As with many design projects, the requirements for the SoS and its related subsystems are very uncertain. While requirements may exist for some of the systems, requirements may not be explicitly defined for the overall SoS or vice versa. For instance, with many military aircraft there are specific requirements for its performance characteristics. But, the requirements for a battlespace SoS are much less specific because the battlespace environment and the involved systems are constantly changing.

Complexity is a major issue.

As mentioned previously, the more systems that are analyzed, the more complex the system will be. Not only may there be a significant number of systems that must be considered, but the complexity of their interfaces and relationships also becomes a major factor. All of the system interactions may not be known or understood fully and unanticipated emergent behaviors also make the problem more complicated. Emergent behaviors are the effects from interacting systems that could not be produced by or localized to an individual system. In other words this is the property where the whole system is greater than the sum of its parts.

Management can overshadow engineering.

In some cases the subsystems will have its own product or project office which results in requirements, budget constraints, schedules, and technology upgrades which are specific to the particular system. The effects from this management structure can affect additional systems in the SoS. An example of this is airports in the National Air Transportation System (NATS). Airports are independently owned and operated. Gate scheduling, luggage transportation systems, renovations, technology upgrades, etc. can cause delays or allow for faster aircraft turn times. The increase or decrease in turn times can propagate through the system affecting the aircraft and other airports in the system. This complex system can be difficult to predict and model making the interactions and the resulting effects uncertain.

Fuzzy boundaries cause confusion.

Often the external system boundaries and interfaces are not defined explicitly or controlled by the designer. This can be a significant source of uncertainty in trying to determine how systems should interact and how they should be operated.

SoS engineering is never finished.

A SoS is always changing either from different operational scenarios, changes in the environment, or changes relating to the life cycles of the systems. This results in constantly evolving requirements. For this reason it is not possible to optimize a SoS. While a SoS may be optimized for one scenario it will more inefficient and less effective than another system for a different scenario. In order to design a SoS for all of these changing requirements and scenarios it is important to focus on designing a robust system for a wide variety of potential scenarios and uncertainties rather than an "optimum" system for a particular scenario.

When reviewing these challenges it is apparent that understanding interactions and interfaces between systems is important to the development of a SoS, and it is also critical to consider all of the different uncertainties present in the problem. To highlight a few of the conclusions from the previous seven challenges, in SoS design there is uncertainty in:

- understanding how the different systems interact,
- understanding what emergent behaviors will result from the interactions of related systems,
- understanding how the systems should be operated,
- understanding how the actions of autonomous agents play into the scenarios, and
- defining current and future requirements.

Each of these bullets represents a different source of uncertainty for a SoS and its design problem. All of these types of uncertainty need to be accounted for in the design process in order to create the most robust and opportunistic design. The different types and sources of uncertainty will be discussed in Chapter 3.

1.3. DARPA Challenges

Other groups are also interested in design of a SoS. Many of DARPA's challenges for developing Command, Control, Communications, Computers, Intelligence, Surveillance and Reconnaissance (C4ISR) technology and determining how this technology should be operated are related to SoS or are concerned with system uncertainty. Four of the key challenges that DARPA identifies are: [184]

1. "Fog of War"
2. Seamless joint operations
3. SoS operations
4. Battlespace nonlinearity

The "Fog of War" deals with ambiguous and incomplete information or knowledge. Examples include unknown enemy doctrine as well as targets which have been concealed. Additionally, denial and deception also affect this category. The seamless joint operations relates to the operation and interaction of multiple systems. Examples include air space de-confliction, including unmanned aerial vehicles (UAVs) and loitering weapons or dynamic sharing of C4ISR assets rather than ownership of these assets. Joint operations suggests a SoS solution which leads into the next challenge, or SoS operations. SoS are not just challenging to design, but they are challenging to operate. For instance, it is difficult to provide seamless access to a common operating picture. Another example is that it is important to increase the lethality and survivability of a system from networked sensors and weapons. The fourth challenge is that battlespaces are nonlinear. For instance they require persistent, continuous sustainment, and they have to operate in urbanized and complex environments. [184]

From these challenges it is apparent that there are difficulties resulting from systems interacting in a SoS. It is also apparent that it is difficult to manage and operate systems in these battlespace environments due to the significant uncertainty, from both the fog of war and because of the nonlinearity of the problem. As a result, deployed systems should be robust and opportunistic for varied and complex terrains as well as sustained deployment.

1.4. Air Force Research Laboratory (AFRL) Challenges

The Air Force Research Laboratory (AFRL) has its own set of challenges relating to SoS. AFRL has developed a number of processes that serve to integrate and balance Science & Technology (S&T) investments, both near and far term. [28] The Focused Long Term Challenges (FLTC), one of these processes, has produced an integrated plan for achieving the AF S&T Vision of: Anticipate, Find, Fix, Track, Target, Engage, Assess, Anything, Anytime, Anywhere. [28]

The FLTCs include: anticipatory synchronized operations; **t**ailored, persistent, collection for predictive battlespace awareness; acquire & engage difficult targets; assured operations in high threat environments; integrated cyber/info effects; responsive adaptive theater operations; and affordable aerospace reliability and readiness. [28] These challenges indicate that the AFRL is interested in solutions that are affordable and reliable, and those that will assure the success of operations in uncertain scenarios. These challenges also emphasize SoS by indicating an interest in synchronized operations and adaptive theater operations.

Identifying challenges is only one step in solving the problem. The Air Force Research Laboratory Vehicles Directorate has identified capabilities to address the FLTCs including: cooperative airspace operations, multi-mission mobility, operational responsive space access, persistent intelligence, surveillance, and reconnaissance (ISR), precision persistent strike, prompt global strike, and long range strike. [196] These capabilities are illustrated in Figure 1-1. While there is a need for all of these capabilities, persistent strike is becoming more and more important and poses a highly complex and uncertain design problem.

Figure 1-1: Capabilities for the FLTCs as defined by the AFRL Vehicles Directorate [196]

1.5. Persistent Strike Capability

This capability is defined as the persistent delivery of lethal and non-lethal precision effects. [46] Persistent strike techniques are used to engage time critical targets such as rapidly moving or 'emerging' targets. There is a limited window of opportunity for striking these targets before they disappear or move to an area that is off limits for a strike. [113] Characteristics of a system with a persistent strike capability include a long duration mission, rapid deployment, survivability, interoperability, affordability, and the ability to produce lethal / non-lethal precision effects. [46]

In addition to AFRL, RAND has a program to design a future long-range/persistent strike capability for the United States Air Force (USAF) where they are looking for the key aspects of persistence and survivability. [152] But, the Air Force is not the only organization interested in this capability. The Navy has also indicated a need for a persistent stealthy reconnaissance platform with the ability to strike. [194] Persistent strike was described as being a capability of a global patrol and homeland defense fleet for the United States in the *The Quadrennial Defense Review: Rethinking the US Military Posture* released by the Center for Strategic and Budgetary Assessments in 2005. [116]

A number of concepts have been suggested for this type of mission from systems with a C-5 class air transport to an unmanned search and strike aircraft to a combination of these two concepts. [178,142] Many of the characteristics (long duration flight, affordability) seem to point designers and decision makers towards a search and strike, or hunter-killer, UAV or Unmanned Aircraft System (UAS). The term UAS is becoming the preferred term for a UAV because it emphasizes that the platforms would be connected to other elements for a SoS solution. [193] Additionally, this type of mission would be either "dull" and /or "dangerous" which would make it ideal for a UAS. [142]

To a certain extent this type of mission is already being done by UASs today. The Predator MQ-1 is known for being a reconnaissance platform while being capable of destroying a couple of ground targets either by itself or in collaboration with other systems. [194] The upgraded version of the Predator is the Reaper MQ-9. The Predator is viewed as a intelligence-surveillance-reconnaissance (ISR) platform and has been called a "killer scout". The Reaper is considered a hunter-killer and due to its lethality is in a similar category as the USAF's other attack aircraft. [194] For instance, the Reaper is in many ways comparable to an F-16. It operates at a similar altitude, has similar sensors, and a comparable weapons load. The Reaper has a significantly greater endurance time (approximately 18-24 hours) than an F-16, but it cannot compare to the F-16 in terms of speed and agility. [194]

These capabilities are only the beginning for persistent strike aircraft. Greater capabilities for remaining on station longer, autonomous flight and operations, and interacting with other UASs are quickly becoming mission requirements. [94] Communication is already a key element for unmanned systems and will only become more and more critical as more unmanned aircraft systems infiltrate the battlespace. One existing issue is that "it has not been figured out yet how to safely deconflict what could be hundreds or even thousands of UASs buzzing around a battlefield." [193] Additionally, stealth is an important capability that both the Predator and the Reaper lack. However, General Atomics, the manufacturer of the Predator and the Reaper, is working on the Predator C. This new aircraft will have all of the capabilities of the Reaper but will also have a new low observable airframe and stealthy qualities. [194]

While Reapers or a future Predator variant might appear to be the default answer to a persistent strike mission, detailed studies have not proven that this is the only option or even the best option. Perhaps a new UAS concept or an existing fighter would be the most robust and opportunistic solution to the problem. Even in Operation Iraqi Freedom (OIF), while the Predator and the Global Hawk received the most publicity there were other UAV platforms which were used. It has been shown that less expensive UASs with more limited capabilities can also provide a strategic advantage. [158] Or, perhaps a new fighter, which has both a manned and unmanned version, should be used in this type of mission. [194]

It is important not to focus on the problem from a vehicle perspective. This will limit the design by only focusing on specific aircraft concepts instead of system concepts. It is critical to look at the problem/solution from a system-of-systems perspective. An aircraft is not operated in isolation. It is part of a complex system which must be integrated to meet the overall system objective. While the conclusion of the design process may be that a new aircraft should be manufactured to meet a specific need, this new aircraft will be more capable of meeting the needs of a robust and opportunistic SoS.

An example of how systems were used to find time critical targets is with locating and striking terrorist leaders in OIF. These targets were extremely challenging because of limited windows of opportunity that in some cases only appear very infrequently. For a strike to occur the targeting data was provided from a combined system of ground-based Special Forces teams and UASs like the RQ-1A Predator and Gnat. [113] Additional data was gathered by reconnaissance satellites and U-2 aircraft and relayed through satellite data links to analysts in the US. The data would be evaluated and air strikes would be called in if appropriate. This is just one example of how a wide variety of systems including satellites, UASs, and ground-based troops maybe used in one scenario. [113]

1.6. Uncertainty in Persistent Strike Mission and System

Both the design of system-of-systems and the operation of these systems involve a significant amount of uncertainty. The persistent strike mission is an excellent example of a mission where this is apparent. From the perspective of operations, there are a significant amount of uncertainties including those pertaining to the environment, how these systems should be operated, how these systems will adapt if part of the system becomes inoperable. There will be uncertainty with different environmental conditions such as weather as well as the nature of the threat. The number and types of targets are often uncertain, as are their locations. In many cases the system may not know how hostile its operating region is. Because of the often modular and interchangeable nature of large scale system-of-systems it may not be certain how best to manage them from an operational perspective. For instance, if multiple aircraft are available for reconnaissance and a possible strike, which aircraft should be used or which sets of aircraft should be used?

An additional consideration is the nonlinearity and noncontiguousness of the battlespace. More and more often, forces are dispersed to many locations. Modern warfare is no longer described by traditional lines, broad fronts, and rear areas. The concept of the wide-open battlefield began integrating itself into Army doctrine back in the 1990s. [83]

The emergence of this type of battlespace is a way of operating in engagement areas containing short-range missiles as well as weapons of mass destruction. Additionally this noncontiguous battlespace is critical when engaging an enemy where there are "pockets of friendly and hostile forces" spread throughout the area. [83] This is an extremely uncertain operating field, resulting in a need for a robust and opportunistic system-of-systems.

1.7. Observations, Gaps and Technical Challenges

1.7.1. Observation / Technical Gap 1

Existing SoS Design Methods are incapable of modeling all of the different types of relevant uncertainty

Because of the scale and complexity inherent to most SoS, there are a wide range of types and sources of uncertainty associated with operation of the SoS. [188] This uncertainty is even more significant in the conceptual design process of the SoS. There are a number of existing design methods for SoS, but none of these methods are capable of addressing all of the different types and sources of uncertainty.[71,179,148] In particular these methods lack the capability to model all of the types of uncertainty. [18] This gap presents an interesting technical challenge, the challenge of developing a technique to model all of the different types of uncertainty in a conceptual design method.

1.7.2. Observation / Technical Gap 2

Existing SoS Design Methods do not specifically address the fact that there can be propitious effects from uncertainty as well as pernicious.

There are two sides to uncertainty. Uncertainty can cause either positive or negative effects on the system.[25] Existing design methods typically focus on developing robust designs that are insensitive to the associated uncertainty. However, these methods do not capitalize on the possibility of maximizing the potential benefits associated with the uncertainty. Based on this concept, there is an existing technical challenge to incorporating both the propitious and pernicious nature of uncertainty in the design process.

1.7.3. Observation / Technical Gap 3

Existing SoS Design Methods focus on identifying the most effective design alternative with respect to the relevant uncertainty. However, none of these methods focus on determining if the uncertainty should be reduced before making the final design decision.

There are two primary types of uncertainty: aleatory and epistemic uncertainty. Aleatory is associated with randomness and cannot be reduced, but epistemic uncertainty is related to a lack of knowledge and can be reduced. [140] Most of the uncertainty in a SoS conceptual design process is epistemic uncertainty and therefore can be reduced.

SoS are often very complex systems consisting of expensive subsystems. When designing a SoS, because of the scale and cost, it is likely to be advantageous to consider reducing uncertainty in a design problem before making a final decision. The main technical challenge for this aspect of the problem is developing a technique that can estimate the value of reducing uncertainty with the available information and processes.

The primary purpose of this research is to address these technical challenges associated with a SoS conceptual design method.

1.8. Research Overview

This document is organized into 13 chapters. Chapters 2 through 4 focus on providing background material necessary for understanding the problem. In this section there is first an extensive literature review discussing what constitutes a system-of-systems and what definitions will be used throughout this research project. The following chapter will provide a general description of uncertainty and more specifically, what the sources of uncertainty are for a SoS. The final chapter in this part discusses the methods and techniques used for modeling uncertainty.

The purpose of Chapter 5 is to benchmark how it has been done in the past. This chapter describes a representative set of methods and identifies the capability gaps associated with them.

Chapter 6 presents an overview of the technical gaps, technical challenges, and the research questions identified in previous chapters. This chapter also presents a number of hypotheses for answering the identified research questions.

Chapters 7 through 9 focus on verifying the hypotheses proposed in Chapter 6. The first chapter in this section presents a process for a Hybrid Uncertainty Modeling Method (HUMM). Chapter 8 provides a discussion of the differences between three different design approaches: Robust Design, Opportunistic Design, and Robust and Opportunistic (RandO) Design. Chapter 9 presents a method called the Value of Reducing Uncertainty Method (VRUM) and a technique called the Expected Value of Reducing Uncertainty (EVRU).

Chapter 10 provides an overview of a conceptual design method for a SoS called CONceptual Design of Opportunistic and Robust System-of-Systems (CONDOR-SS). This method was developed to fill the gaps discussed in Chapter 6.

The purpose of Chapter 11 is to demonstrate all of the concepts in the design method for a SoS. The historical persistent strike scenario of the scud hunt in Operation Desert Storm serves as the base of the example problem. This problem was selected for three reasons: first, because the associated uncertainty made it an excellent test bed for these techniques; second, because similar persistent strike problems are still of interest today; and third, because the scenario, systems, tactics, and outcomes are well documented and in the public domain.

Once all of the concepts have been developed in Chapters 2 through 11, it is necessary to return to the original technical challenges and research questions to determine if the questions have been satisfactorily answered. For this reason, Chapter 12 provides a review discussing the technical challenges, research questions, and hypotheses.

The final chapter summarizes the main conclusions from this research and discusses potential areas for future work.

CHAPTER 2: SYSTEM-OF-SYSTEMS OVERVIEW

While there is a general understanding of what a system-of-systems means, there is no consensus on a specific definition for the term. There is also a fair amount of ambiguity involving the differences between systems, SoS, and a Family-of-Systems (FoS). This chapter provides an overview of the literature involving these terms and describes the definitions which will be used in this research.

2.1. Definition of a System

Before one can consider the purpose or use of a System-of-Systems (SoS), it is important to clarify what is meant by a system. Because of its generality practically every discipline has its own definition for the word system. The word itself originates from the Greek word "systema", which means organized as a whole. [18]. As discussed in Chapter 1, the INCOSE Systems Engineering Handbook defines a system as being a collection of interacting components or elements structured so as to attain a set goal. A similar definition is posed by Ayyub and Klir in Reference 18 . This definition states that, "a system can be defined as a group of interacting, interrelated, or interdependent elements that together form a complex whole that can be a complex physical system, process, or procedure of some attributes of interest. All parts of a system are related to the same overall process, procedure, or structure, yet they are different from one another and often perform completely different functions."

Several sources have defined systems in terms of mathematical equations. Ayyub and Klir assert that a system exists if it can be stated in the form $S = (T,R)$ where T is a set of things and R is a set of relations defined on T. [18] From this definition it is apparent that the critical aspect of systems is the relationships or interactions between the components.

[18] Wymore in Reference 202 provides a mathematical definition of a system that relates the possible states of the system, the possible input states or conditions for the system, the possible input functions for the system, the set of all modes of behavior available to the system, the period of time over which the system exists, and the dynamical behavior of the system.

Not all definitions for a system emphasize the relationships and interactions between components as an important aspect. Reference 172 describes a system as being a combination of matter, parts, and/or components which are within a specified boundary. From the point of view of this research, this definition is not as useful as those provided by INCOSE and Reference 79. If there are no interactions between the different elements, then there is no need to consider the various components in the design process. A much more efficient use of resources would be to focus upon only the important interrelated elements.

Systems according to Reference 18 and 29 are traditionally grouped in various overlapping categories including: natural systems, such as river systems or the human body; man-made systems that can be embedded in natural systems like hydroelectric power systems; or static systems that are without any activity such as bridges under dead loads. Two additional categories include "physical systems that are made of real components occupying space" such as airplanes or buildings, or closed or open loop systems such as with the chemical equilibrium process.

This research focuses on the design of systems under the category of "physical systems that are made of real components occupying space". Although in many cases the systems will also fall under the category of "man-made systems that can be embedded in natural systems". Modern day systems can no longer be designed in isolation of the surrounding natural environment. In the battlespace example, systems are required to operate in a wide variety of weather situations and climates. Precipitation, sand, large temperature gradients, and other elements must be considered when designing systems for a military operation.

2.1.1. Simple, Complicated, and Complex Systems

Systems are often classified as simple, complicated, or complex. Amaral and Ottino in Reference 8 defined a simple system as those that are composed of a small number of elements that operate according to well understood rules. These systems can typically be described with closed-form analytical expressions. An example of a simple system of one main component is a pendulum, but as noted in Reference 8 even this extremely basic system can generate complex dynamics. They also describe a complicated system as one that has a large number of components with well-defined roles and which act on well understood laws. [8]

Often there is a fair amount of confusion between the differences between complicated and complex systems. Unsurprisingly, there is no consensus on what constitutes a complex system. This is easily apparent from the special issue of "*Science"* magazine in 1999 on complex systems where there were five different articles on complex systems each with their own definition. Goldenfeld and Kadanoff said that a complex system is a highly structured system with variations. [82] Whitesides and Ismagilov asserted that, "A complex system is one whose evolution is very sensitive to initial conditions or to small perturbations, one in which the number of independent interacting components is large, or one in which there are multiple pathways by which the system can evolve." [200] A third article by Weng, Bhalla and Iyengar said that this type of system is one that is difficult to understand and difficult to verify. [199] Rind in Reference 154 claims that a complex system has multiple interactions between many separate components. [154] The fifth author, Arthur, said that, "complex systems are systems in process that constantly evolve and unfold over time." [11] While each of these definitions has merit for various applications, it is important to use a general definition that can be applied universally.

From reviewing the literature it is apparent that there are a wide variety of definitions for complex systems; however, much of the literature describes a complex system as one demonstrating emergent behaviors. [16, 95] Emergent behavior can be characterized by the behavior of the system not being able to be inferred from the behavior of its components or parts. [23,37] With this in mind, a complex system can be thought of as a system that is "made up of a large number of parts that interact in a nonsimple way. [134] In such systems the whole is more than the sum of the parts, not in an ultimate, metaphysical sense but in the important pragmatic sense, that, given the properties of the parts and the laws of interaction, it is not a trivial matter to infer the properties of the whole."

Complex systems are also often characterized as demonstrating self-organization. [18, 8, 156, 53] As examples, Ayyub and Klir use the self-organizing behaviors of ant colonies, human brains, and economic markets to illustrate the extremely complex behavior that is significantly greater than the behavior of individual elements in these systems. [18] These examples illustrate the link between self-organization and emergent behaviors of systems. Emergent behavior in complex systems is an important aspect to why it is critical to design a system from a systems or holistic perspective. If the parts are considered independently from the whole it is possible that significant system effects or behaviors will not be considered. [103, 4] This concept can be expanded to one of the reasons why it is also important to consider the design of systems from a SoS approach.

2.2. Definition of a System-of-Systems

2.2.1. Literature Review

Like the term complex systems, system-of-systems is a common term which is well known throughout the system engineering field and is assumed to be understood by most people. However, despite the commonality and propagation of the term there is no one definition that has been agreed upon. [121] This is most likely due to the simplicity of the terms in the phrase. The definition of a system is well understood and it is often assumed that a SoS is simply as it sounds, a system of systems. A survey conducted by the US Air Force in 2005 found that a majority (75%) of the people surveyed believed a SoS to be a large system comprised of many subsystems. The survey found that 20% defined a SoS to be a system of "cooperating specially built systems", and 5% of the surveyed people determined that a SoS is a group of dynamically interacting responsive systems. [64, 197] From the definitions discussed in the previous section, it is straight forward to assume that a SoS is then a combination of interacting systems organized to achieve one or more stated purposes, but the literature shows that a SoS has evolved into a variety of meanings. For instance, some sources claim that SoS are interacting collections of component systems which produce results unachievable by the individual subsystems[1] alone. [79,90] This definition is very similar to that of a complex system and implies that a SoS is a complex system. Others sources state that a SoS is composed of systems with operational independence and managerial independence. [121] Some also emphasize hierarchy and emergence within the overall system and others emphasize a widely distributed overall system. [5, 173] For instance, Kotov claims that, "Systems of systems are large scale concurrent and distributed systems that are comprised of complex systems." [114] Another definition often seen in the literature which combines these

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[1] A subsystem is a set of elements that form a system and belong to a higher level system.

characteristics says that, “Systems of systems exist when there is a presence of a majority of the following five characteristics: operational and managerial independence, geographic distribution, emergent behavior, and evolutionary development.” [163]

Boardman and Sauser in differentiating between a system and a SoS felt that there were five distinguishing characteristics: autonomy, belonging, connectivity, diversity, and emergence. [30] They said that in a SoS, component systems maintain a certain amount of autonomy in order to achieve the purpose of the overall system, while in a system the subsystems give up all autonomy. The subsystems in a system according to their definition did not “choose” to be part of the system, but rather belonging to the system is an inherent characteristic of these systems. Conversely in a SoS the systems “choose” to become part of the overall system. They also said that within the subsystems of a system the connectivity is often “hidden” within the subsystem and that connections are limited. SoS, on the other hand, are not limited by such design requirements. Boardman and Sauser also talk about the battle between the law of requisite variety and the principle of parsimony. The law of requisite variety says that, “the larger the variety of actions available to a control system, the larger the variety of perturbations it is able to compensate.” [92,12] However, the principle of parsimony, more commonly referred to as Ockham’s razor, emphasizes that additional complexity should not be added unless required. [135] Boardman and Sauser assert that SoS should be diverse as opposed to systems which should be “reduced or minimized by modular hierarchy.” Finally, they claim that the emergent behavior in a complex system is designed into it, while the emergent behavior for a SoS is not restricted to the designed or anticipated emergent behavior. It is understood that the level of complexity and uncertainty is such that for a SoS it may not be possible to account for all capabilities or effects. [30]

Often SoS are associated with military applications. For instance Pei states, “System-of-Systems Integration is a method to pursue development, integration, interoperability, and optimization of systems to enhance performance in future battlefield scenarios.” [146]

Manthorpe also wrote that, “In relation to joint warfighting, System-of-Systems is concerned with interoperability and synergism of Command, Control, Computers, Communications, and Information (C4I) and Intelligence, Surveillance, and Reconnaissance (ISR) Systems.” [122]

However, the scope of SoS does not end with military systems. Due to the generality of the term there are applications ranging from the internet to manufacturing product lines. Carlock and Fenton focused upon private enterprise applications for SoS. They claimed that, “Enterprise Systems of Systems Engineering is focused on coupling traditional systems engineering activities with enterprise activities of strategic planning and investment analysis.” [38]

It is apparent that there is no consensus on the definition for a SoS, but that many of the definitions focus on similar themes such as combinations of interacting systems, system autonomy, emergent behavior, etc. The problem with most of the proposed definitions is that they either fall into one of two classes of definitions. The first class of definition is broad and general such as the definition built from the definition of a system. By this definition almost any system composed of complex elements would fit the description. The second class of definitions are much more particular and limit which combinations of systems can be considered a SoS, such as the definitions provided by Sage and Cuppan or the characteristics proposed by Boardman and Sauser.

Considering the different classifications and characteristics proposed in the literature, it is possible to develop a combined general definition for a system-of-systems.

2.2.2. Definition

After reviewing the literature and examples of these systems, it is apparent that there is a hierarchy of SoS levels. The number of levels varies upon how complicated the system is, but there are always at least 2 levels: the Operational Environment and Scenario (OES) and the Base Level. But in many cases there are three categories of levels. The top level is the Operational Environment and Scenario (OES) Level, and the intermediate levels are the subsystems that compose the OES level. There can be (and often are) numerous intermediate levels. These intermediate levels are designated by letters or some other index as appropriate. Literature sources have used a variety of types of indexes to designate the different levels, but for this research the Latin alphabet will be used. The Component Level is the lowest level of the hierarchy and consists of the specific components that make up a system. The Component Level may or may not be the Base Level which consists of the systems or components that compose the OES or intermediate levels. The Base Level is the lowest level that the designer will consider.

It is possible for there to be more than three levels. One example is the National Air Transportation System. The OES Level consists of the airline fleets, airports, air traffic control centers. Intermediate Level A involves the individual aircraft, as well as the individual airports, Air Traffic Control Towers, En Route Centers, and Flight Service Stations that are involved with the specific aircraft throughout its operations. Intermediate Level B for each individual aircraft involves the aircraft systems including the propulsion system, avionics systems, landing gear, environmental systems, etc. For each of these subsystem groups there will be an Intermediate Level C. For the propulsion system this level consists of the different engines. For many aircraft such as the Boeing 737 or 777, there are two engines per aircraft. In cases of the McDonnell Douglas MD-11 there are three engines, and there are fours engines on the Boeing 747. For each engine in Level C, there is an Intermediate Level D which usually consists of the inlet, fan, compressors, combustor, turbines and nozzle. [124] Level E for the compressors consists of the compressor blades. Level E is also the Base Level because the designer for this example will not consider the design beyond the level of the design of the compressor blades.

As another example, consider a battlespace operation for a persistent strike scenario. One option for the OES Level involves reconnaissance satellites, refueling air tankers, hunter/killer UASs, data link systems, remote pilots, and sensor operators. Level A relates to the systems supporting the actions of one UAS such as the aircraft, the data link system, and the remote pilot. Level B consists of the aircraft systems including systems such as propulsion, weapons systems, or sensor systems. For each of these systems there would be a Level C; for the sensors systems the subsystems could be synthetic aperture radar system, infrared sensors, and high definition video cameras. For this example, Level C is the Base Level in regards to sensor systems, because in this case the designer will not consider the design of the sensor packages. However, Level C is not the Base Level for all of the systems. Level C for the propulsion system, assuming there is only one engine, is the different systems and components of the engine such as the inlet, compressor, combustor, turbine, and nozzle. Level D for the combustor would include systems and components such as the inlet diffuser, fuel injector system, igniter, air swirler, and case. [124] For the combustor system, Level D, is the Base Level.

The Base Level may also be the Component Level. This is the level where the components, which cannot further be broken down into subcomponents, are designed. It is important to note that the Base Level for a designer is not always the Component Level. This was illustrated in the battlespace operation example when the designer did not consider the component design of the sensor packages. The Base Level is just a designation for the lowest level of the system that the designer will consider.

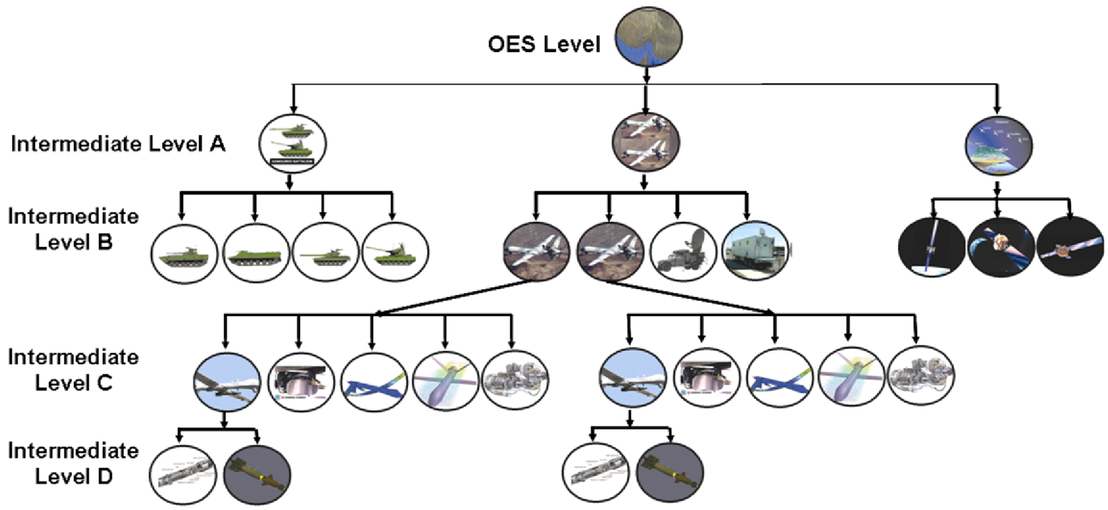

Figure 2-1: SoS Hierarchy for Notional Persistent Strike Battlespace

It is important to break the SoS into different levels because each level has specific characteristics. The Base Level, which can be composed of any of the other levels, will have characteristics of the corresponding type of level. The Component Level is concerned with the simple interactions between the components of the system. This level may be either a simple or a complicated system as long as it is only composed of parts or components. In other words this level cannot be considered a SoS by itself. Intermediate levels, on the other hand are composed of systems and components, and can be considered a SoS. These levels can be composed of simple, complicated, or complex systems and may produce emergent behaviors. The OES Level is also composed of simple, complicated, and complex systems, and generally, some or all of the systems at this level are autonomous. This means that these systems have both operational and managerial independence. [121] These systems are also likely to demonstrate self-organization and result in emergent behaviors. [18]

It is also critical to consider the SoS at each of the different levels because the design of each of the levels builds upon the previous level. The OES Level cannot exist without its subsystems and their respective interactions. But, there may not be a need for the subsystems without the overall objective at the OES Level. Additionally, there are different types of uncertainties that are prevalent in each level of the system. These uncertainties, which will be discussed in Chapter 3, propagate through the system and can significantly affect the system objectives.

Considering this hierarchy and from understanding how the different levels are related the definition for a system-of-systems for this research is considered to be:

A hierarchical set of systems, associated with different operational levels, where each level consists of interacting collections of component systems that produce results unachievable by the individual subsystems.

This definition is very general and emphasizes three important characteristics which are necessary for the design of such a system. First a SoS is hierarchical, and second the systems must interact. It is not enough to simply classify the systems within the same boundary. By recognizing and defining the hierarchy and the relationships between the various elements the designer will be better able to produce and predict the effects of the overall system. Finally if there is no objective or resulting effect for the system, then there is no need to consider the interactions.

2.2.3. Family-of-Systems

There is some confusion over the differences between System-of-Systems and a Family-of-Systems (FoS). This research considers a FoS to be a specific type of SoS. A FoS is a collection of legacy systems which have each been designed for a specific purpose or mission in isolation of the other systems. [129] For this type of system there is an emphasis on the capabilities of the operation of the independent system and on the interoperation of all of the systems. Many existing SoS are FoS. A FoS designer will typically not have control over the design of specific system design parameters. Instead their purpose will be to develop the capabilities of the system based upon the parameters that they can control or select. [59] In the case of a FoS, the Base Level will be the lowest level of detail that the designer can control.

CHAPTER 3: UNCERTAINTY OVERVIEW

Almost every book relating to uncertainty will have a different definition for what constitutes uncertainty and what its sources are. [118,134] In most cases this does not mean that any of these definitions are incorrect, rather that there are many different types of uncertainty and it can be handled in a wide variety of ways. [134] It is important for the designer to identify and determine which types of uncertainty are applicable to their particular problem in order to appropriately quantify the effects of the uncertainty.

The objective of this chapter is to define uncertainty and discuss what sources of uncertainty are present in system-of-systems. The first section of this chapter defines uncertainty by discussing the relationship between knowledge and ignorance, and also discusses how risk and uncertainty are interconnected. Later sections of the chapter provide a taxonomy of uncertainty and indicate how uncertainty affects SoS.

3.1. Knowledge and Ignorance

3.1.1. Knowledge

Knowledge and ignorance are important factors in understanding the concept of uncertainty. Ayyub and Klir in Reference 18 discuss how for engineering and the sciences, knowledge can be defined as a collection of "justified true beliefs" including laws, theories, concepts, and principles. The difference between knowledge and information is that knowledge involves the recognition and understanding of patterns in the information. [24]

Ayyub and Klir divide knowledge into four categories: Episteme, Dianoi, Pistis, and Eikasia. Episteme is cognitive knowledge and is object knowledge and know-how. Know-how knowledge is the knowledge that is required for a person to do a certain activity, procedure, function, etc. [18] Knowing how to fly an airplane or ride a bicycle are examples of this type of knowledge. Object knowledge is from a direct relationship with a person, place, or thing. For instance, a person may know what roads to take to get to the airport from their home. Or, one person may know another person. Dianoi is the type of knowledge relating to mathematics and logic. It is defined as correct reasoning from hypotheses. [18]

The next two kinds of knowledge are propositional knowledge, which mean that they are based on propositions, and it is possible for this knowledge to be true or false. This kind of knowledge is affected by appearances and deception. [18] In the persistent strike SoS example, it may be believed due to intelligence reports that a particular area in the battlefield is clear of targets. This may or may not be true. Pistis is based on belief and pertains to intellectual or emotional acceptance of a proposition. Eikasia is also propositional knowledge, but it is based on conjecture. Eikasia is from prediction or inference which may be from unreliable or incomplete information. Expert judgment will enter into the design process with the Pistis and Eikasia types of knowledge. [18]

3.1.2. Ignorance

Ignorance is defined as a lack of knowledge. The state of being ignorant can be intentional or deliberate, such as when information is purposely ignored or when the information is not obtained or considered due to lack of resources. [18,62] Often uncertainty is ignored in design processes due to limited resources or because of limited exposure to the uncertainty. This is considered conscious ignorance, while unintentional ignorance is blind ignorance. [18]

Ayyub and Klir have identified three types of ignorance: know-how ignorance, object ignorance, and propositional ignorance. In all three of these cases, this type of ignorance relates to either a lack of knowledge or to having erroneous knowledge pertaining to either know-how, object, or propositional knowledge. [18]

3.2. Definition of Uncertainty

The general definition of uncertainty is the condition of being uncertain, meaning indefinite or indeterminate. A more specific definition relating to statistics is that it is the estimated amount or percentage that a determined value may differ from the actual value. [190] Ayyub and Klir in Reference 18 define uncertainty within engineering analysis and design as being incompleteness in knowledge due to deficiencies in knowledge or information. [18]

Examples of uncertainty in design include material properties, program costs, operational costs, vehicle costs, factors related to the operational environment, etc. Specifically for SoS, an example of uncertainty relates to understanding how the various systems interact. For instance with the battlespace example, there is limited knowledge pertaining to how the different aircraft can be used to search and remove targets from a specified coverage area. How will the stealth characteristics of an aircraft play into the scenario? What will result in the most effective and robust scenario for a wide variety of targets and environmental factors such as bad weather or poor visibility terrains?

In order to model uncertainty it is critical to break uncertainty down into types, sources, and factors but first it is important to understand when it is necessary to consider uncertainty in a design problem. This leads to the concept of risk versus uncertainty.

3.3. Risk versus Uncertainty

In general, risk is the possibility of injury or loss and is usually related to a consequence. [61] In the literature, there are also a variety of interpretations to the meaning of risk.

Knight in Reference 112 discusses how risk traditionally is a term used to refer to any sort of uncertainty where there might be a loss. He further discusses how uncertainty is used to discuss the lack of knowledge where there is a possibility of a gain or favorable outcome. [112] Since these terms are vague, he then specifies that the term "risk" is measurable uncertainty while unmeasurable uncertainty should be called "uncertainty". Knight used the example of roulette or dice games where a player could calculate an exact probability or risk, but if there is not an exact probability, such as the odds of your house burning down, then the estimate of probability of this event occurring is the uncertainty pertaining to the event. [112]

Holtan in Reference 96 said that risk is related to probabilities that are real (objective probabilities) and uncertainty is related to subjective probabilities, which are those that are from human beliefs. [96] An example of a subjective probability is an expert saying that based upon intelligence reports, there is a 90% chance that a target will be located in this particular region of the battlefield.

Holtan also provides a different definition where he says that risk includes two essential components, exposure and uncertainty. Exposure relates to if a person cares about the outcome of the event, and uncertainty is when the person does not know what will happen. Risk requires both elements and is the exposure to an uncertainty proposition. An example is a man jumping out of an aircraft without a parachute. There is no risk because while he is exposed, there is no uncertainty. The outcome of this event is well understood. [96] An example relative to the battlespace example is in the personal physical safety of a remote pilot of a UAV. While the UAV is operating in a highly uncertain and dangerous environment, the pilot is not exposed to the danger and therefore there is no risk to the pilot's physical safety.

Holtan also discusses how there are flaws in the definition of risk relating to the subjectivity of these terms. This is why it is impossible to operationally define risk. The author suggests that it is possible to identify the perception of risk but not the risk itself.

The "risk versus uncertainty" debate is long-running and far from resolved at present. [97] For the purposes of this research, the definition where risk implies consequence will be used because this is the most common definition used in design. However, in design it is also important to consider exposure otherwise the results would be indifferent to the uncertainty and risk. These three concepts can be combined to identify a region of concern and a region of indifference.

The UAV pilot's physical safety would fall into the region of indifference. An example that would fall into the region of concern is if a UAV is shot down, how would the other aircraft and system elements be able to adapt to manage the original UAV's coverage area?

Risk, uncertainty, and exposure are important when considering the impact of uncertainty. All of these elements are critical in quantifying the effect of the uncertainty and the cost of additional knowledge to reduce the uncertainty. Before the uncertainty can be quantified we must first better understand how uncertainty is classified as well as the types and sources of uncertainty.

3.4. Taxonomy of Uncertainty

Uncertainty can be classified in several ways, either from the types of uncertainty, the sources of uncertainty, or factors relating to the specific type of ignorance that the uncertainty falls under.

3.4.1. Types of Uncertainty

3.4.1.1. Reducible and Irreducible Uncertainty

There are several different ways to classify uncertainty types in the literature. The first method of classifying uncertainty is by identifying whether it is reducible or irreducible.

With irreducible uncertainty it is inherent to the system or problem and there is no way that additional information can reduce the uncertainty. It can only be quantified in statistical sense. [44, 102, 58]

An example of this type of uncertainty is material properties that will be used in the structure of an aircraft. While aluminum alloys have average tensile strengths, the specific piece of alloy used in the structure may not actual have this exact strength. It may vary by a small percentage, and without testing the individual piece it is not possible to exactly know its properties.

Reducible uncertainty on the other hand can be reduced by investing additional resources to run tests or gain additional knowledge. For instance, in the battlespace scenario, the location or capabilities of targets are unknown. However with additional surveillance or other forms of intelligence gathering it is possible to reduce the uncertainty pertaining to their locations or capabilities. This form of uncertainty can also be lowered by improvements in measurements or by creating a more detailed model. [44]

Isukapalli, et al. in Reference 102 claimed that this type of uncertainty is related to two sources. This first source is model parameter uncertainty. This is where there is incomplete knowledge of model parameters/inputs either from insufficient or inaccurate data. The source of this uncertainty can also be called "data uncertainty". The other source of reducible uncertainty according to the authors is model structural uncertainty which is from approximations and simplifications in the model formation. This is also known as "model uncertainty".

Der Kiuregian claimed that model uncertainty is from estimation error which is related to the incompleteness of sampling information and the designer's inability to accurately estimate the parameters of the probabilistic models that describe inherent system variability. [58] This is similar to Isukapallli et al's parameter uncertainty. Der Kiurgian also claims that another source is imperfections in both probabilistic and mechanical models. This is related to ignorance of physical phenomena and errors of simplification from the use of simplified structural and probabilistic models. The errors in the probabilistic models are from errors in the selection of a parameterized probability distribution. This is similar to Isukapalli et al's model structural uncertainty. Der Kiuregen considered human error to fall into this type of uncertainty. [58]

3.4.1.2. Uncertainty and Variability

Some authors such as Rowe classify uncertainty as either "uncertainty" or "variability". [161] Uncertainty is the absence of information that may or may not be obtainable and is related to the reducible uncertainty. Variability is more related to the irreducible type of uncertainty.

The main sources of variability are underlying variants which are inherent in natural systems and include: underlying variants, collective/individual membership assignment, and value diversity. Underlying variants are inherent in natural systems and are due to the randomness of nature. Inconsistent human behavior and nonlinear dynamic behavior also fall into this category. [161] The collective/individual membership assignment type of variability is from distinctions between individuals and collectives. The example provided by Rowe is that data can be obtained about some parameters with a significant amount of precision, such as the average weight of people in the United States. However, this information does not provide information about the weight of a particular individual. [161] Value diversity relates to variability through the different value systems and varying perspectives. This type of variability can relate to knowledge from subject matter experts.

3.4.1.3. Aleatory Uncertainty, Epistemic Uncertainty, and Error

The most common way of expressing the different types of uncertainty is with aleatory and epistemic uncertainty and error. [140, 136, 18] As discussed in the following sections, these terms are closely related to the concept of irreducible and reducible uncertainty.

Aleatory Uncertainty

The term aleatory means that it is something which is dependent on chance or an uncertain outcome. It is derived from the latin word āleātōrius, from āleātor, which means gambler or from ālea which was a game of chance. [192] In literature aleatory uncertainty relates to the random or irreducible uncertainty. [171, 44] It is also often considered the variability of the problem, as well as the inherent or the stochastic uncertainty. [140] Oberkampf et al. in Reference 138 further describes this as the inherent variation associated with the related environment or the physical system. It is usually represented probabilistically by a random variable. [18, 140] Often sources of this type of uncertainty can be identified from other factors contributing to the uncertainty by representing these factors as randomly distributed quantities within an established range. [140]

Epistemic Uncertainty

Epistemic is defined by the *American Heritage Dictionary* as relating to knowledge. It is from the greek words "*επιστήμη or episteme*" which mean knowledge or science. [190] In literature the epistemic uncertainty is related to the level of ignorance of the system or the related environment. [140] Oberkampf et al. defines it is as “any lack of knowledge or information in any phase or activity of the modeling process”. [138, 140]

Examples of this type of uncertainty include a lack of data for a physical parameter, limited understanding of a process or function, or unmodeled environmental conditions.

[140] In the persistent strike battlespace problem an example of epistemic uncertainty is the lack of knowledge pertaining to the actual performance capabilities of the involved aircraft. Another example is the lack of knowledge involving the location, capabilities, and numbers of enemy threats and targets.

This type of uncertainty is reducible and is traditionally represented probabilistically and modeled as a random variable. [44,18,140] When there is limited data the common practice is to model the likelihood of possible values occurring with a familiar probability distribution. [140] Since probability distributions are typically used when modeling aleatory uncertainty, this tends to cause some confusion as to the differences between the two types of uncertainty. [143]

While there are some cases when it is acceptable to model epistemic uncertainty using probability distributions, there are also several weaknesses to this technique. In many cases designers care about events or values that will rarely occur. This means that the tails of the probability distributions are of particular interest. If distributions such as Weibull or Normal probability distributions are selected, it is extremely difficult to properly estimate the tails. [140] Oberkampf et al. in Reference 140 point out how treating epistemic uncertainty as though it is aleatory uncertainty can result in strongly compounded discrepancies. Chapter 4 discusses several methods for modeling the different types of uncertainty.

Error

Error is generally defined as a deviation from correctness or accuracy, and error can also be considered a form of uncertainty because the true value is unknown. [60] However error is not from lack of knowledge. A more specific definition from the literature states that error is the "recognizable deficiency in any phase or activity of modeling and simulation that is not due to lack of knowledge." [140]

The defining characteristics of error include that it can be reduced or increased by the designer, that error distributions are usually not well known, and that it usually involves minor changes in values. [127] Typically uncertainty is much more significant than the related error. An example from McDonald in Reference 127 points out how the seasonal uncertainty of temperature is considerably larger than the error related to measuring the temperature.

Error is usually corrected or allowed to remain when it is deemed acceptable because of the analysis requirements or because of the resources required to improve or eliminate it. [140]

Aleatory and epistemic uncertainty and error are the most common descriptions of uncertainty in design. Error is usually insignificant when compared to uncertainty and can be corrected by the designer when identified. Additionally the subject of error modeling and propagation for the design of systems is thoroughly discussed by McDonald in Reference 127. For these reasons, this research will focus on aleatory and epistemic uncertainty.

3.5. Sources of Uncertainty

One of the most common techniques is to classify uncertainty by source. Depending on the type of problem, there are a variety of different sources of uncertainty and a variety of different ways of interpreting the sources.

Rowe describes four different types of uncertainty: temporal, structural, metrical, and translational. [161] Temporal sources of uncertainty are either related to uncertainty in future states or uncertainty from past states. Uncertainty in future states can be a priori. For example, when playing roulette it is possible to calculate the odds from examining the game, even though the actual outcome is uncertain. Future uncertainty can also be evaluated from a frequentist perspective, where the uncertain parameters are estimated by sampling test events or past events, or from a subjective perspective when the likelihood of uncertain parameters or values occurring are estimated. Uncertainty in past states is when there was a failure to record past state conditions. Structural uncertainty is due to complexity in the system. The sources of this are systematic fluctuations, parameter interactions, subjective interpretations of models, and the selection of a model. The metrical uncertainty, which relates to how something is measured, is from empirical observations or the interpretation of observations or measurements. Translational uncertainty, according to Rowe, is related to explaining uncertain results. This is related to conflicting goals and values and different perspectives. [161]

Many authors in the literature list some of the sources of uncertainty as parameter uncertainty and model uncertainty. [143, 68, 206] Du and Chen in Reference 68 describe input parameter uncertainty as the variability of input values and model parameter uncertainty as the uncertainty relating to the lack of information from estimating model parameter characteristics. Model uncertainty, or model structure uncertainty, pertains to the fact that any model, despite its fidelity, will be an approximation of the actual system. [143, 206] Parameter uncertainty is then the combination of input parameter uncertainty and model structure uncertainty. [68] There will always be some epistemic uncertainty relating to the formulation of the model and how well it predicts reality. [143] This is where the uncertainty relating to the validity of assumptions built into the model is considered. [68]

3.5.1. Sources of Uncertainty for Systems-of-Systems

As described in Chapter 1, there are several types of uncertainty pertaining to SoS design that relate to the seven challenges described by INCOSE. [79] These types of uncertainty include: understanding how the different systems interact; understanding what emergent behaviors will result from the interactions of related systems; understanding how the systems should be operated; and understanding how the actions of autonomous agents play into the scenarios. There is also uncertainty pertaining to defining current and future requirements for these systems. While identifying sources of uncertainty such as parameter uncertainty and model structure uncertainty are applicable to systems of systems, they are very broad and it is more useful to further isolate the sources. The designer must consider sources of system uncertainty relating to component interaction, system interaction, emergent behavior, the environmental situation, operations, independent agents, and new systems.

Component interaction uncertainty focuses on the relationships and interfaces between the different components in the system. This kind of uncertainty is in any system design problem, but it should also be considered for a SoS problem. There are two types of interactions which must be considered: direct interactions and indirect interactions. [117] Direct interactions are those interactions that can be identified by considering the inputs and outputs from the interdependent components. With these interactions the cause and effect relationship between components is usually well understood by the designer. Indirect interactions are the non-obvious and generally unintended interactions between components. [117]

System interaction uncertainty deals with uncertainty pertaining to how the different systems interact. For instance in many cases the inputs and outputs from one system to the next may be ambiguous or completely unknown. For a persistent strike mission, in some scenarios, both sensor aircraft and fighter aircraft are used to identify and strike targets. There is some uncertainty between how these two systems should be operated together. Should the fighter aircraft be waiting at the base? Should the fighter aircraft be loitering around the engagement area waiting to strike? Or, should the fighter aircraft be actively searching as well as the sensor aircraft? There may also be secondary interaction effects, or byproducts from indirect interactions, that are not generally considered. For example, waste heat from the propulsion system or avionics components might be affecting aircraft sensor systems. Another example would be noise from the propulsion system of the aircraft alerting potential targets that a sensor or strike aircraft may be in the area.

Emergent behavior uncertainty pertains to uncertainty in unmodeled effects from system interactions. Behavior relating to self organization that was not explicitly designed into the system also falls into this category. Situational uncertainty is uncertainty relating to the environment of SoS. Examples include: weather factors or factors relating to unknown enemy threats or targets. In the battlespace example, an unknown number of threats will occur at unknown times in unknown areas of the engagement area. Usually these are uncontrollable factors that the SoS must be robust and opportunistic against.

Operational uncertainty is the uncertainty dealing with how systems will be operated. When designing a SoS the exact coverage area and engagement time are generally not known, because they will vary from mission to mission. Another example could be the maintenance schedules for long term engagements. Perhaps an aircraft or other system will not be available when desired due to maintenance. These are additional factors that a SoS must be designed for in order to result in the most opportunistic and robust system.

Independent agent uncertainty deals with fact that systems can be autonomous. Often systems will be forced to make a quick decision in the middle of a mission, but which decision will be made is uncertain as is the effects of this decision. An example relating to the battlespace problem, is that if a strike aircraft has two potential targets, it is uncertain which target it will strike first. The actions of this aircraft affect the rest of the system and these effects should be considered.

Another important source of uncertainty relates to new systems. It is uncertain as to what future systems will be integrated into the SoS. It is common that particular systems on an aircraft will become obsolete and will be replaced throughout that aircraft's life cycle. The main platform (system) must be designed as to allow for technologies to be replaced, upgraded, removed, or added.

It is possible to map these types of uncertainty back to the challenges posed by INCOSE. This is illustrated in Figure 3-1.

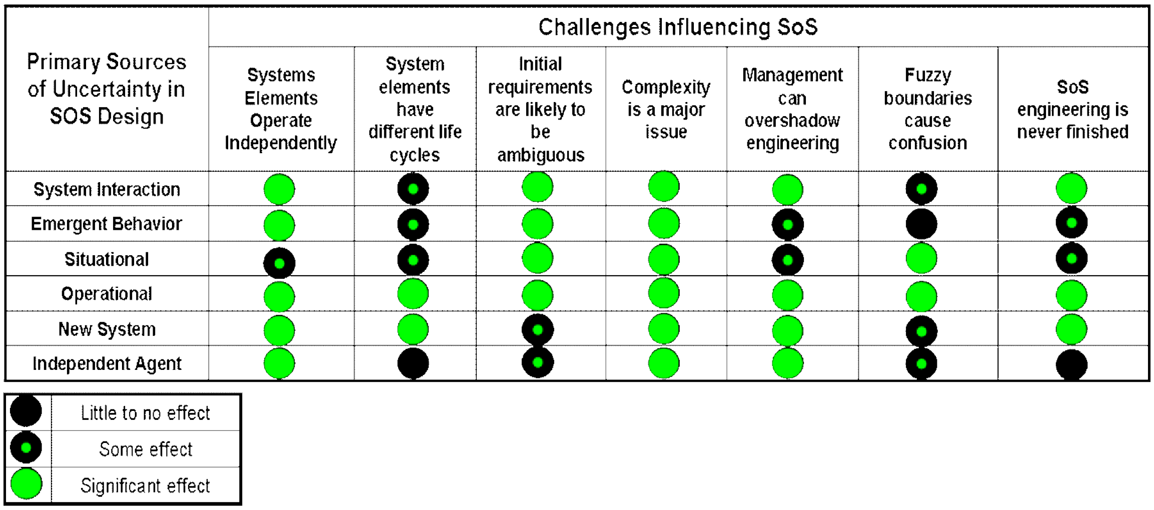

Figure 3-1: Relationship between INCOSE Challenges and Sources of Uncertainty in SoS

From Figure 3-1 it is evident that the major sources of uncertainty that affect the challenges as listed by INCOSE are: system interaction, operational interaction, and the uncertainty associated with integrating new systems into the SoS. But the uncertainty from emergent behavior, situational, and independent agents can also lead to significant effects and should be considered. An interrelationship diagram between the different sources of uncertainty is shown in Figure 3-2, and can be used to provide insight into the relationships between the different sources of uncertainty.

As illustrated in Figure 3-2: The sources of uncertainty are heavily interrelated. Several of the sources are related to all of the other sources such as emergent behavior, operational uncertainty, and system interaction. New system uncertainty is related to the uncertainty from system interactions, the situation, and the operational environment.

Independent agent uncertainty is related to system interaction, situational, operational, and emergent behavior uncertainty. It is interesting to note that while situational uncertainty affects most of the other sources, it is not affected by them. Its uncertainties come from outside sources, and it is important in modeling noise factors.

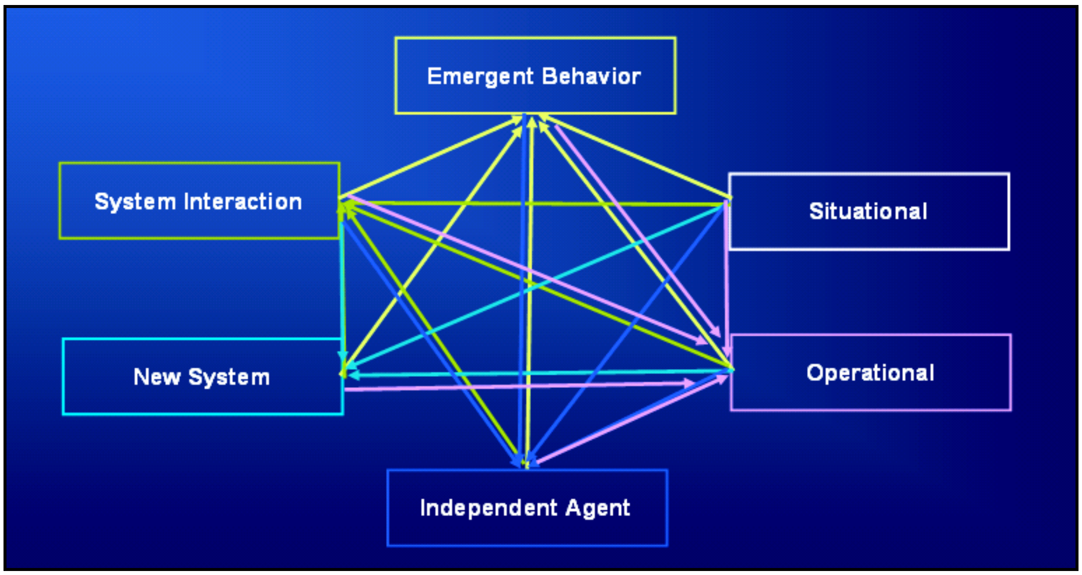

Figure 3-2: Interrelationship Diagram between Sources of Uncertainty for SoS

A notional relationship between the different levels of the SoS Hierarchy and the various sources of uncertainty is illustrated in Figure 3-3. While this chart highlights which sources of uncertainty are typically significant for each level, it does not apply to every SoS. For instance, in some cases there might be a large amount of emergent behavior uncertainty within the intermediate levels, or a particular SoS may have very little operational uncertainty at the OES level. The Base Level will be either a Component Level or an Intermediate Level, and its sources of uncertainty vary accordingly.

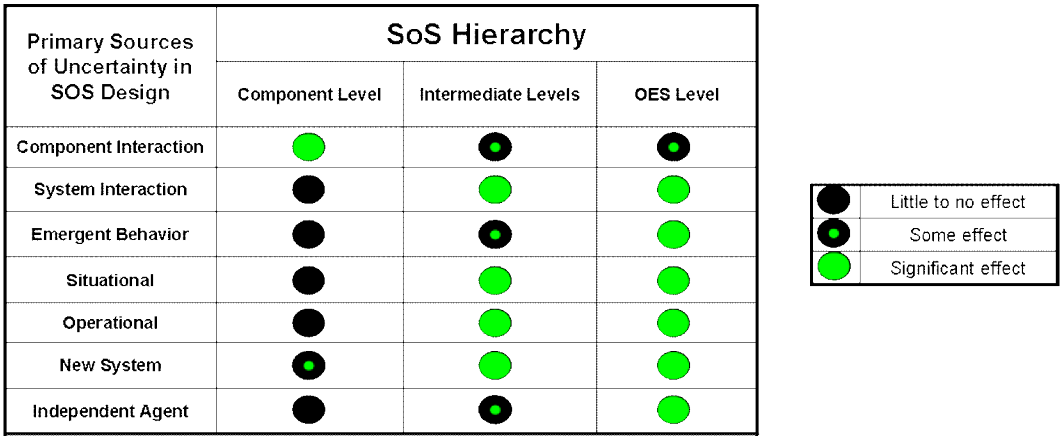

Figure 3-3: Relationship between Sources of Uncertainty and SoS Hierarchy

3.6. Factors of Uncertainty

While it is possible to identify the source of uncertainty, it is often useful to break the problem down further and look at the different factors contributing to the uncertainty. These factors are: randomness, sampling, confusion, conflict, inaccuracy, ambiguity, vagueness, coarseness, and simplification. A taxonomy of uncertainty is provided in Figure 3-4.

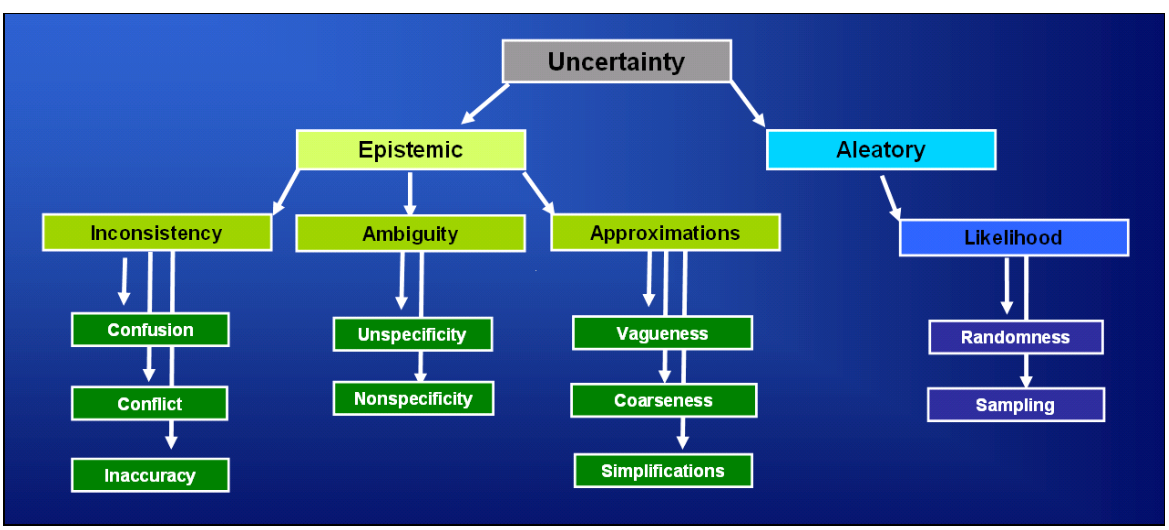

Figure 3-4: Taxonomy of Uncertainty [18]

Randomness is related to the non-predictability of outcomes. [18] An example of this in the persistent strike scenario is not knowing when a target will emerge. Sampling is another factor and is related to the uncertainty of using a sample to characterize a population. [18] An example of how this factor contributes to the uncertainty, could be in estimating the length of a window of opportunity to strike for a particular target from previous data on that type of target. It is likely that no matter how much data was sampled there will still be some variation between the estimated time and the actual time. Confusion is the third factor and pertains to incorrect substitutions. [18] An example of confusion would be operational orders being mixed up or misunderstood. For instance, perhaps ground based sensors were mistakenly setup in location A instead of location B. Conflicting or contradictory information can also lead to uncertainty. [18] Conflicting sources of intelligence would be an example of how this would play into the battlespace scenario. Uncertainty relating to inaccuracy, or bias and distortion, can considerably affect the results. [18] For instance, inaccuracy in targeting data could result in weapon systems missing the target. Ambiguity is a common factor of uncertainty and is from the possibility of having multiple outcomes. [18, 19] This is especially prevalent in the battlespace example. There might be numerous targets that appear or no targets that appear. These targets may emerge at one time in one location or spread across the region. There are an infinite number of possibilities which is why it is impossible to optimize for one particular scenario. The best option is to select the most robust and opportunistic SoS. Vagueness is another common factor. [18, 19] It deals with imprecision in identifying whether or not a particular element belongs to a set. For example, when a potential target emerges it may not be certain without gathering additional information about the potential target if it is or is not a target. Coarseness pertains to situations when approximations are made because it is difficult to identify which elements belong in a particular set or factor. [18] When a soldier or pilot is trying to decide if a potential target is an actual target, they may have to make a decision in situations where they cannot be completely certain. They will consider the known characteristics of the potential target to determine if it should be grouped into the target category or not. The last factor is simplifications and occurs when assumptions are made to make the problems modelable. This relates to model structure uncertainty. Considering the complexity of the persistent strike battlespace scenario, a number of simplifications will be made to model the problem. The uncertainty becomes whether or not the model will be a good approximation of reality despite the simplifications or if the results will be useless. Figure 3-5 illustrates the relationships between these factors and the different sources of uncertainty for a SoS.

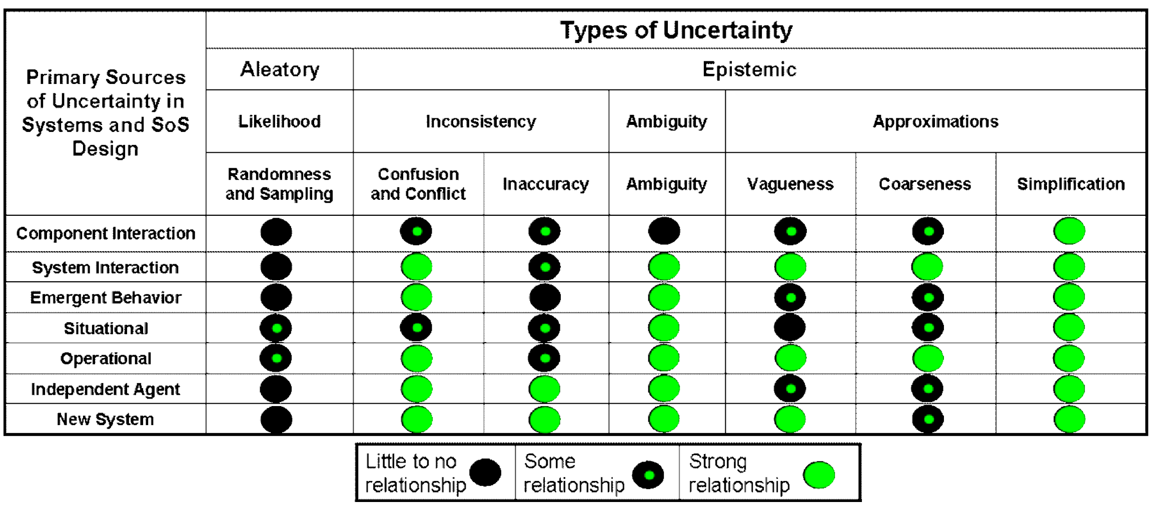

Figure 3-5: Relationship of between Factors of Uncertainty and Sources of SoS Uncertainty

CHAPTER 4: METHODS FOR MODELING UNCERTAINTY

There have been a wide variety of methods and techniques for modeling uncertainty. Historically, statistics is the most commonly used tool for quantifying uncertainty in engineering and the sciences. [17] Other classic methods include Set Theory and Probability Theory. [110] Additionally, since the 1960s there have been a considerable number of other techniques including Fuzzy Sets Theory, Rough Sets Theory, Possibility Theory, Evidence Theory, Interval analysis, and Info-Gap Theory. [18,25] Each of these theories has its particular strengths; however, they should not be blindly used to model all types of uncertainty. [18,140]

Often there are multiple types of uncertainty within the same design problem. To appropriately analyze these different types of uncertainty, hybrid uncertainty modeling techniques are becoming popular in some engineering fields. This is when multiple techniques are used together to capture different aspects of the uncertainty. This is particularly useful when working with engineering systems, since there are often multiple types of uncertainty involved from numerous sources. [17] In these cases it is typically useful to model each of the different types of uncertainty separately within the same process in order to determine how it propagates through the system and how to possibly reduce it.

However, the most common practice in the aerospace community is to utilize Probability Theory. In many cases, this can be a very useful technique. When the appropriate probability distributions are known for modeling particular uncertain variables it is highly likely that this is the best approach to use. Also, since most people are somewhat familiar with probabilities, it is fairly easy to understand and to communicate the results. But, as discussed briefly in the previous chapter, if the uncertainty is not aleatory in nature, Probability Theory is not always the best technique to use. [140, 143] Since a significant portion of the uncertainty pertaining to SoS problems is epistemic in nature, designers should consider additional techniques before attempting to model the uncertainty of the system.

The purpose of this chapter is to review several of the different uncertainty modeling techniques and to describe when they should be used in the design process. However, this chapter is not meant to be a comprehensive description of the described methods. It is merely meant to be an overview or review for the reader.

4.1. Overview of Uncertainty Modeling Theories

There are numerous techniques for modeling uncertainty. While this chapter does not provide an overview of all the existing tools and theories, it does discuss a number of the more commonly used techniques. In order to provide a general overview these theories are briefly described and then evaluated in terms of their effectiveness in addressing the uncertainty factors described in Chapter 3. The theories that will be discussed include:

- Probability,
- Statistics,
- Bayesian,
- Classical Sets,
- Fuzzy Sets,
- Rough Sets,
- Possibility,
- Evidence,
- Interval probabilities,
- Interval analysis,

- and Info-Gap.

4.1.1. General Terminology and Notation

Before discussing the various theories it is useful to discuss some of the common terminology between the methods. A set is a collection of elements or objects. The notation $s \in S$ means that s is an element of the set S, and $s \notin S$ means that s is not a member of S. [183] The universal set, is also called the sample space, and refers to the set of all possible outcomes. [18,106] The sample points are individual possible outcomes from the sample space. [106]

From the set it is also possible to identify subsets. Subsets are indicated by the notation: $A \subset B$. This notation means that the set A is contained in the set B, meaning that it is a subset of B. [106] An event is a particular subset of the sample space. [132]

Sample points that are in A or B (or both) can be described by the union of these two sets. The union is usually represented by the notation in Equation 4-1. The sample points which are in both A and B are in the intersection of A and B. This is written in Equation 4-2. If there is a set that has no sample points it is called a null set and is signified by the symbol $\varnothing$. [106] The power set refers to all of the possible subsets of the universal space, including the null set.

$$A \cup B = \{s : s \in \mathrm{A} \text{ or } \mathrm{s} \in B\} \qquad \textbf{Equation 4-1}$$

$$A \cap B = \{s : s \in \mathrm{A} \text{ and } \mathrm{s} \in B\} \qquad \textbf{Equation 4-2}$$

Two sets are mutually exclusive if the intersection of the two sets is a null set as shown in Equation 4-3. A combination of sets is considered to be exhaustive when the union of these sets is equivalent to the sample space. This is represented by Equation 4-4. [132]

$A \cap B = \varnothing$ **Equation 4-3**

$E_1 \cup E_2 \cup ... \cup E_n = S$ **Equation 4-4**

4.1.2. Probability Theory

Probability Theory models the uncertainty pertaining to random events. [70] For a set of alternative events, the measure of uncertainty is called the probability, which numerically expresses the likelihood of a particular alternative from that set occurring. [111] The likelihood is assessed on a scale from 0 to 1, where 0 indicates that it would be impossible for the event to occur and 1 indicates that it is absolutely certain that the event will occur. [70]

4.1.3. Statistics

The mathematical base for statistics developed from Probability Theory, and pertains to the collection, analysis, and presentation of data to make inferences about future events. Statistics is best used to understand the uncertainty related to randomness or variability. [132] While probability pertains to predicting the likelihood of potential events, statistics, on the other hand, deals with analyzing the occurrence and frequency of past events. [175]

4.1.4. Bayesian Statistics

Bayesian Statistics Theory is based on Bayes' Theorem and relates to updating uncertainty estimates based upon additional evidence. As described by Bernardo in Reference 27 this theory considers a probability "as a conditional measure of uncertainty associated with the occurrence of a particular event, given the available information and the accepted assumptions." Unlike in Probability Theory, with this definition there is no absolute probability because it is always a function of the event whose uncertainty is being determined and the conditions of the event. [27]

4.1.5. Interval probabilities

This theory is based on Probability Theory and is under a category of techniques called imprecise probabilities. These techniques have been developed for situations where exact probabilities have not been determined. Interval probabilities use a probability measure which belongs to the set of values encapsulating the lower and upper estimates of the actual probability of a particular event occurring. This theory can incorporate conditional probabilities and can account for dependencies between events.

4.1.6. Classical Sets

This theory models the uncertainty associated with inherent nonspecificity in a particular problem and expresses this uncertainty through the use of sets of mutually exclusive potential alternatives. [98, 111] These sets are defined using characteristic functions where alternatives are either labeled 0 or 1. A value of 0 indicates that the alternative does not belong to the set while a value of 1 indicated that it does. The values 1 and 0 are purely symbolic and can be represented by other pairs of symbols. [17]

4.1.7. Interval analysis

Interval analysis models the potential range of values that could be the quantity of interest. There is ambiguity concerning which value is correct, but it is certain that the value lies within the interval. Within this theory, arithmetic is performed on the intervals and the intervals themselves can be treated as numbers. [133]

4.1.8. Rough Sets

This technique was specifically developed as a way to describe classical sets[2] when there is limited resolution capability. These sets are represented through two subsets of the universal set: the lower approximation and the upper approximation. The lower approximation contains the elements that are definitely within the classical set, while the upper approximation consists of all of the elements that are at least partially in the classical set. [17]

4.1.9. Fuzzy Sets

Fuzzy sets use a membership function to distinguish between the degree of an element (or alternative) belonging to a set. In contrast to classical sets, this technique does not require sharp boundaries to determine if an element or alternative belongs to a set. Instead it generalizes the classical theory to allow for partial memberships. [203] This function is typically expressed using the unit interval of real numbers [0,1]. For this set, 0 designates that the element has no compatibility with the set and 1 means that the element has the highest level of compatibility. [17]

4.1.10. Possibility

This theory is used to identify the degree of how possible it is for an element to take a particular value. Probability Theory and Possibility Theory are often confused despite the fact that they model two different types of uncertainty and are complementary theories. [69] Possibility Theory models imprecision[3] as opposed to Probability Theory which

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[2] Classical sets are also known as crisp sets and indicate that it is possible to uniquely determine if individual element belong to a particular set. [18]

[3] Imprecision relates to the vagueness factor of uncertainty.

models the likelihood of occurrence of an element. [203] The uncertainty is modeled through two functions called a possibility measure and a necessity measure. The possibility measure identifies if a particular parameter could have a certain value, while the necessity measure quantifies if it is required for that parameter to have the certain value. [203]

4.1.11. <u>Evidence</u>

Evidence Theory, also called Dempster-Shafer Theory (DST), models situations where evidence supports multiple possible events. In scenarios where there is sufficient evidence to support assigning probabilities to a single event, this theory condenses into traditional Probability Theory. [170] For this reason, Probability Theory can be considered a special case of DST. [17] It was developed to manage varying levels of precision or vagueness and does not require additional information to model the available information. [170] It utilizes two measures of the "likelihood" of a set called the plausibility and the belief function, which basically represent the lower and upper probabilities. [140]

4.1.12. <u>Info-Gap</u>

This theory was specifically designed for problems with limited amounts of information. An Info-Gap model groups sets of potential values based upon each sets level of uncertainty, and uses two different functions to express the uncertainty: Robustness Function and the Opportunity Function. As described in Reference 25 the Robustness Function quantifies the largest level of uncertainty without violating a constraint, and the Opportunity Function is the lowest level of uncertainty where there is still the possibility of "sweeping success". [25]

All of these theories have their particular strengths and can be used in modeling the different uncertainty factors. While it is possible for these theories to model several if not all of the uncertainty factors discussed in the Chapter 3, it may not be as effective or accurate as another theory. Figure 4-1 illustrates the most appropriate factors for each theory to model. [18]

Theory and Methodologies	Types of Uncertainty						
	Aleatory	Epistemic					
	Randomness	Confusion and Conflict	Inaccuracy	Ambiguity	Vagueness	Coarseness	Simplification
Classical Sets	Poor	Poor	Poor	Good	Fair	Poor	Fair
Probability	Good	Fair	Good	Good	Fair	Fair	Good
Statistics	Good	Fair	Fair	Good	Fair	Fair	Fair
Bayesian	Good	Poor	Fair	Good	Fair	Fair	Good
Fuzzy Sets	Fair	Fair	Fair	Fair	Good	Good	Good
Rough Sets	Poor	Poor	Fair	Fair	Fair	Good	Good
Possibility	Fair	Good	Good	Fair	Good	Fair	Fair
Evidence	Fair	Good	Fair	Good	Fair	Fair	Fair
Interval probabilities	Fair	Good	Good	Good	Good	Fair	Good
Interval analysis	Fair	Good	Good	Good	Fair	Fair	Good
Info-gap	Good	Fair	Good	Good	Fair	Fair	Good

Poor | Fair | Good

Figure 4-1: Appropriateness of a Theory in Modeling different Uncertainty Factors

As evident from Figure 4-1, each uncertainty factor has several different types of theories which would be appropriate to use. Based upon this information a possible matrix of alternatives for selecting a potential uncertainty modeling theory is shown in Figure 4-2.

Uncertainty Factor	Potential Theory							
Randomness and Sampling	Probability	Statistics	Bayesian	Info-Gap				
Confusion and Conflict	Possibility	Evidence	Interval Probabilities	Interval Analysis				
Inaccuracy	Probability	Possibility	Interval Probabilities	Interval Analysis	Info-gap			
Ambiguity	Classical Sets	Probability	Statistics	Bayesian	Evidence	Interval Probabilities	Interval Analysis	Info-gap
Vagueness	Fuzzy Sets	Possibility	Interval Probabilities					
Coarseness	Fuzzy Sets	Rough Sets						
Simplification	Probability	Bayesian	Fuzzy Sets	Rough Sets	Interval Probabilities	Interval Analysis	Info-gap	

Figure 4-2: **Uncertainty Modeling Matrix of Alternatives**

If all seven uncertainty factors were involved in a design problem, there are over 26,880 possible ways to model the uncertainty in this problem. To add to the complexity of the problem, all of these techniques model different types of uncertainty and model different levels of knowledge. Some of these techniques are applicable for situations with limited knowledge, such as Info-Gap Theory, and other techniques such as Probability Theory are more useful in situations when the problem has been reasonably well characterized. Most uncertainty modeling techniques are capable of modeling multiple types of uncertainty, so it is possible to reduce the number of required uncertainty modeling techniques that need to be utilized for a design problem of interest.

For a general SoS conceptual design problem it is possible that all seven types of uncertainty exist and need to be modeled. From Figure 4-2 there are a number of uncertainty modeling techniques that could be utilized to model all of the different types of uncertainty. Three of the most prevalent theories are Probability Theory, Evidence Theory and Fuzzy Set Theory. If these three techniques are used in conjunction they can model all of the seven types of uncertainty.

However, while all of the different types of uncertainty are accounted for, the situation when very little is known about the uncertainty cannot be effectively modeled with these techniques alone. This is because when little is known about an uncertain variable additional assumptions need to be made before the uncertainty can be modeled. In the conceptual design phase it is highly probable that very little information will be known about some of the uncertain variables within the problem. In some cases, there will be no historical evidence suggesting an appropriate distribution function or even enough information to bound the problem with any certainty.

For certain types of ignorance, such as ambiguity, Evidence Theory and Info-Gap Theory can be used to model the variables when there is not enough information to run a probabilistic analysis. Figure 4-3 pictorially illustrates how for certain types of ignorance, as information/knowledge increases, another uncertainty modeling technique may be more appropriate for modeling the uncertainty. When very little is known about the uncertainty, then Info-Gap Theory can be a useful technique for evaluating the data. When it is possible to identify bounds to the uncertainty then Evidence Theory can model this information without applying any additional assumptions. And, finally if there is enough statistical information to define the range and distribution of the uncertainty, Probability Theory is often the best uncertainty modeling technique.

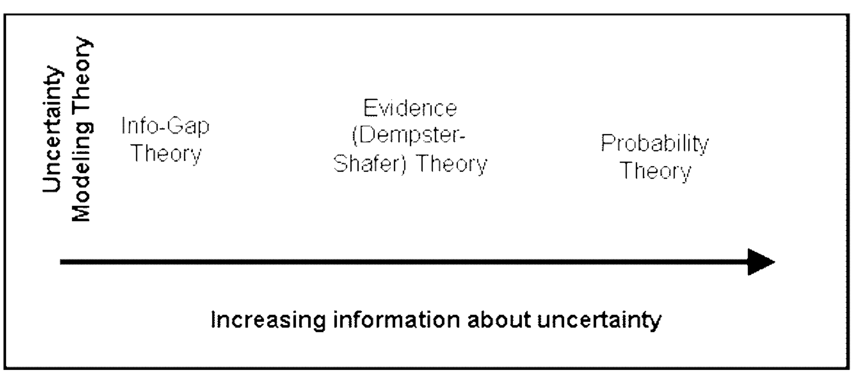

Figure 4-3: Representation of Relationship between Uncertainty Modeling Techniques and Level of Knowledge

A designer should consider all of the available design methods and select the theory which is most appropriate for their problem of interest. As illustrated in Figure 4-1 the following four theories when combined are capable of modeling all of the different uncertainty factors at varying levels of knowledge: Probability Theory, Fuzzy Set Theory, Evidence Theory, and Info-Gap Theory. Probability Theory, Fuzzy Set Theory, and Evidence Theory are well known and have demonstrated their applicability to a wide range of problems. These techniques were selected because in conjunction they can model the different types of uncertainty: randomness and sampling, confusion and conflict, inaccuracy, ambiguity, vagueness, coarseness, and simplification. Info-Gap theory was selected because of its unique capability of modeling uncertainty for cases of severe uncertainty. By incorporating these four techniques into one hybrid uncertainty modeling technique it is possible to not only model different types of uncertainty but also to model different levels of knowledge.

For this reason these four theories were utilized throughout this research and are discussed in greater detail in the following sections of this chapter. To fully illustrate the main properties and the utility of using these techniques an example problem where the acquisition cost for a persistent strike UAV is considered.

4.2. Probability Theory

The most widely used techniques for quantifying uncertainty come from Probability Theory. [134] It dates back to the 17th century and was developed by people such as Pascal, Fermat, Bernoulli, Bayes, and Laplace for modeling odds pertaining to games of chance. [106, 18] Numerous advances to the field were then made throughout the 19th and 20th centuries. [106]

Within the theory there are several approaches: the classical, the frequentist, and the subjectivist, which is also known as the Bayesian or Personalist perspective. [134,165,56] The classical perspective is based upon the concept of equal outcomes. It models the probability as the ratio between the number of outcomes and the total number of potential outcomes. A common example is a coin toss. Since the coin will only be tossed once the number of outcomes is 1. The total number of potential outcomes is 2, to model the possibility that the coin could land "heads up" or "heads down". The resulting probability of getting either heads or tails on any given toss is ½. [181] Frequentists consider probability as the relative frequency that would be obtained by repeating a process a large number of times under similar conditions. [18] Those who follow the Bayesian perspective consider probability to be the degree of belief of the person that an event will occur. This belief depends upon the information that is available to the person. [134] However, all of these different approaches to probability theory must follow the axioms of probability.

4.2.1. Axioms of Probability

From a mathematical viewpoint the definition of probability comes from three axioms, where for an event A in the universal set X, the notation P(A) denotes the probability of occurrence of A. [18,181,160]

AXIOM 1

$$0 \leq P(A) \leq 1 \qquad \textbf{Equation 4-5}$$

AXIOM 2

$$P(X) = 1 \qquad \textbf{Equation 4-6}$$

AXIOM 3

$$P\left(\bigcup_{i=1}^{n} A_i\right) = \sum_{i=1}^{n} P(A_i), \quad n = 1,2,...,\infty$$ **Equation 4-7**

The first axiom, as shown in Equation 4-5, states that the probability of the occurrence of event A must be between 0 and 1. Axiom 2 the probability of the universal set occurring is 1, meaning that the outcome will be a member of the universal set. The third axiom, as stated in Equation 4-7, says that the probability of one member from a set of mutually exclusive events occurring is equal to the sum of the respective probabilities of all of the events in the set. [18, 181,160]

4.2.2. <u>Conditional Probability and Bayes Theorem</u>

Conditional probability is one of the most important concepts in Probability Theory. [160] It is useful when calculating the probability of an event when only partial information about that event is available and when it is possible to condition the likelihood of the event based upon the occurrence of another event. If there are two events, A and B, where the probability of A is greater than zero (P(A)>0), then the probability of B given that A has occurred is denoted by: $P(B|A)$. This conditional probability is calculated by comparing the probability of the intersection of the probabilities of A and B as well as the individual probability of A as shown in Equation 4-8. [132,160] Or, if the conditional probability is known it is possible to calculate the probability of both A and B occurring as shown in Equation 4-9 if either the probability of A or B occurring is known. [181,132]

$$P(B|A) \equiv \frac{P(A \cap B)}{P(A)}$$ **Equation 4-8**

$$P(A \cap B) \equiv P(B|A)P(A) = P(A|B)P(B)$$ **Equation 4-9**

If the conditional probability of B given the occurrence of A is not equal to the probability of B, then B is dependent on A. Or, if $P(B|A) = P(B)$ then B is independent of A. [160] Additionally, if A and B are independent, Equation 4-10 will be true. [181]

$$P(A \cap B) = P(A)P(B) \qquad \textbf{Equation 4-10}$$

Bayes' Theorem is an extension of the concept of the conditional probability theorem. Consider the mutually exclusive and exhaustive set[4]: A_1, A_2, …A_n . For this set, "A" represents any individual event. Bayes' Theorem is given in Equation 4-11. [181, 160]

$$P(A_k|A) = \frac{P(A|A_k)P(A_k)}{\sum_{j=1}^{n} P(A|A_j)P(A_j)} \qquad \textbf{Equation 4-11}$$

This theorem is also referred to as the theorem on the probability of causes, because it allows a user to determine the specific probabilities for related events that will cause A to occur. [181]

4.2.3. Probability Distributions

As discussed earlier in this chapter, Probability Theory is especially useful for quantifying the randomness uncertainty factor. This is because Probability Theory considers random events and random variables. Reference 106 states that a random event can have only two possible outcomes: true (1) or false (0). This event is random because

[4] Indicating that this is the exhaustive set means that this is the set of all alternatives and one of these events will occur.

it is uncertain which outcome will occur. A random variable, also called a random quantity, can take a number of different values representing a different outcome from a sample space. [106, 132] If the random variable has a finite or countably infinite number of values (such as the set of integer numbers) it is called a discrete random variable. If it could be a value within a set of noncountably infinite numbers then it is a nondiscrete or continuous variable. [181] Probability distributions describe the probabilities associated with these random variables. [132]

Probability distributions are used to manipulate the data so that it is easier to analyze. [106] There are both discrete and continuous probability distributions[5], depending upon the related type of random variable.

The probability distribution for a random variable X, with the possible set of values x, is often represented by the probability density function (pdf), f(x), if Equations 4-12 and 4-13 are true. Equation 4-13 applies if the random variable is discrete, and Equation 4-14 applies if the variable is continuous. [181]

$$f(x) \geq 0$$ **Equation 4-12**

$$\sum_{x} f(x) = 1$$ **Equation 4-13**

$$\int_{-\infty}^{\infty} f(x)dx = 1$$ **Equation 4-14**

An alternative technique for describing the probability distribution of a random variable is with a cumulative probability. The cumulative distribution function (CDF) is denoted by F(x) and is defined in Equation 4-15 for a discrete variable and Equation 4-16 for a continuous variable. [132, 181]

[5] Probability distributions functions are also referred to as probability mass functions.

$$F(x) = P(X \leq x) = \sum_{x_i \leq x} f(x_i)$$ **Equation 4-15**

$$F(x) = P(X \leq x) = \int_{-\infty}^{x} f(x_i) du$$ **Equation 4-16**

These techniques can be generalized to two or more random variables. For the discrete random variables X and Y, the probability density function is given in Equation 4-17 where Equations 4-18 and 4-19 are true. [181]

$$P(X = x, Y = y) = f(x, y)$$ **Equation 4-17**

where

$$f(x, y) \geq 0$$ **Equation 4-18**

$$\sum_{x} \sum_{y} f(x, y) = 1$$ **Equation 4-19**

If the variables are continuous, the pdf is also given in Equation 4-17, but for Equations 4-20 through 4-21.

$$f(x, y) \geq 0$$ **Equation 4-20**

$$\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(x, y) dx dy = 1$$ **Equation 4-21**

4.2.4. Mathematical Expectation

One of the most important concepts of this theory pertains to the mathematical expectation, or the expected value, of a random variable. [160,181] The expectation is often called arithmetic mean (or mean), and is one of the three measures of central tendency. The additional two measures are the mode and the median. [18]

The most commonly used central tendency measures is the arithmetic mean or average value and is often denoted by $\overline{X}$ or μ. This is the weighted average of the possible values of a random variable, and is the 'centre of mass' of the distribution. [18,106] If all of the potential values are weighted equally and if there are n values to be considered, the equation for the expectation is below: [18]

$$\overline{X} = \frac{1}{n}\sum_{i=1}^{n} x_i$$ **Equation 4-22**

Additionally, the expectation can also be determined from the probability distributions for discrete and continuous variables as shown in Equations 4-23 and 4-24, respectively. [18] The formula for the expectation as shown in Equation 23 is very common and utilized extensively in techniques such as Expected Utility Theory.[6]

$$\mu = \sum_{x} x_i P(x_i)$$ **Equation 4-23**

$$\mu = \int_{-\infty}^{\infty} x f(x) dx$$ **Equation 4-24**

The second measure of central tendency is the median which is defined as the value which divides the data into two equal parts. The main advantage of using the median value over the mean is that it is relatively insensitive to extreme points, such as outliers. [18] The third measure is called the mode, and describes the most likely or most probable value. [106]

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[6] A brief summary of Expected Utility Theory is presented in Chapter 9 and later in this chapter.

However, it is not enough to determine the mean or any other central tendency measure. It is also important to consider the dispersion or the variability of the data. [18] The variability is described by a value called the variance which quantifies the scatter in the data around the central tendency point. Values such as the standard deviation and the coefficient of variation are derived from the variance. [18] Jordaan in Reference 106 described the variance as being analogous to the moment of inertia of a body about the center of mass.

The variance, denoted by σ^2, is a nonnegative number and is calculated by either Equation 4-25 (discrete variable) or Equation 4-26 (continuous variable). [18]

$$\sigma^2 = \sum_x (x_i - \mu)^2 P(x_i)$$ **Equation 4-25**

$$\sigma^2 = \int_{-\infty}^{\infty} (x - \mu)^2 f(x)dx$$ **Equation 4-26**

The standard deviation, represented by σ, is the square root of the variance. Jordaan in Reference 106 describes how the standard deviation "corresponds to the radius of gyration in mechanics". The standard deviation is often used in place of the variance because it has the same unit as the random variable. [181]

Another useful parameter is the coefficient of variation. This term indicates the variation about the mean and is defined by Equation 4-27. [18,106]

$$\gamma \equiv \frac{\sigma_x}{\mu_x}$$ **Equation 4-27**

The covariance is the variance for two or more random variables with a joint density function. For the discrete variables, v and w with a joint density function of f(v,w), Equations 4-28 and 4-29 represent the means for each of these variables and Equation 4-

30 is the covariance. For the continuous variables, X and Y with a joint density function of f(x,y), Equations 4-31 and 4-32 represent the means for each of these variables and Equation 4-33 is the covariance. [106, 181]

$$\mu_V = \sum_v \sum_w v f(v,w)$$ **Equation 4-28**

$$\mu_W = \sum_v \sum_w w f(v,w)$$ **Equation 4-29**

$$\sigma_{VW} = \sum_v \sum_w (v-\mu_V)(w-\mu_W) f(v,w)$$ **Equation 4-30**

$$\mu_X = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} x f(x,y)\,dxdy$$ **Equation 4-31**

$$\mu_Y = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} y f(x,y)\,dxdy$$ **Equation 4-32**

$$\sigma_{XY} = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} (x-\mu_X)(y-\mu_Y) f(x,y)\,dxdy$$ **Equation 4-33**

If the random variables are independent, the covariance is equal to zero, and equal to Equation 4-34 when dependent. Equation 4-35 is the equation for the correlation coefficient[7]. [181] Both the covariance and the correlation coefficient describe any linear relationship between the random variables. [106]

$$\sigma_{XY} = \sigma_X \sigma_Y$$ **Equation 4-34**

$$\rho = \frac{\sigma_{XY}}{\sigma_X \sigma_Y}$$ **Equation 4-35**

---

[7] This is also known as coefficient of correlation. [181]

4.2.5. <u>Probability Theory in Design</u>

Aspects of this theory are commonly used to model uncertainty within the design community. The objective with these techniques is to determine the expected value for a design metric while considering the associated uncertainty. The two most common techniques for accounting for the uncertainty are to determine the expected value of the metric by using the mathematical expectation and to conduct a Monte Carlo Simulation (MCS). Each of these techniques is discussed in the following sections.

4.2.5.1. Uncertainty Modeling with the Mathematical Expectation

A common technique for accounting for uncertainty in design and decision making processes is based upon the concept of the expectation as discussed earlier in this chapter. This concept forms the backbone of Expected Utility Theory where different outcomes are evaluated based upon the expected utility as calculated in Equation 4-36. [20] In this equation, x represents a specific outcome, u(x) is the utility of that outcome and π(x) is the probability of that outcome occurring. [20]

$$E[u(x)] = \sum_{x} \pi(x) u(x) \qquad \textbf{Equation 4-36}$$

For the general design problem where the focus is on determining the value of a specific design metric this equation can be rewritten with respect to the design metric of interest. Consider the scenario where for a design problem there is one uncertain variable whose characteristics can be modeled by a known probability density function (pdf). To determine the metric value for this scenario consider Equation 4-37. In this equation the pdf is either discrete or can be modeled as a discrete function by dividing the continuous pdf into intervals that can be individually evaluated. In Equation 4-37, "NI" represents the different interval values. $Metric_i$ is the value for the design metric that results from using the value of the uncertain variable that is selected based upon the interval (i) of interest. The probability associated with the likelihood of the interval of interest for the uncertain variable is indicated by π_i.

Equation 4-37

$$Metric = \sum_{i=1}^{NI} \pi_i \cdot Metric_i$$

The number of intervals used to model the uncertainty should be based on the qualities of uncertainty variable, the sensitivity of the metric to the uncertainty variable, and the computational resources available for the analysis. While the accuracy of the calculation will increase as the number of intervals increases, in many situations it is only necessary to model a few of the intervals. For instance, if there is only a small variance with the change in uncertain variable value, then it is reasonable to approximate this variable with only a few interval values. Or, often there is a natural discretization that is appropriate for an uncertain variable. All of these considerations should be taken into account when determining the appropriate number of intervals for the analysis.

Consider the design problem where there are multiple uncertain design variables, each of which can be modeled discretely. It is possible to determine the expected metric value for this type of scenario by using a full factorial Design of Experiments (DOE) that compares all possible combinations of the values from each interval for each uncertain variable. [35] The combined probability ($Л_j$) can be determined for each DOE run. For the case when the uncertain variables are independent, Equation 4-38 can be used to calculate the combined probability. If the variables are not independent the conditional probability of the variables must be considered as discussed earlier in this chapter. In Equation 4-38 “NPV” represents the number of uncertain variables to be modeled with Probability Theory, and “k” designates the DOE run of interest.

$$\Pi_k = \prod_{j=1}^{NPV} \pi_{j,k}$$ **Equation 4-38**

The expected metric value for the case with multiple uncertain variables can be calculated using Equation 4-39. The term "NR" represents the number of DOE runs.

$$Metric = \sum_{k=1}^{NR} \left(Metric_k \cdot \Pi_k \right)$$ **Equation 4-39**

This approach is also sometimes referred to as the probability tree approach as described in Reference 134. Each full factorial DOE run represents a different branch of a "probability tree" that models all of the possible discrete outcomes.

Depending on the number of uncertain variables modeled by probability theory and the number of intervals used in modeling the uncertainty, modeling the uncertainty can be a very powerful technique and be less computationally expensive than a Monte Carlo Simulation. However when there are a large number of uncertain variables that are to be modeled by Probability Theory often a Monte Carlo Simulation can be the more appropriate technique. [134]

4.2.5.2. Monte Carlo Simulation

A Monte Carlo Simulation (MCS) is a powerful technique for solving problems numerically and is a common method for conducting uncertainty analyses in the engineering industry today as shown in References 66 and 85. [106] Furthermore, this technique is often used in the design community to model uncertainty. For example see References 21 and 107. There are also several examples of how this technique has been used for system-of-systems design problems such as Reference 179, 71, and 148.

This method models uncertain variables with a pdf and involves running a large number of simulations of the analysis code or experiment where the uncertain input values have been randomly sampled from the pdfs. The results are tracked for every simulation run and usually either displayed as a histogram or averaged to determine the expected values. The primary components of a MCS include: probability density functions (pdfs), random number generator, sampling rule, scoring algorithms, error estimation, variance reduction techniques, and algorithms for parallelization. [65]

This can be an effective technique provided the pdfs are appropriately selected. Often normal, uniform, or log-normal distributions are used in this process. The pdfs are usually formed from statistical data. As an example Reference 10 discusses how to quantify probabilities from the perspective of a risk analysis for expert info, historical data, and modeling.

A common practice when there is a lack of information about which distribution to use, is to assume that the uncertain variable should be modeled as uniformly distributed. [93] This is known as Laplace's principle of insufficient reasons, which was first introduced in Keynes Treatise on Probability in 1921. [55, 93] However, this technique adds an additional uncertain assumption to the already uncertain process. It is important for designers to select distributions carefully so that the results are not affected by further sources of uncertainty. Reference 106 lists additional information on common distributions including characteristic functions, equations for the mean and variance, and suggestions for which types of problems these distributions should be used.

There are a number of benefits to using a Monte Carlo Simulation (MCS) for modeling uncertainty, where appropriate. For instance, it is easy to conceptualize, easy to incorporate with existing modeling and simulation tools, and the user can set the maximum number of runs (samples taken). The accuracy of the MCS increases with the number of samples taken for the analysis. In other words the more MCS runs the more accurate the results. One benefit to this technique is that the accuracy of the analysis can be estimated using standard statistical techniques. [134] Another benefit that is discussed in Reference 134 is that while the number of runs that is required is based upon the required accuracy of the output distribution, but it generally is independent of the number of uncertain variables.[8] This makes the technique particularly appealing to design problems where there are a large number of uncertain variables that are to be modeled using a pdf.

For situations where the analysis code is computationally expensive or the experiment requires a significant (or even a moderate) amount of time, this technique can become infeasible. However this can be mitigated through the use of surrogate models. [134, 35, 54, 31] The surrogate model is an equation that approximates the behavior of the original model by relating the value of a response (or output metric) to the value of the input variables. Three of the most common types of surrogate models are Response Surface Equations (RSEs), Neural Nets (NN), and Kriging.

4.2.6. Probability Theory Example Problem

To demonstrate this concept, consider a simple example where the objective is to determine the aircraft acquisition cost for a Persistent Strike UAV. The cost is calculated in the following equation where the aircraft acquisition cost is the product of the Aeronautical Manufacturers Planning Report (AMPR) Weight and the cost per pound of the aircraft. The AMPR Weight is the product of the AMPR weight factor and the empty weight of the aircraft as shown in Equations 4-40 through 4-42. [153] Finally, the empty weight is calculated from the empty weight fraction and the takeoff gross weight (TOGW) of the aircraft.

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[8] For the case when the variance of the output changes significantly with the number of uncertainty variables, the number of required runs for the MCS is not independent to the number of uncertainty variables, and the number of required runs will need to be increased.

$ACCost = Costperpound \cdot W_{AMPR}$ **Equation 4-40**

$W_{AMPR} = AMPR\_Weight\_Factor * W_{empty}$ **Equation 4-41**

$Wempty = We/W0*TOGW$ **Equation 4-42**

Probability Theory Process Task 1 – Define the uncertainty

For each possible design alternative, the uncertain variables need to be identified and their characteristics determined.

Determining AMPR Weight Factor

The AMPR weight factor ($AMPR\_Weight\_Factor$) is said to be between 60-70% in Reference 153. No probability density function is provided for these values.

Determining Cost per pound

There is limited cost information available for UAVs, but it is possible to estimate the cost per pound by considering data for the Predator A, Predator B, Heron TP, and the Global Hawk.[9] [40,149,162] The data is presented in Table 4-1. While the AMPR weight factor is an uncertain variable for this example problem, in order to determine the relationship between the aircraft cost and the weight, the AMPR weight is estimated to be 65% of empty weight.

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[9] The data for the Global Hawk did not include development costs or sensor packages.

Table 4-1: Cost per Pound Data [40,149,162]

	Empty Weight (lb)	AMPR Weight Factor	Cost ($Million)
Predator A	1150	747.5	4.5
Heron TP	1764	1146.6	6.5
Predator B	3700	2405	8.3
Global Hawk	9200	5980	~30

Using this information it is possible to linearly regress the data to estimate the relationship between the aircraft cost and the weight of the aircraft. This is shown in Figure 4-4 and the resulting estimate for the cost per pound is approximately $4850 per pound of AMPR weight. Considering the lack of data for this type of aircraft and different potential system packages (weapon, sensors, etc), for this problem the cost per pound is estimated to be between $4500-5000.

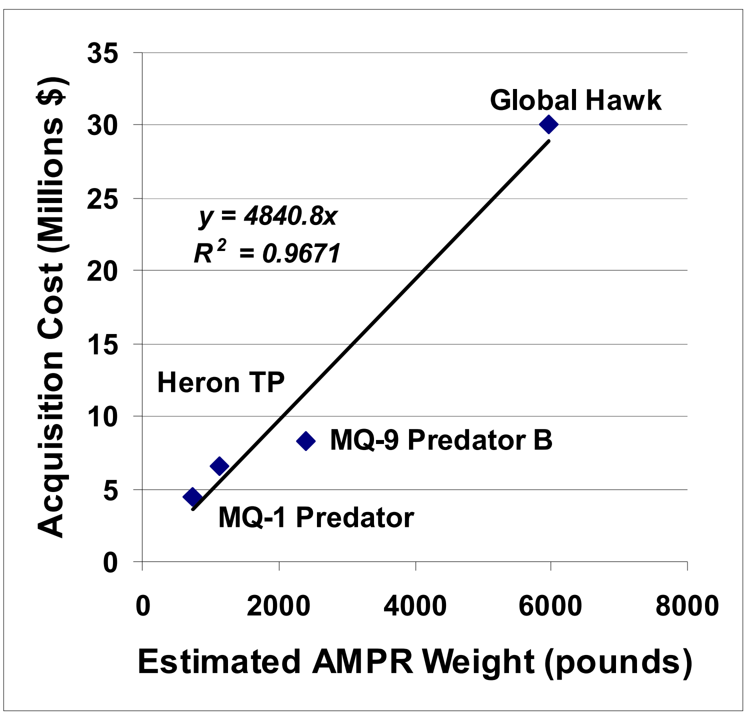

Figure 4-4: Cost per Pound Estimation

Determining Empty Weight Fraction of Aircraft

It is possible to estimate the empty weight fraction for this class of aircraft based upon data for existing aircraft. Data was collected from Reference 40 and organized by engine for three different potential engine types: piston, turboprop, and jet.

As illustrated by Figures 4-5 and 4-6, an empty weight fraction was determined based on linear regression of the empty weight data to the TOGW for each engine category.[10] Table 4-2 provides a summary of the empty weight fraction values for each engine type.

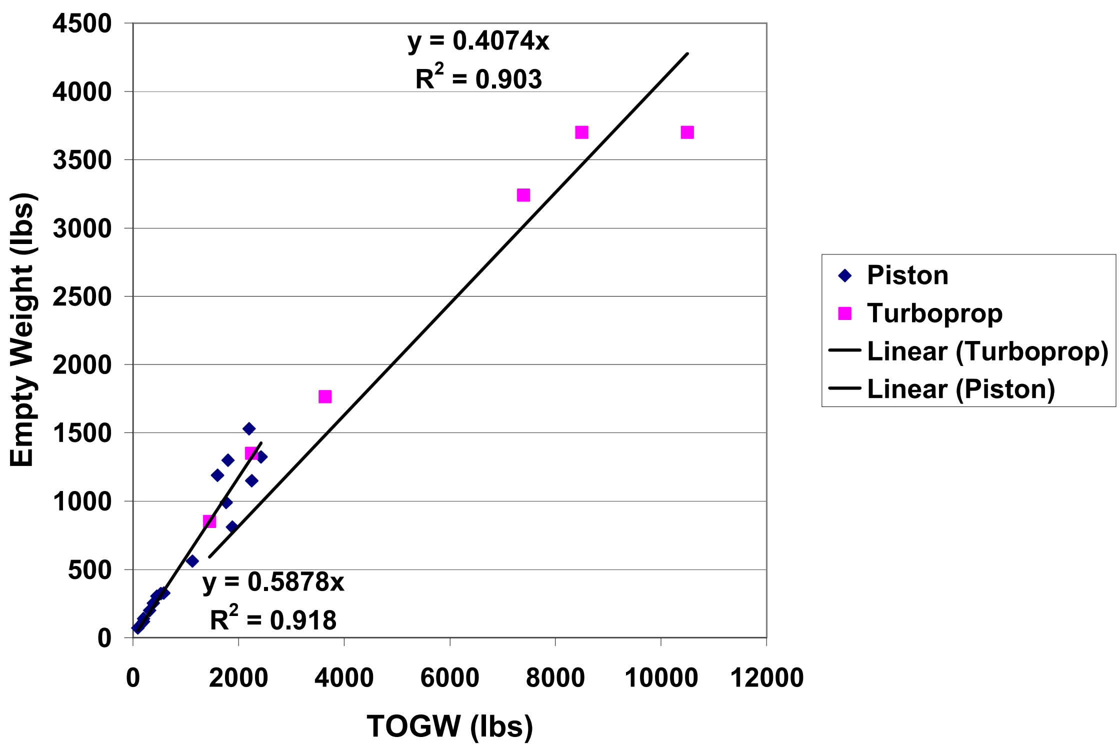

Figure 4-5: Empty Weight versus TOGW for Unmanned Aircraft (Piston/Turboprop)

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[10] Typically as discussed in Reference 159 there is a linear relationship between the $\log_{10}$ (TOGW) and the $\log_{10}$ (W_{empty}). However, the best fit for the UAV data was found from the linear relationship between the TOGW and the W_{empty} .

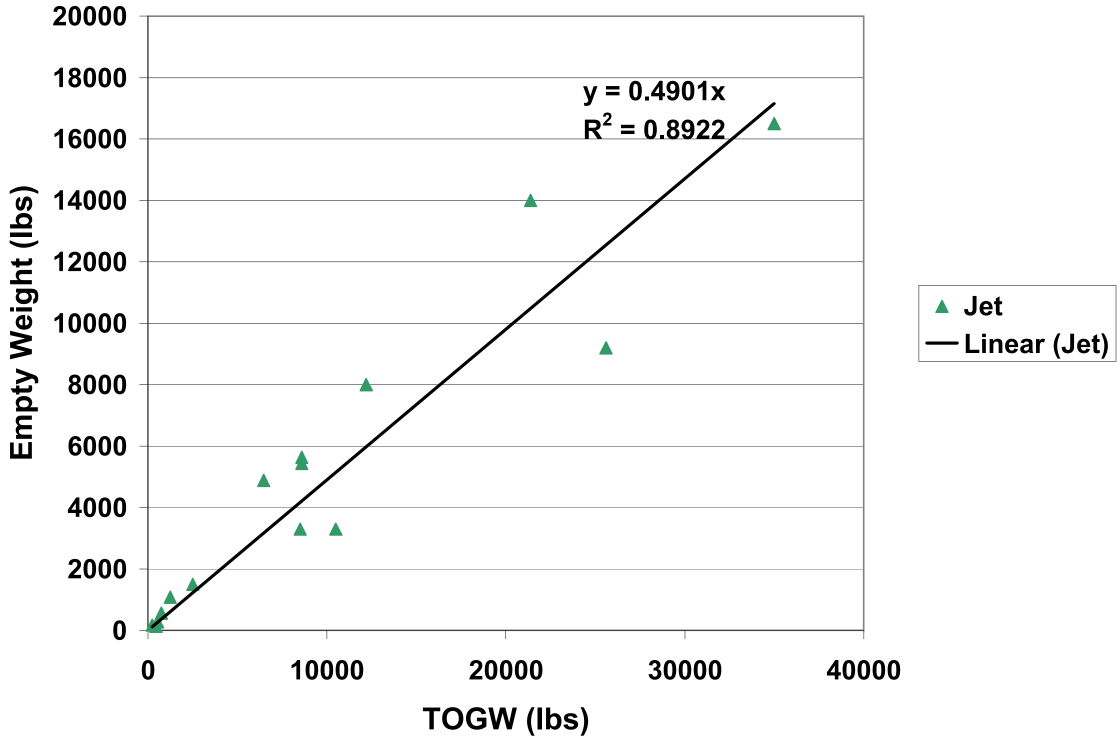

Figure 4-6: Empty Weight versus TOGW for Unmanned Aircraft (Jet)

Table 4-2: Unmanned Aircraft Empty Weight Fractions

Engine Type	**Empty Weight Fraction**
Piston	0.5878
Turboprop	0.4074
Jet	0.4901

Determining TOGW

It is uncertain what the value of the TOGW will be for this aircraft, or even the specific boundary values for the range of this variable. For this particular example problem it is assumed that the aircraft will have increased performance over the MQ-9 Predator B, which has a TOGW of 10,500 lbs, and it is also assumed that its performance is under that of conventional fighters such as the F-16 (TOGW ~35,000 lbs). [40,75] Based upon this information a range of 15,000 to 25,000 lbs for the TOGW will be assumed for this problem. Additionally, because of the estimated range of TOGW values, it is assumed that a jet engine is used.

Probability Theory Process Task 2 – Discretize the range for each uncertain variable and determine the corresponding probability for each discrete point

Determination of Probability Analysis Parameters

The uncertainty in the problem is associated with three variables:

- AMPR Weight Factor
- Cost per pound
- TOGW

All of these variables have an estimated range of values but no set distributions. In order to model these uncertainty variables with Probability Theory it is necessary to assume a distribution. Reference 93 discusses that a common practice is to assume that the uncertain variable should be modeled with a uniform distribution. However this technique adds an additional uncertain assumption to the already uncertain process. The question becomes what distributions should be used to model the uncertainty variables and how does assuming a distribution affect the final result?

To explore this problem, three different distributions were used to model the pdfs of the uncertainty variables. Because of the limited number of uncertainty variables, it is appropriate to determine the resulting design metric by using the mathematical expectation technique discussed previously in this chapter. For this example problem, the number of intervals was also considered as a variable to illustrate the affect of incorporating additional design analysis runs in the process.

This example problem was broken down into four separate test groups. The number of intervals for each uncertainty variable is constant for each test group, and each test group considers three different distributions for each of the uncertainty variables. Each of the uncertainty variables was assumed to have the same distribution. The distributions were a uniform distribution, an approximated normal distribution, and an asymmetric beta distribution. All three distributions were actually modeled with a beta distribution with different distribution parameters. The approximated normal distribution (beta distribution with parameters: α=4,β=4) was used in place of a traditional normal distribution because only a specific range was of interest and it was not desired to model the tails of the distribution. The parameters of the asymmetric beta distribution were: α=4,β=2. This distribution was used to illustrate affect of using asymmetric distribution.

Probability Theory Process Task 3 – Determine the combined Probability for each Discrete DOE run

Using Equation 4-37, the combined probability value for each DOE run is calculated.

Probability Theory Process Task 4 – Analysis

An analysis is conducted for each full factorial DOE run. The output from each run is the metric of interest. For the example problem, the aircraft acquisition cost is calculated for each of the uncertainty variable value settings dictated by the DOE.

Probability Theory Process Task 5 – Calculate the Expectation

Using the information from Task 3 and Task 4 with the equation for the expectation, Equation 4-36, the final metric value for problem can be calculated. Table 4-3 presents the test matrix for each of the test groups and the final calculated aircraft acquisition cost results. Figure 4-7 presents the final cost results graphically.

Table 4-3: Probability Theory Example Problem Test Matrix

	Type of Distribution Used	Number of runs for AMPR Weight Factor	Number of runs for Cost per Pound	Number of runs for TOGW	Aircraft Acquisition Cost
Test Group 1	Uniform	5	5	5	49.58
	Approximated Normal	5	5	5	30.77
	Beta	5	5	5	26.93
Test Group 2	Uniform	10	10	10	37.80
	Approximated Normal	10	10	10	30.28
	Beta	10	10	10	32.60
Test Group 3	Uniform	10	20	100	33.94
	Approximated Normal	10	20	100	30.27
	Beta	10	20	100	33.57
Test Group 4	Uniform	10	20	250	33.80
	Approximated Normal	10	20	250	30.27
	Beta	10	20	250	33.57

From Figure 4-7 it is evident that the distribution does affect the metric value. While the results of the uniform distribution and the asymmetric beta distribution converge as the number of intervals considered increased, the final value differs from that of the approximated normal distribution. Traditionally only one type of uncertainty distribution is used to model an uncertainty variable in a design problem and the selection of this distribution can lead to unwarranted confidence in the results if the selection is not based upon relevant statistical data.

While this technique can be very useful and generally should be used if there is enough information about the uncertain variable, there are other uncertainty modeling techniques that can be used without the application of unnecessary assumptions for situations when there is limited information about the uncertainty variable.

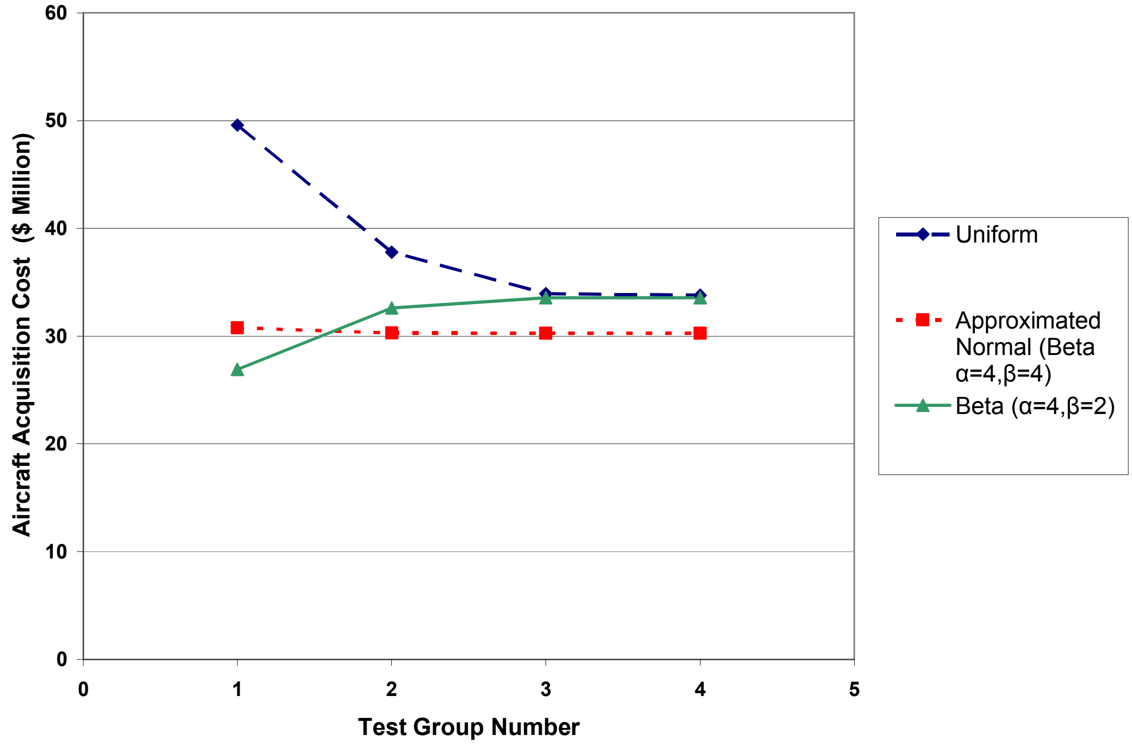

Figure 4-7: Results from Probability Theory Example

4.3. Evidence Theory

In Probability Theory the evidence pertains to only one possible event, while in Evidence Theory[11], the evidence can be related to multiple possible events, or sets of events. If it is possible to assign probabilities to a single event due to the available evidence, this theory becomes traditional Probability Theory. [170] This is why Ayyub and Klir in Reference 17 state that Probability Theory can be considered a special case of Evidence Theory. For comparison of Evidence Theory versus Probability Theory see Reference 140.

[11] Also called Dempster-Shafer Evidence Theory or Evidence Theory

4.3.1. Basic Elements of Theory

The three important functions in this theory: the Basic Probability Assignment (BPA) function, the Belief function (Bel), and the Plausibility function (Pl). [170]

The BPA is a basic assignment which is given to sets of elements, and is also referred to as the Möbius (m) representation. [140] While this function does not technically refer to the probability, some researchers have found it useful to look at it from this perspective. [170,45] The Möbius is used to map the power set to the interval from 0 to 1, and the summation of all of the subsets in the powerset of m is 1. For a set A, the Möbius or m(A) represents the evidence that supports an element of the universal set belonging to the set A. [170,109] In other words m(A) describes the amount of "likelihood" that is assigned to A. [93, 140]

In some cases it may be uncertain which set a particular element of interest belongs to due to imprecision in its boundaries. The BPA (Möbius) can be used to define the upper and lower bounds of a possible interval for the element. These bounds are called the Belief and Plausibility. [170] These functions measure the "likelihood" of a subset from the sample space. The Plausibility function and the Belief function can be thought of as the highest and lowest probability that is consistent with the available evidence. [140] The relationship between the Belief (Bel), Plausibility (Pl), and Probability(P) of a set is shown in Equation 4-43.

$$Bel(A) \leq P(A) \leq Pl(A) \qquad \textbf{Equation 4-43}$$

Because of this relationship, in the case where the Belief is equation to the Plausibility, the Probability is uniquely defined. The Belief function (Bel) for a set A, as shown in Equation 4-44, can be defined as the sum of the BPA of the subsets. In Equation 4-44 the subsets are represented by B. [170]

$$Bel(A) = \sum_{B|B \subseteq A} m(B)$$ **Equation 4-44**

The Plausibility function (Pl) for the set A is defined as the sum of the BPA of the sets that intersect set A. In Equation 4-45, B represents the sets that intersect set A. [170]

$$Pl(A) = \sum_{B|B \cap A \neq \varnothing} m(B)$$ **Equation 4-45**

Both the Belief and Plausibility measures are nonadditive, which means that neither of these measures must sum to 1. It is possible to obtain the BPA from the Belief Measure as shown in Equation 4-46.

$$m(A) = \sum_{B|B \subseteq A} (-1)^{|A-B|} Bel(B)$$ **Equation 4-46**

In this equation the term $|A - B|$ represents the difference in cardinality[12] of the two sets. Evidence Theory specifically models the types of uncertainty: confusion and conflict, ambiguity, and coarseness. [18, 170] There are two aspects of this technique that will be particularly useful for SoS design method. First, it is possible to model uncertainty when only a range of values is known but there is not enough information or data to determine an appropriate pdf. Second, it is possible to use this theory to combine information (even conflicting information) in the same analysis from different sources.

For example again consider a simple aircraft acquisition cost example.

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[12]The cardinality for a set is the number of elements in that set.

4.3.2. Evidence Theory Example Problem A

To first illustrate how this technique can be utilized the original example problem from Section 4.2.6 is simplified to only consider two uncertainty variables, cost per pound or TOGW. The full example problem will be demonstrated later in this chapter so that this theory and Probability Theory may be compared.

For Evidence Theory Example Problem A assume that the AMPR Weight Factor is 65% and that the propulsion source is a jet. Based upon Figure 4-6 this results in an empty weight fraction (We/W0) value of 0.4901.

No additional information has been identified to determine what distributions would best model the cost per pound or TOGW. Previously distributions were assumed from each of these ranges, but with Evidence Theory it is possible to model these variables without applying the assumption of a specific distribution. Instead of having a distribution for the uncertainty variables, there is a maximum and minimum value for each uncertain variable.

Evidence Theory Process Task 1: Gather information from sources

For this example there are two notional sources that provide information about the cost per pound and the TOGW.[13] Source 1 indicates that the actual value of the cost per pound of the aircraft lies in the interval [4500,4700] with a 30% confidence, or in the interval [4700,5000] with a 70% confidence. Source 2, on the other hand, has 100% confidence that the cost per pound is within the interval [4750,5000].

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[13] Note: that this information was developed for the example problem and is notional. It was not taken from literature.

Table 4-4: Cost per Pound Uncertainty Characteristics

Source	Minimum Value ($/lb)	Maximum Value ($/lb)	Confidence of Range
1	4500	4700	30%
1	4700	5000	70%
2	4750	5000	100%

Similarly, two notional sources provide information about their belief on the value of the TOGW of the aircraft. Source 1 states that the TOGW will be within [15000,18000] with 25% confidence, or that the TOGW will be within the interval [18000,23000]. Another source indicates that the value could be within [15000,17500] with 30% confidence, [17500,20000] also with 30% confidence, or [20000,25000] with 40% confidence.

Table 4-5: TOGW Uncertainty Characteristics

Source	Minimum Value (lbs)	Maximum Value (lbs)	Confidence of Range
1	15,000	18,000	25%
1	18,000	23,000	75%
2	15,000	17,500	30%
2	17,500	20,000	30%
2	20,000	25,000	40%

Evidence Theory Process Task 2: Create Basic Probability Assignments (BPA) Matrices for each uncertainty variable and for each source

The information from the sources is used to create the Basic Probability Assignment (BPA). The BPA is a function that maps the set (E) of all subsets of a universal set S to the interval [0,1] such that:

$$BPA(\varnothing) = 0 \qquad \textbf{Equation 4-47}$$

$$\sum_{E \subset S} BPA(E) = 1$$ **Equation 4-48**

Based on the interval bounds from the sources, sets of possible intervals can be determined. Each possible interval can be considered a separate set and in essence, the BPA represents likelihood of occurrence of each set (E). A convenient way to visualize these sets is by using matrixes to represent the information as shown in Table 4-6. A similar technique was first used by Luo and Caselton and is also demonstrated by Oberkampf and Helton in Reference 139.

Each column indicates the lower values from the uncertainty ranges provided by the sources. L_1 represents the value for the low value from interval 1, L_2 represents the value for the low value from interval 2, and so on. These columns are organized in increasing order, where $L_1 \leq L_2 \leq \ldots \leq L_N$. "N" is the number of subintervals or the number of low values from the original intervals. Each row is related to one of the upper values from the uncertainty ranges provided by the sources. U_1 represents the value for the upper value from interval 1, U_2 represents the value for the upper value from interval 2, etc. As with the columns the rows are also organized in increasing order (from top to bottom). For example, $U_1 \leq U_2 \leq \ldots \leq U_N$.

For every different combination of L_X and U_X, where X represents the number of the subinterval of interest, there is a resulting BPA. In other words, there is a particular likelihood of occurrence for each different combination of L_X and U_X, based upon the information provided by the sources. This is represented in Table 4-6 by BPA($[L_i, U_j]$) for the subinterval $[L_i, U_j]$.

Table 4-6: BPA Matrix for Generic Uncertainty Variable

	Low value Interval 1 (L_1)	**Low value Interval 2 (L_2)**	**Low value Interval 3 (L_3)**
Upper value Interval 1 (U_1)	BPA([L_1, U_1])	BPA([L_2, U_1])	BPA([L_3, U_1])
Upper value Interval 2 (U_2)	BPA([L_1, U_2])	BPA([L_2, U_2])	BPA([L_3, U_2])
Upper value Interval 3 (U_3)	BPA([L_1, U_3])	BPA([L_2, U_3])	BPA([L_3, U_3])

In cases where intervals had the same upper value, multiple rows would represent the same value. In this case, the duplicate rows can be condensed to one row representing this value. If multiple columns represent the same lower value, they can also be condensed into one column.

The technique described by Oberkampf and Helton in Reference 139 is similar but creates subintervals for both the columns and the rows of the matrix based upon all of the upper and lower values. This technique is equally valid, but results in the creation of additional unnecessary non-zero subintervals. Because this process only considers the intervals with non-zero BPA values, the results are the same from both techniques.

For the example problem, the BPA matrix for Source 1 for the cost per pound uncertainty is shown in Table 4-7. The BPA matrix for Source 2 for the cost per pound uncertainty is shown in Table 4-8. In this step, the BPA for each interval in the matrix is taken directly from the information provided by the notional sources.

Table 4-7: BPA Matrix for Cost Per Pound for Source 1

	4500	**4700**	**4750**
4700	0.3	0	0
5000	0	0.7	0

Table 4-8: BPA Matrix for Cost Per Pound for Source 2

	4500	**4700**	**4750**
4700	0	0	0
5000	0	0	1

The BPA matrices for the TOGW uncertainty are shown in Table 4-9 and 4-10. It is important to note that the BPA value is only assigned when there is an exact match between the lower and upper values for the interval. No overlap or inexact matches is considered in constructing these matrices. Also, note that there are a large number of sub-intervals with a BPA of 0. In future steps of this process only the non-zero intervals will be considered.

Table 4-9: BPA Matrix for TOGW for Source 1

	15,000	**17,500**	**18,000**	**20,000**
17,500	0	0	0	0
18,000	0.25	0	0	0
20,000	0	0	0	0
23,000	0	0	0.75	0
25,000	0	0	0	0

Table 4-10: BPA Matrix for TOGW for Source 2

	15,000	17,500	18,000	20,000
17,500	0.3	0	0	0
18,000	0	0	0	0
20,000	0	0.3	0	0
23,000	0	0	0	0
25,000	0	0	0	0.4

Evidence Theory Process Task 3: Create combined Basic Probability Assignments (BPA) Matrices for each uncertainty variable

To combine the available information in this problem the averaging combinatorial rule is utilized. For this example problem, all of the sources are considered equally credible and thus are equally weighted. As shown in by Equation 4-49, for set S, the combined BPA for each subinterval is the weighted average of the original BPAs from the separate sources.

$$BPA(CS_{k,j}) = \frac{1}{NS}\sum_{i=1}^{NS} w_i BPA(S_{i,j})$$ **Equation 4-49**

NS: Number of sources

NSI: Number of sub-intervals

CS: Combined set

j: 1..NSI

k: 1..Number of uncertainty variables

For a detailed comparison of additional combinatorial rules see Reference 170.

For the uncertainty related to the cost per pound, the combined BPA matrix is shown in Table 4-11. The combined BPA matrix for the TOGW uncertainty is shown in Table 4-12.

Table 4-11: Combined BPA Matrix for Cost Per Pound

	4500	**4700**	**4750**
4700	0.15	0	0
5000	0	0.35	0.5

Table 4-12: Combined BPA Matrix for TOGW

	15,000	**17,500**	**18,000**	**20,000**
17,500	0.15	0	0	0
18,000	0.125	0	0	0
20,000	0	0.15	0	0
23,000	0	0	0.375	0
25,000	0	0	0	0.2

Evidence Theory Process Task 4: Determine the Basic Probability Assignments for the Product Space

The uncertainty variables do not occur in isolation and it is necessary to determine the product space of the combination of the uncertainty variables. A simplifying assumption is that the uncertainty variables are assumed to be independent. If the variables are not independent, it will be necessary to consider the conditional probability of the uncertainty variables. This research assumes that the uncertainty variables are independent in both this example problem and throughout this document.[14]

To determine the BPA values for the entire product space a full factorial DOE can be used to combine all possible combinations of all the sub-intervals from each variable. Now there is a product space (PS) interval for each of the full factorial design runs.

The BPA for each product space interval can be determined from Equation 4-50.

$$BPA(PS_m) = \prod_{k}^{NUV} BPA(CS_{k,m})$$ **Equation 4-50**

NPSI: Number of product space intervals

NUV: Number of uncertainty variable

CS: Combined set

k: 1..Number of uncertainty variables

m:=1..Number of product space intervals

Only the intervals with non-zero BPA values are of interest in this step. For automation purposes this process can be done while considering the non-zero intervals, but if it is possible to easily identify the non zero intervals there is no need for the additional calculations. For the example problem, there are three intervals from the cost per pound BPA matrix that are non-zero and five non-zero intervals from the TOGW matrix.

The non-zero intervals for cost per pound variable are [4500,4700], [4700,5000], and [4750,5000]. The non-zero intervals for TOGW variable (lbs) are [15000,17500], [15000,18000], [17500,20000], [18000,23000], and [20000,25000].

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[14] This assumption was made because one of the main objectives of this research is to determine if it is possible to combine four different uncertainty methods (PT, ET, FST, and IGT) together. It is logical to focus on combining these theories with the core fundamentals. Additional concepts, such as dependent uncertainty variables, are opportunities for future research in this area.

The BPA is determined for the product space for every combination of the non-zero intervals. This process is shown in Table 4-13.

Table 4-13: BPA Calculation for Evidence Theory Example Problem

		Cost Per Pound ($/lb) BPAs		
		[4500,4700]	**[4700,5000]**	**[4750,5000]**
TOGW (lbs) BPAs	**[15000,17500]**	0.15 * 0.15 = 0.0225	0.35 * 0.15 = 0.0525	0.5 * 0.15 = 0.075
	[15000,18000]	0.15 * 0.125 = 0.01875	0.35 * 0.125 = 0.04375	0.5 * 0.125 = 0.0625
	[17500,20000]	0.15 * 0.15 = 0.0225	0.35 * 0.15 = 0.0525	0.5 * 0.15 = 0.075
	[18000,23000]	0.15 * 0.375 = 0.05625	0.35 * 0.375 = 0.13125	0.5 * 0.375 = 0.1875
	[20000,25000]	0.15 * 0.2 = 0.03	0.35 * 0.2 = 0.07	0.5 * 0.2 = 0.1

Evidence Theory Process Task 5: Determine the Belief and Plausibility

The BPA for each combination of intervals represents the likelihood of occurrence of that set, but does not indicate the likelihood of occurrence of any proper subset. Additionally while it is possible to determine metric values based upon the upper and lower variable values for every set, this technique does not lead to the speculation of other values. Table 4-13 lists the resulting BPAs for the product space for the example problem. Table 4-14 lists the resulting metric (cost) values for the product space. For instance in the example problem consider the set of values that is defined by cost per pound values between [4500,4700] and TOGW values within the interval [15000,17500]. The resulting values, calculated from Equations 4-40 through 4-43, from every combination of these sets is: $21.5 million, $22.5 million, $25.1 million, or $26.2 million. For this set it is possible to identify the lowest resulting aircraft acquisition cost. In other words, it is plausible that the aircraft will cost as low as $21.5 million. But, it is also possible to determine the upper lever for this set. For this set of uncertainty variable values, it is believable that the aircraft will cost $26.2 million or less. Table 4-15 identifies the minimum and maximum values for each of the sets. After considering all of the values from the sets it is possible to bound the plausible and believable values for the final metric, the aircraft acquisition cost. It is plausible that the aircraft could cost as little as $21.5 million, but not less. It is believable that the aircraft will be $39.8 million or less.

Table 4-14: Aircraft Acquisition Cost values for intervals

	[4500,4700]	[4700,5000]	[4750,5000]
[15000,17500]	21.5, 22.5, 25.1, 26.2	22.5, 23.9, 27.9, 26.2	22.7, 23.9, 26.5, 27.9
[15000,18000]	21.5, 22.5, 25.8, 27.0	22.5, 23.9, 27.0, 28.7	22.7, 23.9, 27.2, 28.7
[17500,20000]	25.1, 26.2, 28.7, 30.0	26.2, 27.9, 29.9, 31.9	26.5, 27.9, 30.3, 31.9
[18000,23000]	25.8, 27.0, 33.0, 34.4	27.0, 28.7, 34.4, 36.6	27.2, 28.7, 34.8, 36.6
[20000,25000]	28.7, 29.9, 35.8, 37.4	29.9, 31.9, 37.4, 39.8	30.3, 31.9, 37.8, 39.8

Table 4-15: Minimum and Maximum Aircraft Acquisition Cost values for intervals

	[4500,4700]	[4700,5000]	[4750,5000]
[15000,17500]	Min: 21.5 Max: 26.2	Min: 22.5 Max: 27.9	Min: 22.7 Max: 27.9
[15000,18000]	Min: 21.5 Max: 27.0	Min: 22.5 Max: 28.7	Min: 22.7 Max: 28.7
[17500,20000]	Min: 25.1 Max: 30.0	Min: 26.2 Max: 31.9	Min: 26.5 Max: 31.9
[18000,23000]	Min: 25.8 Max: 34.4	Min: 27.0 Max: 36.6	Min: 27.2 Max: 36.6
[20000,25000]	Min: 28.7 Max: 37.4	Min: 29.9 Max: 39.8	Min: 30.3 Max: 39.8

For a constraint on the design problem it is possible to calculate the Belief and Plausibility associated with failing that constraint. For the example problem, consider the constraint shown in Equation 4-51.

Aircraft Acquisition Cost <= $30 Million **Equation 4-51**

For this problem the aircraft must be $30 million or less. The question becomes how much does the designer believe that the cost will be under this value and how plausible is it for the cost to be under this value. The Belief is calculated using Equation 4-44 and the Plausibility is calculated using Equation 4-45.

Table 4-16: Numbered Subsets of Product Space

	Interval 1 Var 1	Interval 2 Var 1	Interval 3 Var 1
Interval 1 Var 2	1	2	3
Interval 2 Var 2	4	5	6
Interval 3 Var 2	7	8	9
Interval 4 Var 2	10	11	12
Interval 5 Var 2	13	14	15

$$C = \{Cost : Cost \leq constraint\}$$ **Equation 4-52**

$$Cost_j = f(E_j)$$ **Equation 4-53**

Consider the various subsets to be numbered as indicated by Table 4-16. For one of the subsets to be included in the Belief all four values from this subset must meet the constraint. For the example problem only seven of the sub intervals: 1,2,3,4,5,6, and 7, satisfy the constraint. The BPA from these sub intervals are then combined through a summation operation. As Equations 4-54 through 4-56 demonstrate for the problem, there is a 29.75% belief that the constraint will be met.

$$Bel(C) = \sum_{j \in \{j : Cost_j \subset C\}} BPA(E_j)$$ **Equation 4-54**

$Bel = 0.0225 + 0.0525 + 0.075 + 0.01875 + 0.04375 + 0.0625 + 0.0225$ **Equation 4-55**

$Bel = 0.2975$ **Equation 4-56**

The plausibility has less restrictive requirements. Instead of all four possible values satisfying the constraint, only one of the values needs to met the constraint. For the example problem all but one of the subsets meets the constraint. The subsets 1,2,3,4,5,6,7,8,9,10,11,12,13, and 14 all possibly meet the constraint. The BPA values from these subsets are then combined in a summation operation to result in the final Plausibility value. Equations 4-57 through 4-59 demonstrate this operation for the example problem. The final result is that there is a 90% plausibility of satisfying the constraint.

$$Pl(C) = \sum_{j \in \{j : Cost_j \cap C \neq \varnothing\}} BPA(E_j)$$ **Equation 4-57**

$Pl = 0.0225 + 0.0525 + 0.075 + 0.01875 + 0.04375 + 0.0625 + 0.0225 + 0.0525 + 0.075 + 0.05625 + 0.13125 + 0.1875 + 0.03 + 0.07$ **Equation 4-58**

$Pl = 0.9$ **Equation 4-59**

The Belief and Plausibility for a range of possible constraint values in shown in Figure 4-8. This chart illustrates how the Belief and Plausibility bound the problem without requiring additional assumptions to be made about the uncertainty. Now for the design problem, instead of only having the aircraft acquisition cost as the metric for comparison, there are two metrics: the Plausibility and the Belief of achieving a specific aircraft cost.

These two metrics would then be used in a Multi-Attribute Decision Making (MADM) process to evaluate alternatives.

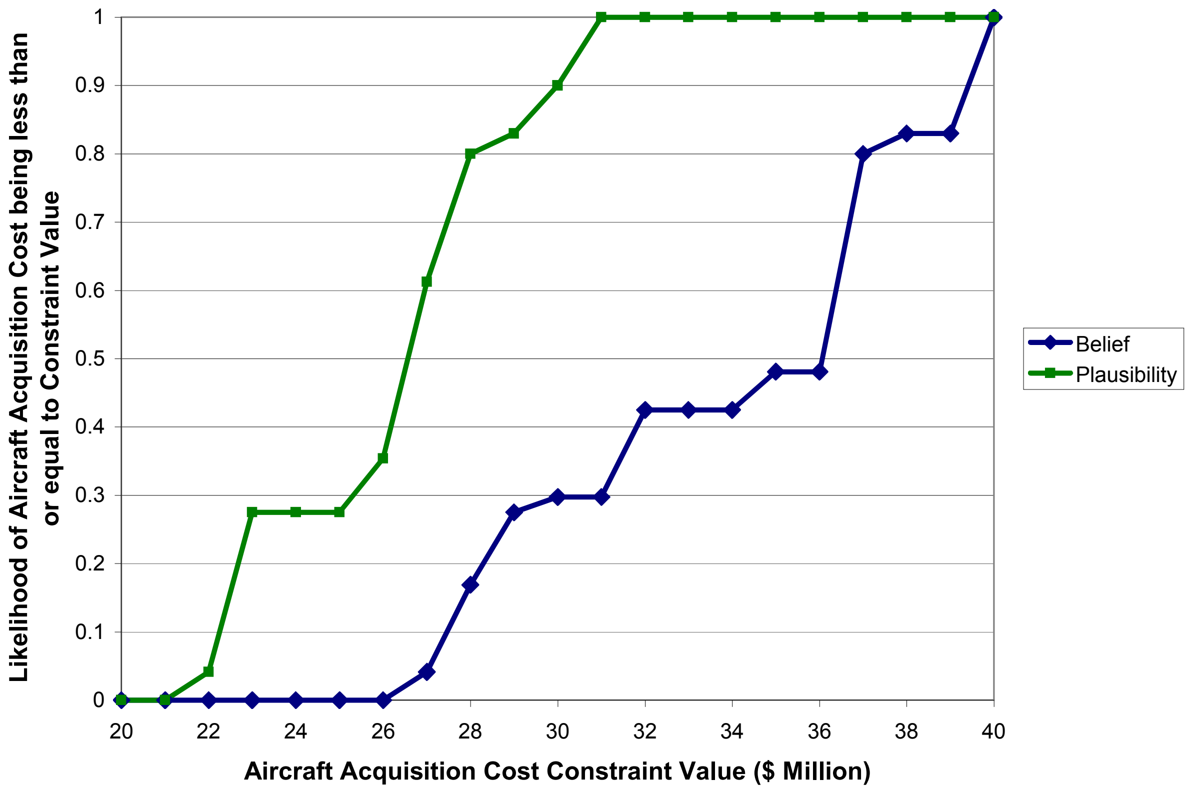

Figure 4-8: Results from Evidence Theory Example A

4.3.3. <u>Evidence Theory Example Problem B</u>

To demonstrate the one of the main differences in results between Evidence Theory and Probability Theory consider the example problem from Section 4.2.6, where there are three uncertainty variables: AMPR weight factor, cost per pound, and TOGW. The AMPR weight factor is estimated to have a range of 60-70% and its probability distribution is unknown. [153] From the discussion in Section 4.2.6, the cost per pound has a range of 4500-5000 $/lb and again its probability distribution unknown. The

TOGW, based on similar aircraft, is estimated to have a range of 15,000-25,000 lbs. As with the other two uncertainty variables, there are no statistics providing a probability distribution for the TOGW. There is only one source and one interval range for all of the uncertainty variables.

By following the five Evidence Theory process steps it is possible to: create Basic Probability Assignments (BPA) Matrices for each uncertainty variable, determine the Basic Probability Assignments for the Product Space, and finally to determine the Belief and Plausibility for the problem. The results from the analysis are shown in Figure 4-9 and are superimposed on top of the results from the analysis in Section 4.2.6.

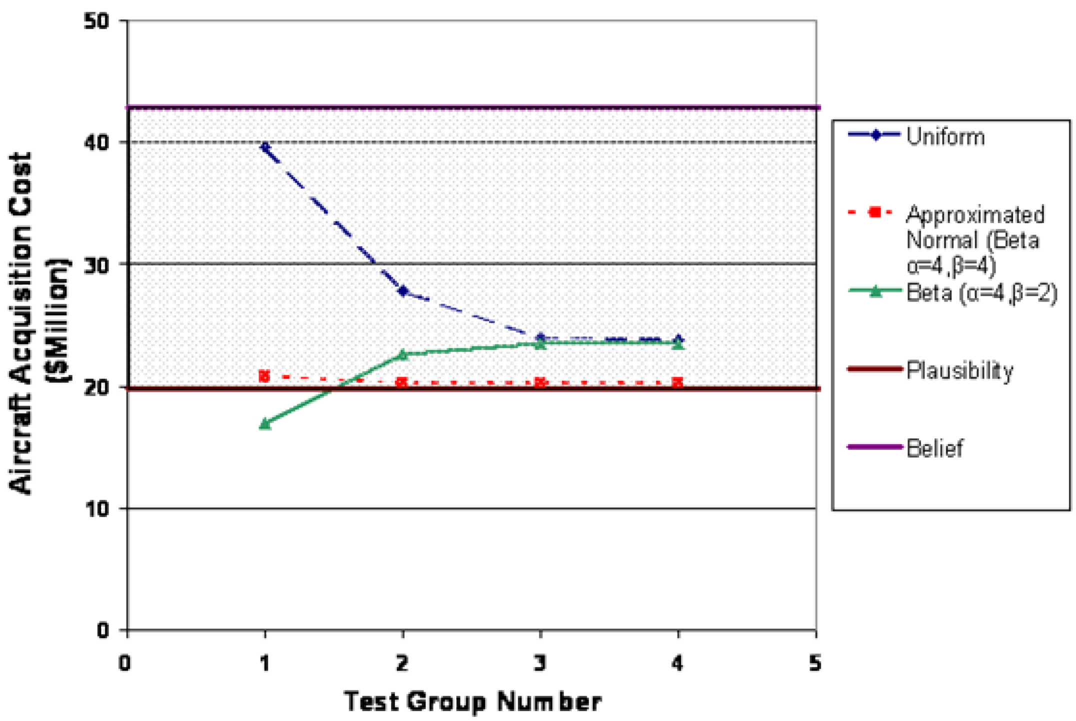

Figure 4-9: Results for Example Problem for Probability Theory and Evidence Theory

From the analysis, the Belief is calculated to be $42.9 million and the Plausibility is calculated to be $19.8 million. In effect the Belief and Plausibility provide and upper and lower bounds on the problem without having to assume a probability distribution. Within this bounded region there is no assumption as to the actual value that will occur, there is not enough information to calculate this value. By assuming a probability distribution, it is possible to calculate a final result, but without the necessary information this process gives the designer an unwarranted confidence in the final value. In a design process, it is possible to use the Belief and the Plausibility as metrics instead of the calculated metric value from a probability analysis.

If it is necessary to determine the potential values for a design alternative, then it is necessary to gain additional information about the uncertainty variable and then repeat the process.

4.4. Info-Gap Theory

Info-Gap Theory is a theory that was developed for situations when design decisions are to be made when there is a severe lack of information. For instance, this theory is particularly appropriate for design problems at the conceptual design stage when there is little to no information available about the system (or system-of-systems) being designed. Info-Gap Theory was originally developed to provide a tool for the decision making community that could overcome some of the limitations of Probability Theory. [25] Info-Gap Theory was developed for a class of problems where very limited amounts of information are available, and there is not enough information to create probability distributions or membership functions (as in fuzzy logic) for uncertain parameters. As stated in Reference 25, Info-Gap theory quantifies uncertainty “as the gap between what is known and what could be known.” Info-Gap Theory emphasizes the epistemic type of uncertainty as opposed to Probability Theory, which is more appropriate for aleatory uncertainty. [25]

Because of the lack of information for an associated design problem, this technique models uncertainty very differently than Probability Theory or Evidence Theory. No traditional measure functions such as probability density functions or fuzzy set membership functions are used. Instead Info-Gap Theory uses two new parameters as design metrics called the Robustness Function and the Opportunity Function. The Robustness Function, α, describes the greatest amount of uncertainty when a constraint is always met. The Opportunity Function, β, represents the smallest amount of uncertainty where it is possible to meet the desired value of a design metric. By utilizing these two functions, α and β, it is possible to account for the pernicious or propitious aspects of relevant uncertainty in the design problem. [25]

The focus is now on the Robustness Function and the Opportunity Function instead of traditional design metrics. For instance consider the aircraft acquisition cost example that has been used throughout this chapter. Instead of determining the variation of the traditional design metric, the aircraft cost, the focus with Info-Gap Theory now becomes how much variation can be present in the uncertainty variables (AMPR weight factor, cost per pound, and TOGW) before either a constraint will be reached or before it is possible to achieve some value of success.

To determine the Robustness Function (α) and the Opportunity Function (β), Info-Gap Theory utilizes uncertainty parameters $\hat{\alpha}$ and $\hat{\beta}$.[15] The uncertainty parameter $\hat{\alpha}$ represents the variation in the values of uncertain variable for every possible scenario before a design constraint is failed. The Robustness Function (α) is then the maximum

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[15] Ben-Haim in Reference 25 actually reverses the notation where the robustness function is designated by $\hat{\alpha}$ and the uncertainty parameter is α. Additionally Ben-Haim only uses one uncertainty parameter, α instead of $\hat{\alpha}$ and $\hat{\beta}$.

value of $\hat{\alpha}$ where the constraint is always satisfied. This represents the "degree of resistance to uncertainty and immunity against failure" for the design concept. The uncertainty parameter $\hat{\beta}$ represents the variation in the values of uncertain variable for every scenario where it is possible that a desirement (desired value signifying success) is met. The Opportunity Function is then the minimum value of β where it is possible to succeed.[16]

α = max{ $\hat{\alpha}$: constraint always satisfied} **Equation 4-58**

β = min{ $\hat{\beta}$: desirement can be achieved} **Equation 4-59**

Within a design problem the uncertainty parameters $\hat{\alpha}$ and $\hat{\beta}$ are unknown and hence "the horizon of uncertain variation is unbounded". [25] One possible technique for determining these uncertainty parameters is by selecting an Info-Gap model as described in Reference 25 to describe the characteristics of the uncertainty parameter. The Info-Gap model is selected based upon available information about the uncertainty. However it is also possible to determine the uncertainty parameters from modeling and simulation thereby eliminating the need to select an Info-Gap model. The uncertainty parameters $\hat{\alpha}$ and $\hat{\beta}$ can be directly found from a simulated model of the design problem. This technique is emphasized and utilized throughout this research. It was chosen to use the modeling and simulation technique over the traditional Info-Gap model technique

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[16] The letters α, β which represent the robustness function and the opportunity function, respectively, have no relation to the beta distribution parameters which are also defined by α, β. If α, β are in reference to a beta distribution this will be explicitly explained in the text.

because the complexity of most system-of-systems design problems makes it difficult to capture all of the critical interactions with a simple model.

4.4.1. Info-Gap Process with Modeling and Simulations

The objective of this process is to determine the Robustness and Opportunity Functions for each design alternative. These metrics can then be used in a Multi-Attribute Decision Making (MADM) process to evaluate different design alternatives and ultimately select the most robust and opportunistic design. For a detailed discussion over the differences between robust design, opportunistic design, and robust and opportunistic design see Chapter 8.

The following tasks can be utilized to determine these functions:

- Identify constraints and desirements
- Identify uncertainty variables
- Identify the nominal value and potential ranges for the uncertain variables
- Select an appropriate model for the design problem
- Determine the difference in values of the constraint and the desirement from the nominal value in terms of the uncertain variables
- Determine the opportunity and robustness function
- Identify the alternative with the largest Robustness Function and the smallest Opportunity Function

This process is only a part of an overall design problem. For example, this process is part of Step 8 in the method described in Chapter 10. This technique itself does not determine the different design alternatives to be compared, and nor does it do the final comparison. This technique is for one specific alternative and therefore the design variables are constant throughout this design process. The only variables are those relating to the uncertainty.

To demonstrate this technique, consider the aircraft cost acquisition problem discussed previously in this chapter.

Info-Gap Process Task 1: Identify constraints and desirements

The constraints and desirements are set by stakeholders or derived from economic or technical requirements. Constraints and desirements do not have to be constant values. It is possible for these parameters to be functions of the uncertainty or the design variables within the design problem.

For this persistent strike aircraft the overall objective is to develop an aircraft with the lowest acquisition cost that meets specific performance requirements. The design has been constrained such that an aircraft that is $30 million or more is not acceptable. While the overall objective is to minimize the cost, it is desired for the aircraft to have an acquisition cost of $10 Million or less.

Info-Gap Process Task 2: Identify uncertainty variables

This step is where the designer considers the design problem and determines the relevant uncertainty variables based upon existing literature, experimental data, or expert opinion. The available information about the variables is considered. If the variable falls into one of the following types of uncertainty: randomness, inaccuracy, ambiguity, coarseness, and simplification, and if no information pertaining to distributions or ranges is available, then it is acceptable to model the variables using Info-Gap Theory.

The uncertainty in the problem is associated with three variables: AMPR weight factor, cost per pound ($/lb), and the TOGW. For this problem, very little is known. There are no known distributions for these variables and the associated ranges are also unknown.

Info-Gap Process Task 3: Identify the nominal value and potential ranges for the uncertain variables

While no information is available about the uncertain variables, it is necessary to have a starting point for application purposes. The nominal, or expected, value of the uncertainty variable needs to be identified based on literature, experimental data, or expert opinion. Additionally, a range of possible values for the uncertainty variables also needs to be determined. This range should be the maximum and minimum expected values for the uncertainty variables based upon literature, experimental data, or expert opinion.

AMPR Weight Factor

- Nominal Value = 65%
- Minimum = 55%
- Maximum = 75%

Cost per pound

- Nominal Value = 4700 $/lb
- Minimum = 4000 $/lb
- Maximum = 5500 $/lb

TOGW

- Nominal Value= 20,000 lb
- Minimum = 10,000 lb
- Maximum = 30,000 lb

Info-Gap Process Task 4: Select an appropriate model for the design problem

Different potential models should be considered and evaluated. The model to be utilized in modeling the problem needs to be capable of modeling all of the relevant uncertainty. The model for the example problem is Equations: 4-40 through 4-42.

Info-Gap Process Task 5: Determine the difference in values of the constraint and the desirement from the nominal value

The challenge underlying this step is that it is unknown how far the constraint or desirement is from the nominal value (in terms of the uncertainty variable). The objective is to determine how much the uncertainty variable can vary before the constraint is violated or the desirement is achieved. To determine this distance, first consider the value of the nominal value. Utilizing Equations 4-40 through 4-42 and the nominal values for the AMPR weight factor, cost per pound, and the TOGW, the aircraft acquisition cost for the uncertainty variables is $29.9 Million. This value meets the constraint but it does not satisfy the desirement. This information indicates which side of the desirement and the constraint that the nominal value is located. For instance consider the notional charts shown in Figure 4-10. These charts show that in essence the potential variation of the uncertainty variables, represented by the values on the x axis, represents a different dimension that must be considered. These charts show how in each of these “dimensions” the nominal value meets the constraint but fail the desirement. These charts are also used to emphasize the fact that the distance from the nominal value to the constraint (or desirement) differs depending upon the uncertainty variable under consideration.

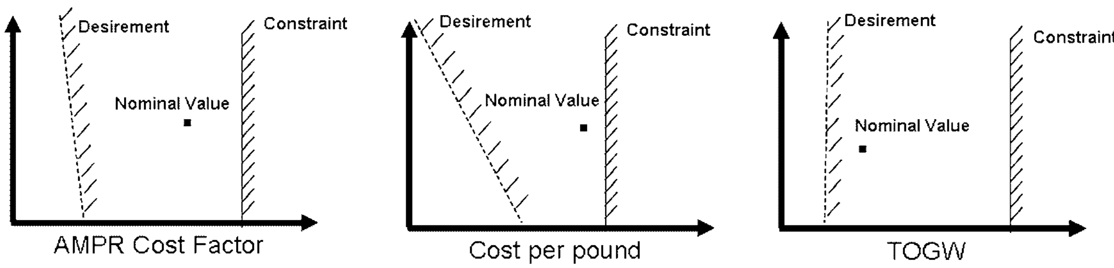

Figure 4-10: Notional Illustration of Design Space with respect the Uncertainty Variables

Once the "location" of the nominal value for each uncertainty variable has been determined, the task becomes to determine to location of the constraint and the desirement thereby determining the distance from the nominal value to the constraint or desirement.

The distance from the nominal value to the metric constraint is the Robustness Function (α). This objective is to maximize this value so that the expected value is as far from the constraint as possible. The distance from the nominal value to the metric desirement is called the Opportunity Function (β). The objective is to minimize the Opportunity Function so that the expected value is as close to possibly achieving success as possible. Unlike other uncertainty modeling theories, the resulting design metrics of interest, the Opportunity Function and the Robustness Function are in terms of the uncertainty variable. The larger the value of the function the larger the variation in the uncertainty variable that is possible before the constraint or desirement is reached.

In many cases it is difficult to estimate the location of the constraint and the desirement. This can be accomplished by running a set of cases in the modeling and simulation environment. The set of cases is determined by dividing the range of the maximum and minimum possible values for each uncertainty variable into subintervals. A design analysis, in this case Equations 4-40, 4-41, and 4-42 will be run for each sub-interval.

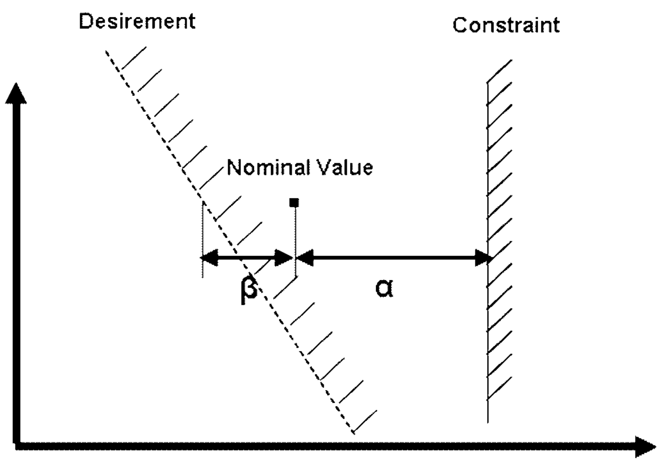

Figure 4-11: Illustration α and β Relationship to Constraint and Desirement

This concept is illustrated in Figure 4-12. The horizontal blue line represents the range of possible values and each tic mark indicates the interval value.

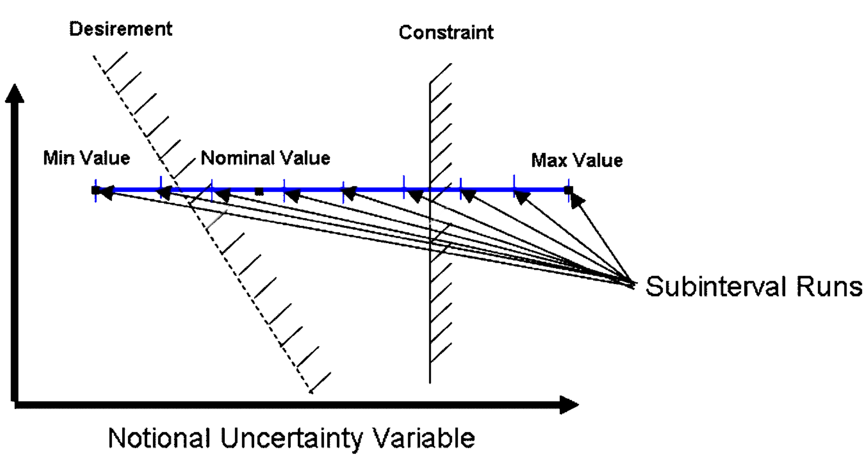

Figure 4-12: Illustration of Info-Gap runs

The number of intervals to divide the range into depends upon the maximum computational resources and time available for the analysis. For the example problem the uncertainty variable ranges have been divided into 5 intervals. This interval number was selected because it was appropriately large enough to introduce the interval concept while still allowing the reader to easily reproduce the results if desired.

AMPR weight factor (%) Interval Runs: 55, 60, 65, 70 & 75

Cost per pound ($/lb) Range: 4000, 4375, 4750, 5125, & 5500

TOGW (lbs) Range: 10000, 15000, 20000, 25000, & 30000

To evaluate all possible combinations of these variables and their interval values a full factorial design of experiments (DOE) is created. [35]

The model will be run for each DOE case. Figure 4-13 illustrates how each of the interval values is associated with a separate DOE case. The horizontal blue line represents the range of possible values and each tic mark indicates the interval value. However, this figure only represents one dimension of the problem. Every interval value is compared with every other possible interval values from the additional uncertainty variable dimensions.

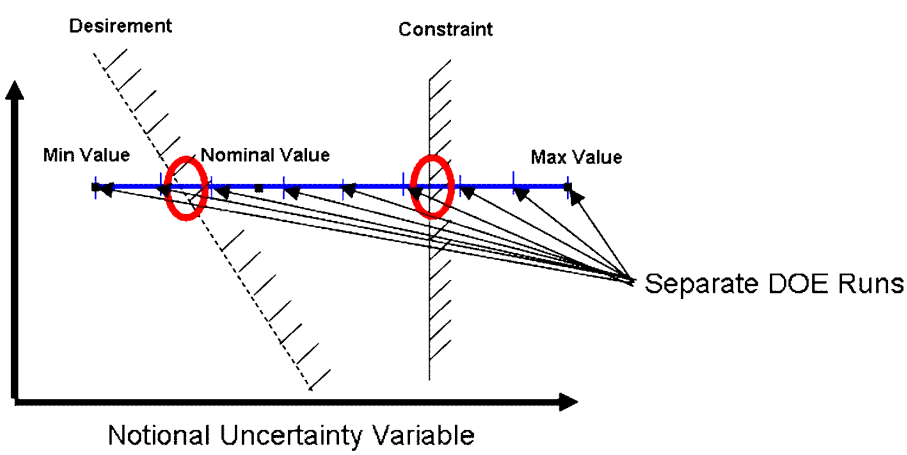

Figure 4-13: Illustration of Info-Gap Full Factorial DOE runs

The entire purpose behind running the DOE is to determine the approximate locations of the constraint and the desirement with respect to the nominal value of each uncertainty variable so that the Robustness and Opportunity Functions can be determined.

To determine the location of the constraint, for each DOE run, determine if the resulting metric violates the constraint or meets the desirement. It is possible to determine where the constraint or desirement is located by identifying when the metric switches from meeting the constraint to failing the constraint, or when the metric switches from failing to meet the desirement to satisfying the desirement. This “switch” identifies that the boundary line of interest is located between the current uncertainty variable value and the uncertainty variable value from the previous analysis.

For example consider the notional scenario illustrated in Figure 4-14. For each interval run the potential α is identified. For run 1 the potential value is A1, for A2 the potential value is A2, etc. For the scenario in the figure, runs 4, 5, and 6 fail to meet the constraint. The final value of α for this scenario is A3 because of two reasons. First, the actual location of the constraint is only known to be between the values of A3 and A4 from the nominal value. Second because the objective is to maximize the value of α, the final value of α for this scenario is A3.

The resulting α values for the example problem are presented in Appendix A. A sample of the data is presented in Table 4-17. The data has been sorted so that the constraint values listed in column 1 are increasing. The problem has been set up so that a negative constraint value means that the constraint has been violated. The negative values have been highlighted in blue. The row in the table where the constraint value changes from negative to positive indicates that the constraint is located between the values of the two bounding rows.

The nominal values for the uncertainty variables are as follows: AMPR weight factor: 65%, cost per pound: 4700 $/lb and TOGW: 20000 lb. The values listed in columns 2-4 of Table 4-17 are the actual values of each uncertainty variable used for the particular run. The values listed in columns 5-7 are the potential values of α, or in other words, these are the values of A1, A2, etc.

Table 4-17: Sample Data from Info-Gap Analysis

	Actual Values			Potential α Values		
Constraint ($Million)	AMPR Weight Factor (%)	Cost($)/lb	TOGW (lbs)	AMPR Weight Factor (%)	Cost($)/lb	TOGW (lbs)
-2.01	55	4750	25000	10	50	5000
-1.86	65	4000	25000	0	700	5000
-0.32	75	5500	15000	10	800	5000
-0.26	65	4750	20000	0	50	0
-0.14	60	5125	20000	5	425	0
-0.02	70	4375	20000	5	325	0
0.35	55	5500	20000	10	800	0
0.52	55	4375	25000	10	325	5000
0.59	75	4000	20000	10	700	0
0.59	60	4000	25000	5	700	5000
1.70	70	5500	15000	5	800	5000
1.74	75	5125	15000	10	425	5000

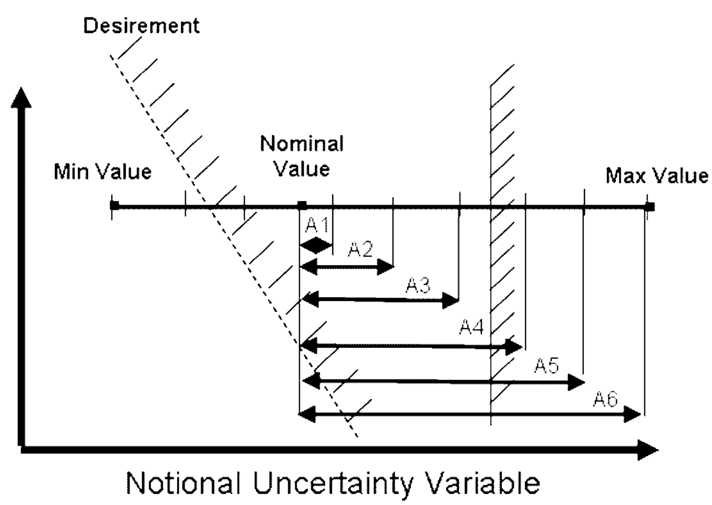

Figure 4-14: Illustration of Potential α Estimates

The value of β is determined from a similar process. Again for each interval run a potential β is identified. For the scenario in Figure 4-15 the potential β values are: B1, B2, and B3. For this scenario, runs associates with B2 and B3 meet the desirement, which indicates that the desirement is located between B1 and B2 with respect to the nominal value of the uncertainty variable. The objective is to minimize the Opportunity Function, therefore the final value of β is B2.

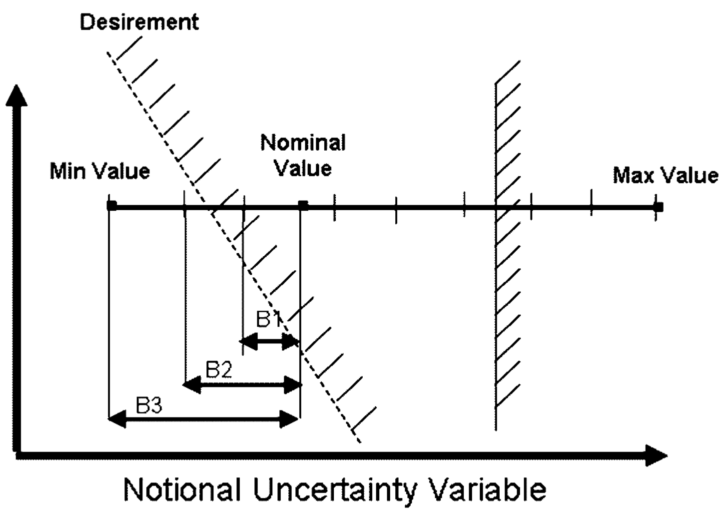

Figure 4-15: Illustration of Potential β Estimates

The value of α and β were selected to be conservative. For situations where it is necessary to use a minimum number of intervals to identify the location of the desirement and constraint, due to the computational limitations, the selected value of α and β are not misrepresented. The nominal value is located, at minimum, a distance of α from the constraint, and the nominal value is located, at most, a distance of β from the desirement. If it is desired to have a more accurate and less conservative estimate for the location of the desirement and constraint, more intervals should be used.

For some scenarios the constraint or the desirement will never be reached by using the uncertainty variable values from the maximum expected range. In general, this means that either the designer was not aware of the actual range that is possible for the uncertainty variable or that the desirement or the constraint will never be reached. However, it is necessary to have a α and β for every design alternative for comparison. For situations where no α and β were able to be located, it is possible to extrapolate their location by modeling the data as a line through the minimum resulting metric value and the maximum resulting metric value. This line can be extrapolated to the estimated location of the constraint and desirement. This concept is illustrated in Figure 4-16.

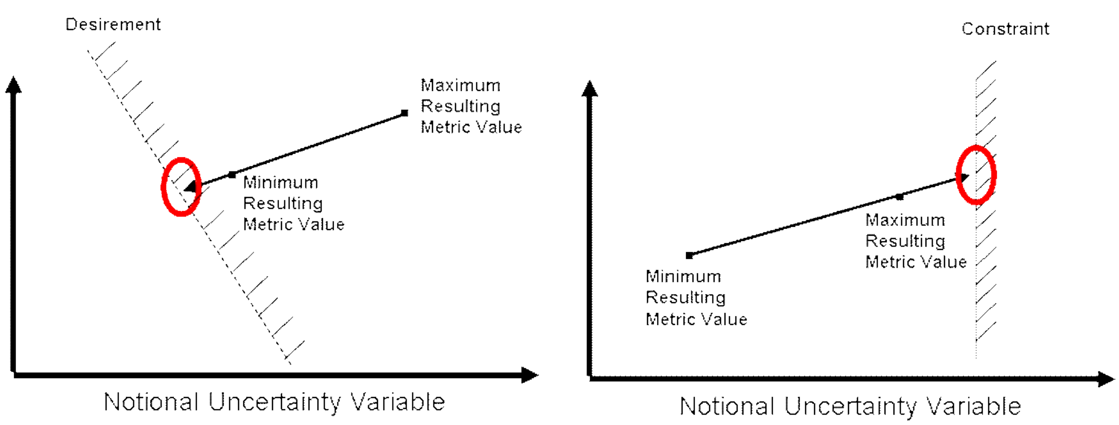

Figure 4-16: Illustration Illustrating Extrapolation Technique for Estimating α and β

The value can be used directly or an additional penalty or bonus can be applied to the value if desired to emphasize the fact that the constraint or desirement was never located within the original range of uncertainty values. For example consider Equations 4-60 and 4-61 that represent options for the case when the constraint is never reached. For this scenario, the designer may want to incorporate a bonus function to indicate that for the given alternative the constraint is never failed. In the equations, $\tilde{x}_v$ represents the nominal value of the v^th uncertainty variable and $\hat{x}_{c,v}$ represents the extrapolated location of constraint c in the dimension of the v^th uncertainty variable. Equation 4-60 provides the equation for α with no bonus function using the traditional definition for α. Equation 4-61 is the equation for the α value with the bonus function. The bonus function is adapted from a typical penalty function as described in Reference 198. The bonus is based upon the distance from the maximum metric value of the uncertainty variable range to the extrapolated location of the constraint. Another option would be to base the penalty or weight on the distance from the nominal value to the extrapolated location of the constraint. While this second option would likely require one less calculation, since the distance from the maximum metric value to the extrapolated constraint is not calculated; however, this option may over penalize or reward the function.

$$\alpha_{c,v} = \left|\hat{x}_{c,v} - \tilde{x}_v\right| \qquad \textbf{Equation 4-60}$$

$$\alpha_{c,v} = \left|\hat{x}_{c,v} - \tilde{x}_v\right| + w_c \cdot \left(\left|\hat{x}_{c,v} - \tilde{x}_{v,\max}\right|\right)^P \qquad \textbf{Equation 4-61}$$

c is for each metric constraint

v is for each Info-Gap uncertainty variable

~ indicates nominal value

w_c and P are penalty/bonus weights

A similar exercise as shown in Equations 4-62 and 4-63 could be done to penalize a β for never reaching the desirement. Since the objective is to minimize the Opportunity Function, increasing the value of this function results in a penalty. Assuming the objective is to minimize the original design metric, the penalty is based upon the distance from the minimum metric value of the uncertainty variable range to the extrapolated location of the constraint.

$$\beta_{d,v} = \left|\hat{x}_{d,v} - \tilde{x}_v\right| \qquad \textbf{Equation 4-62}$$

$$\beta_{d,v} = \left|\hat{x}_{d,v} - \tilde{x}_v\right| + w_d \cdot \left(\left|\hat{x}_{d,v} - \tilde{x}_{v,\min}\right|\right)^P \qquad \textbf{Equation 4-63}$$

d is for each metric constraint

v is for each Info-Gap uncertainty variable

~ indicates nominal value

w_d and P are bonus weights

For the example problem, the desirement was never reached within the estimated maximum potential range for the uncertainty variables. The location of the desirement is estimated by creating a line between the value with the maximum success value and the minimum success value for each of the uncertainty variables.

$$Slope_{AMPRWeightFactor} = (55-75)/(0.78-50.65) = 0.40 \qquad \textbf{Equation 4-64}$$

$$\beta_{AMPRWeightFactor} = 10.31 \qquad \textbf{Equation 4-65}$$

$$Slope_{CostperPound} = 30.1 \qquad \textbf{Equation 4-66}$$

$$\beta_{CostperPound} = 723.5 \qquad \textbf{Equation 4-67}$$

$Slope_{TOGW} = 401.1$ **Equation 4-68**

$\beta_{TOGW} = 10313.7$ **Equation 4-69**

Info-Gap Process Task 6: Determine the normalized value of α and β

The next step is to normalize all of the α and β values. The Robustness Function should be normalized based on the value of the maximum and minimum α value for each uncertainty variable.[17] Similarly the Opportunity Function should be normalized based upon the value of the maximum and minimum β value for each uncertainty variable.[18] Table 4-18 lists the normalized values for the α values.

<u>β for AMPR Weight Factor (and Aircraft Cost Metric)</u>

The value of β would typically be normalized by the maximum and minimum possible β values for the entire problem. For this problem, since there is only one design alternative being considered, there is only one possible β value so the resulting normalized value is 50 for all three of the uncertain variables.

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[17] Note: this includes any α that has been penalized or rewarded by the constraint being located outside of the expected range.

[18] Note: this includes any β that has been penalized or rewarded by the desirement being located outside of the expected range.

Table 4-18: Sample Data from Info-Gap Process with Normalized α

	Potential α Values			Normalized α Values		
Constraint ($Million)	AMPR Weight Factor (%)	AMPR Weight Factor (%)	AMPR Weight Factor (%)	AMPR Weight Factor (%)	Cost($)/lb	TOGW (lbs)
-2.01	10	10	10	100	0.00	50
-1.86	0	0	0	0	86.67	50
-0.32	10	10	10	100	100.00	50
-0.26	0	0	0	0	0.00	0
-0.14	5	5	5	50	50.00	0
-0.02	5	5	5	50	36.67	0
0.35	10	10	10	100	100.00	0
0.52	10	10	10	100	36.67	50
0.59	10	10	10	100	86.67	0
0.59	5	5	5	50	86.67	50
1.70	5	5	5	50	100.00	50
1.74	10	10	10	100	50.00	50

Info-Gap Process Task 7: Apply appropriate penalty or bonus functions

For cases when the nominal value violates the constraint, as shown in Figure 4-17, it is likely that the constraint will fail most, if not all, of the cases. In this situation it is appropriate to apply a penalty function such as shown in Equations 4-70 and 4-71. The penalty function is based on a typical penalty function as described in Reference 198.

$$\alpha_{c,v} = \left| x_{c,v} - \tilde{x}_v \right|$$ **Equation 4-70**

$$\alpha_{c,v} = -\left| x_{c,v} - \tilde{x}_v \right| - r_c \cdot \left(\left| x_{c,v} - \tilde{x}_v \right| \right)^P$$ **Equation 4-71**

c is for each metric constraint

v is for each Info-Gap uncertainty variable

~ indicates nominal value

r_c and P are penalty weights

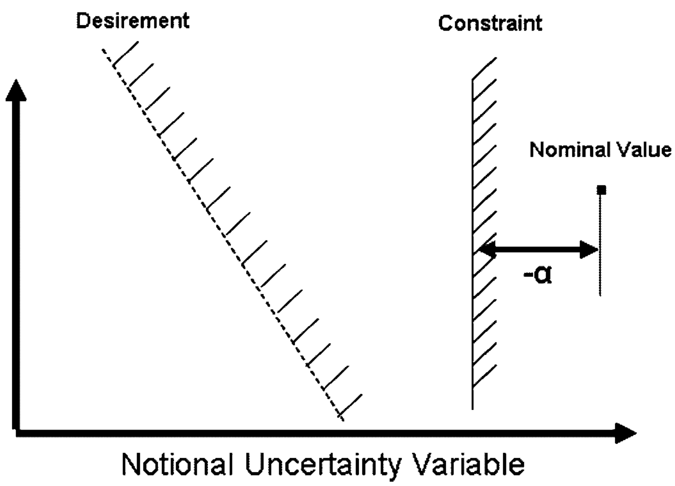

Figure 4-17: Illustration of α when Constraint Failed

However, in many situations the distance between the nominal value and the constraint may be such a small distance that the penalty is very minor. For cases where it is acceptable for the constraint to be slightly violated, this type of penalty function will be satisfactory. However for situations where the constraint should not be violated, even by a small amount, it is suggested to incorporate an additional penalty factor (PF) to the penalty function. This penalty factor as shown in Equation 4-72 applies a set penalty for any constraint violation. Equation 4-73 provides an option for a general penalty factor where PF% is a penalty factor percentage provided by the designer.

$$\alpha_{c,v} = -\left|x_{c,v} - \tilde{x}_v\right| - PF - r_c \cdot \left(\left|x_{c,v} - \tilde{x}_v\right|\right)^P$$ **Equation 4-72**

$$PF = \left|\alpha_{\max} - \alpha_{\min}\right| \cdot PF\%$$ **Equation 4-73**

For the opposite situation when the nominal value satisfies the desirement, it is highly likely that most, if not all, of the cases will be considered successful. A notional illustration of this situation is provided in Figure 4-18. For this case it is appropriate to apply a bonus function. This function as shown in Equation 4-71 is based upon the penalty function described in Reference 198. It is also possible to apply a bonus factor, in a similar manner as done for the penalty factor in Equation 4-72, if it is desired for an alternative to be heavily rewarded if the success value is met.

$$\beta_{s,v} = \left|x_{s,v} - \tilde{x}_v\right| \qquad \textbf{Equation 4-74}$$

$$\beta_{s,v} = -\left|x_{s,v} - \tilde{x}_v\right| - r_s \cdot \left(\left|x_{s,v} - \tilde{x}_v\right|\right)^P \qquad \textbf{Equation 4-75}$$

s is for each metric desirement

v is for each Info-Gap uncertainty variable

~ indicates nominal value

r_s and P are bonus weights

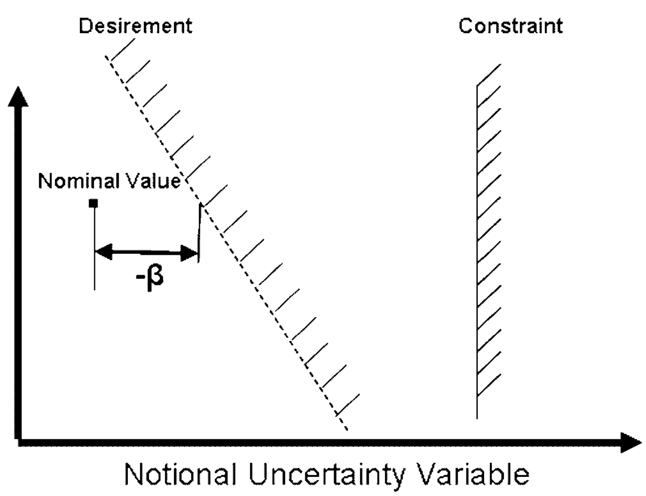

Figure 4-18: Illustration of β when Desirement satisfied

Info-Gap Process Task 8: Create a combined Robustness Function and Opportunity Function for each metric

At this stage in the design process there is a α and β for every traditional metric (Ex: aircraft cost) *and* for every uncertainty variable (Ex: AMPR weight factor, cost per pound, TOGW). For the example problem with one traditional metric (aircraft cost) there are now 6 design metrics ($\alpha_{AircraftCost,AMPRCostFactor}$, $\alpha_{AircraftCost,CostperPound}$, $\alpha_{AircraftCost,TOGW}$, $\beta_{AircraftCost,AMPRCostFactor}$, $\beta_{AircraftCost,CostperPound}$, $\beta_{AircraftCost,TOGW}$). For large design problems this number will exponentially grow and quickly become prohibitively cumbersome.

To counter this issue, an option is to create a combined α and β for each of the original metrics. Now for the example problem there would only be two metrics ($\alpha_{combined}$ and $\beta_{combined}$) instead of six. The combined α and β values are created by determining the n-dimensional Euclidean distance based upon the various α and β for each metric.

$$\alpha_c* = \sqrt{\sum_{v=1}^{n}\left(\alpha_{c,v,norm}\right)^2}$$ **Equation 4-76**

$$\beta_s* = \sqrt{\sum_{v=1}^{n}\left(\beta_{s,v,norm}\right)^2}$$ **Equation 4-77**

c is for each metric constraint

s is for each metric desirement

v is for each Info-Gap uncertainty variable

norm indicates that the original alpha and beta values have been normalized

n is the number of Info-Gap uncertainty variables

Results of Info-Gap Example Problem

$\alpha_c* = 141.42$ **Equation 4-78**

$\beta_s* = 86.6$ **Equation 4-79**

While the robustness and opportunity functions can be very useful design tools, the combined and normalized α and β values no longer have any physical meaning. For this reason, it is difficult for a designer to look at the α_c* or β_s* for one alternative individually and understand how this relates to the original design metric. However, none the less, Info-Gap Theory can be an extremely useful technique for situations where there is very little information available about the uncertainty.

4.5. Fuzzy Set Theory

In many cases, Probability Theory and Evidence Theory can model the same type of uncertainty. The appropriate uncertainty modeling theory is then based upon the level of knowledge about the uncertainty variable. But there are other types of uncertainty such as vagueness and coarseness that are better modeled by using a theory such as Fuzzy Set Theory.

Fuzzy Set Theory models a different kind of uncertainty. Instead of modeling a variables likelihood of achieving a specific value, it instead indicates to which group a variable or alternative belongs.

Fuzzy Set Theory is based upon the use of fuzzy sets. In contrast to the use of classical sets, where there is a sharp boundary which determines if a value is included within the set or not, a fuzzy set has a fuzzy boundary that considers possible membership in the set as options. [203] A function called a membership function is used to determine to what degree the value is expected to belong to the set. [203]

There are a number of different types of membership functions. Yen and Langari in Reference 203 states that the most common membership functions are: the triangle, the trapezoid, the bell curve, the Gaussian, the signoidal, and the S membership functions. For example, following the notation provided in Reference 203, the triangle membership function is defined by three parameters: {a,b,c}. [203]

$$triangle(x:a,b,c) = \begin{cases} 0, x < a \\ \dfrac{(x-a)}{(b-a)}, a \le x \le b \\ \dfrac{(c-x)}{(c-b)}, b \le x \le c \\ 0, x > c \end{cases}$$

Equation 4-80

The trapezoidal function is defined by four parameters: {a,b,c,d}. [203]

$$trapezoidal(x:a,b,c,d) = \begin{cases} 0, x < a \\ \dfrac{(x-a)}{(b-a)}, a \le x < b \\ 1, b \le x < c \\ \dfrac{(d-x)}{(d-c)}, b \le x \le c \\ 0, x \ge d \end{cases}$$

Equation 4-81

The bell-shaped membership function, which is a generalization of the Cauchy distribution, is defined by three parameters: {a,b,c}. [203]

$$bell(x:a,b,c) = \frac{1}{1+\left|\dfrac{x-c}{a}\right|^{2b}}$$

Equation 4-82

Another common membership function is the Gaussian function, which is defined by two parameters: {m,σ}.[19] [203]

$$Gaussian(x:m,\sigma) = e^{\left(\frac{-(x-m)^2}{\sigma}\right)}$$ **Equation 4-83**

The sigmoidal membership function is specified by the parameters: {a,b}. [203]

$$Sigmoidal(x:a,c) = \frac{1}{1+e^{-a(x-b)}}$$ **Equation 4-84**

The S membership function also is defined by two parameters: {a,b}. [203]

$$S(x:a,b) = \begin{cases} 0, x < a \\ 2\left(\frac{x-a}{b-a}\right)^2, a \le x < \frac{a+b}{2} \\ 1-2\left(\frac{x-b}{b-a}\right)^2, \frac{a+b}{2} \le x < b \\ 1, x \ge b \end{cases}$$ **Equation 4-85**

For a particular value, x, these membership functions can be used to determine the degree of membership of this value. For example consider that there is the possibility of three types of engines being used as propulsion sources for an unmanned persistent strike aircraft: piston, turboprop, and jet.

[19] Note: often the first parameter is designated by 'μ' instead of 'm'. However, the membership function itself is commonly designated by the symbol 'μ', so to avoid confusion 'm' is used for this function.

While typically there are a number of factors that are involved in the selection of an engine type for an aircraft, the problem has been simplified to where the engine type is selected purely based upon the relationship between TOGW and propulsion source. Based upon historical data from Reference 40 for unmanned aircraft the relationship between the aircraft's TOGW and the propulsion source can be specified using a membership function for each of the three propulsion sources. The membership functions, designated by μ, are specified in Equations 4-86 through 4-97 and are shown in Figure 4-19.

$TOGW <= 2500$ lbs **Equation 4-86**

$\mu_{Piston} = -0.000267*TOGW+0.66675$ **Equation 4-87**

$\mu_{Turboprop} = 0.000196*TOGW+0.20275$ **Equation 4-88**

$\mu_{Jet} = 0.000071*TOGW+0.1305$ **Equation 4-89**

2500 lbs $<=$ TOGW $<=$ 12,250 lbs **Equation 4-90**

$\mu_{Piston} = 0$ **Equation 4-91**

$\mu_{Turboprop} = -0.000071*TOGW+0.8695$ **Equation 4-92**

$\mu_{Jet} = 0.000071*TOGW+0.1305$ **Equation 4-93**

TOGW $>=$ 12,250lbs **Equation 4-94**

$\mu_{Piston} = 0$ **Equation 4-95**

$\mu_{Turboprop} = 0$ **Equation 4-96**

$\mu_{Jet} = 1$ **Equation 4-97**

For example if the TOGW is 8,000 lb:

$\mu_{Piston} = 0$ **Equation 4-98**

$\mu_{Turboprop} = 0.3015$ **Equation 4-99**

$\mu_{Jet} = 0.6985$ **Equation 4-100**

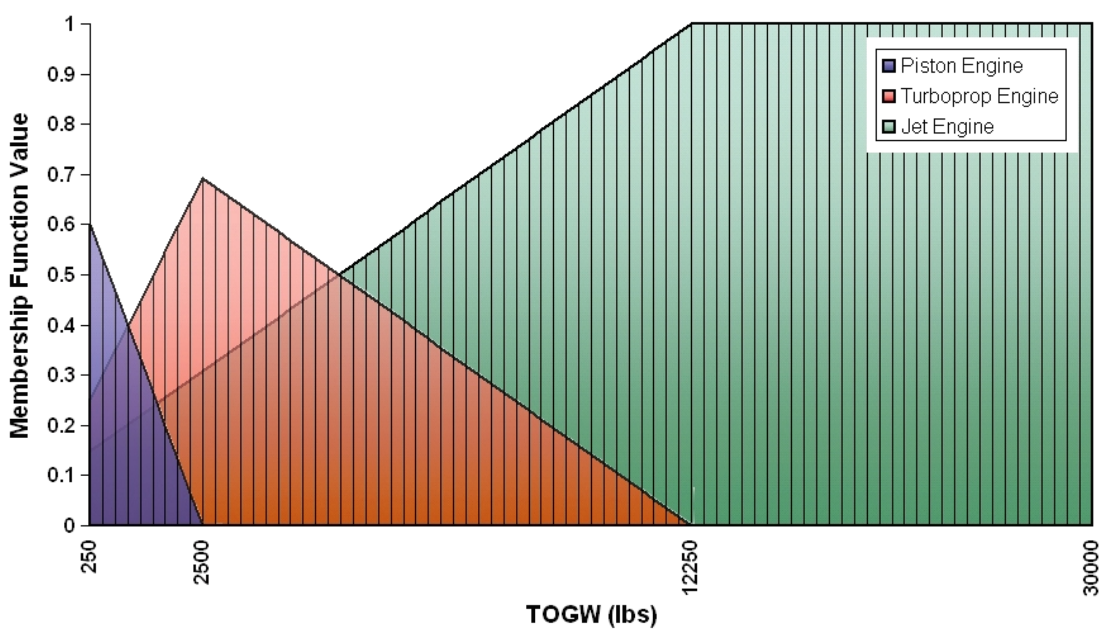

Figure 4-19: Fuzzy Set Membership Functions for Simplified Aircraft Acquisition Cost Example Problem

If it is estimated that the TOGW is to be 10,000lbs then according to the membership functions it could either be powered by a Turboprop or a Jet engine. Considering it is not certain which engine will be used at this time, both options must be considered. However if the aircraft TOGW is 2,000 lbs then all three propulsion sources are possible.

To demonstrate the use of membership functions in calculating the expected aircraft cost, consider a situation where the AMPR weight factor is 65%, the cost per pound is 4750 $/lb, and the empty weight of the aircraft is calculated from the empty weight fraction and the TOGW. The empty weight fraction differs based upon the propulsion source used. An empty weight fraction has been determined for each of the engine types based upon historical data from Reference 40 and was determined based upon linear regressions as shown in earlier in this chapter.

Consider that the TOGW is 8,000 lbs. For a Piston the Aircraft Acquisition Cost is $14.5 million and is calculated in Equations 4-101 through 4-103.

$$ACCost_{Piston} = Costperpound \cdot AMPR\_Weight\_Factor \cdot \left(\frac{W_e}{W_0} \cdot TOGW \right)$$ **Equation 4-101**

$$ACCost_{Piston} = 4750 \cdot 65\% \cdot (0.5878 \cdot 8000)$$ **Equation 4-102**

$$ACCost_{Piston} = \$14{,}500{,}000$$ **Equation 4-103**

For a Turboprob the Aircraft Acquisition Cost is estimated to be $10.1 million and is determined in Equations 4-104 through 4-106.

$$ACCost_{TP} = Costperpound \cdot AMPR\_Weight\_Factor \cdot \left(\frac{W_e}{W_0} \cdot TOGW \right)$$ **Equation 4-104**

$$ACCost_{TP} = 4750 \cdot 65\% \cdot (0.4074 \cdot 8000)$$ **Equation 4-105**

$$ACCost_{TP} = \$10{,}100{,}000$$ **Equation 4-106**

Finally, for a Jet the Aircraft Acquisition Cost is calculated with Equations 4-107 through 4-109 and is found to be $12.1 million.

$$ACCost_{Jet} = Costperpound \cdot AMPR\_Weight\_Factor \cdot \left(\frac{W_e}{W_0} \cdot TOGW \right)$$ **Equation 4-107**

$$ACCost_{Jet} = 4750 \cdot 65\% \cdot (0.4901 \cdot 8000)$$ **Equation 4-108**

$$ACCost_{Jet} = \$12{,}100{,}000$$ **Equation 4-109**

The aircraft acquisition cost can now be estimated for the problem by taking the product of the estimated aircraft acquisition cost for each of the types of propulsion sources and the respective membership value. This process is shown in Equations 4-110 through 4-112 for the example problem.

$$ACCost_{Combined} = \sum_{i}^{O} \mu_i \cdot ACCost_i$$ **Equation 4-110**

$$ACCost_{Combined} = \mu_{Piston} \cdot ACCost_{Piston} + \mu_{TP} \cdot ACCost_{TP} + \mu_{Jet} \cdot ACCost_{Jet}$$ **Equation 4-110**

$ACCost_{Combined} =$ **Equation 4-111**

$$0 \cdot \$14{,}500{,}000 + 0.3015 \cdot \$10{,}100{,}000 + 0.6985 \cdot \$12{,}100{,}000$$

$ACCost_{Combined} = \$11{,}500{,}000$ **Equation 4-112**

For the above equations, "O" is the number of fuzzy set options. The final value for the aircraft acquisition cost is estimated to be $11.5 million.

In the example problem note that values for the original uncertainty variables: AMPR weight factor, cost per pound, and TOGW were all given as inputs to the problem. This is necessary because, Fuzzy Set Theory does not model the same type of uncertainty as can be modeled by the other uncertainty modeling theories previously discussed in this chapter. It instead handles the case where there are multiple options that need to be considered in the problem and the likelihood of a value being in a particular option set. For this reason this theory can compliment the other theories by modeling coarseness and vagueness in the problem.

CHAPTER 5: DESIGN OF SOS

For the persistent strike battlespace design problem, and other SoS design problems, it was proposed in Chapter 2 that to effectively consider the interactions and potential emergent behaviors associated systems cannot be considered in isolation and that they must be developed as part of an integrated SoS design process. Additionally, as discussed in Chapter 3, because of the inherent complexity and lack of knowledge about the system in the conceptual design stage, it is critical that the SoS uncertainty is integrated into the design process. Chapter 4 briefly described a range of different techniques that can be used in a design method to quantify the uncertainty.

The first objective of this chapter is to use the information from the previous chapters to identify requirements for a conceptual SoS design process. The second part of this chapter discusses existing robust system design methods and SoS design methods. These methods are compared against the proposed requirements and existing capability gaps are identified in Chapter 6.

5.1. Requirements for a System-of-Systems Conceptual Design Method

There are certain elements which are required in any design method: processes for defining the problem, identifying the design decisions, analyzing and evaluating the relevant information, and making decisions. With this in mind one of the first requirements for this method should be a:

1. **Process for defining system elements, functions, variables, requirements, objectives, and constraints**

In addition to identifying system design parameters, there are a number of requirements that just apply to the development of a SoS. For instance, due to the interrelatedness of SoS, subsystems cannot be modeled in isolation. This results in the second requirement. The method must include a:

2. **Process for considering the design from a hierarchical SoS perspective**

Many SoS involve subsystems that were not developed by the designer. In these instances the job of the designer becomes determining which systems to incorporate into the overall system and how to include them. For this reason it is necessary that a SoS design method should also be a:

3. **Process that is capable of designing a FoS**

Due to the complexity of the problem, the method must also involve a:

4. **Process that is capable of modeling independent elements**
5. **Process that is capable of modeling a variety of potential scenarios**
6. **Process for designing system with multiple objectives**
7. **Process that is capable of identifying and modeling potential emergent behaviors of the SoS and its subsystems**

Because uncertainty is so significant in both the development and operation of SoS, a design method for such as system must include:

8. **Process for modeling SoS uncertainty**
9. **Process for propagating uncertainty through problem**

10. Process for identifying robust and opportunistic solutions

While considering uncertainty provides the designer with additional knowledge to aid in the decision making process, it also requires additional resources and time. It is important to identify when in a design process additional information is needed and when a decision should be made without expending additional resources. This is why the methodology should include the following two processes:

11. Process for determining the value of reducing uncertainty before a final decision is made

12. Process for evaluating and selecting designs under uncertainty

Considering these requirements, it is now possible to evaluate existing design methods to identify useful techniques and current capability gaps.

5.2. Robust Systems Design Methods

There are a number of methods for the design of a robust system. This section briefly reviews a representative sample of the existing methods.

5.2.1. Taguchi Robust Design Method

One of the most well known robust design method is the Taguchi Robust Design Method. [186, 187] It has been used for a variety of problems in both the engineering industry and academia. In this method values for design variables (control factors) are identified to meet design requirements despite variation in the noise factors (uncontrollable factors). [44] Because this method requires that the variation in noise factors be quantified numerically, it is traditionally used more for detail design stages than conceptual design processes. [44]

5.2.2. Suh's Axiomatic Design

Purpose of this method is to select a robust design concept for the conceptual stage of the design process from a set of available candidates. [44] This technique was developed by Suh as described in Reference 185 and uses two axioms to select the concept. The first axiom, the Independence Axiom, maintains the independence of function requirements and can be used to select the best design configuration from the available candidates. The Information Axiom is then used to evaluate the quality of the designs to aid in the selection process. [44]

5.2.3. Robust Concept Exploration Method (RCEM)

As described in Reference 44, RCEM is "a systematic approach for finding a ranged set of design specifications that produce robust performance in variations of noise and control factors by integrating statistical experiments, approximate models, robust design techniques, multidisciplinary analyses, and multiobjective decisions." It was specifically developed to address the deficiencies of the Taguchi and Suh's methods. Variations of this method have also been developed called: Robust Concept Exploration Method with Design Capability Indices (RCEM-DCI) and Robust Concept Exploration Method with Error Margin Index (RCEM-EMI). [44, 42, 43] RCEM-DCI uses Design Capability Indices (DCIs) as metrics for the robustness and system performance to make the design process more efficient and to improve the overall understanding of the design concept. RCEM-EMI uses Error Margin Indices (EMIs) to specify how reliable a system is in meeting the problem constraints. [44]

5.2.4. Robust Optimization Incorporating Worst Case Uncertainty Propagation

Gu et al. in Reference 84 describes a method which was specifically designed to reduce the computation load of analyses in multi-disciplinary design problems. It trades accuracy in the results for a reduction in the required computational resources by only considering the "worst case" design points under uncertainty instead of considering the entire uncertain design space. This technique includes the variability of the input variables as well as errors from any bias in the numerical procedure. The overall objective of this method is to provide a procedure for the propagation of the uncertainty through the multi-disciplinary design analysis process. [44]

5.2.5. System Uncertainty Analysis (SUA) and Concurrent Subsystem Uncertainty Analysis (CSSUA)

As with the previous method, these combined techniques involve the quantification of propagated uncertainty for multi-disciplinary design problems. [68] The process for determining the variability in design variables is similar to the approach used in the Worst Case Uncertainty Propagation method, but instead of using a "worst case" scenario approach, this method uses a First Order Reliability Method (FORM). This results in this approach being less conservative than the method proposed by Gu and his coauthors. [44,84] However, as discussed in Reference 44, this method can result in inaccurate values for the variables that are passed between the subsystems, which then leads to inaccurate overall system outputs. [44]

5.2.6. Inductive Design Exploration Method (IDEM)

This method proposed by Choi in Reference 44 was developed to identify feasible design spaces in a hierarchical top-down manner while maintaining as much design freedom as possible. Choi defines design freedom as "the ratio of the feasible ranges versus the entire design space". The design spaces of the overall hierarchical environment are selected using an inductive design exploration process and by evaluating the degree of achieved robustness with the model structure uncertainty of the involved subsystems. [44] The hierarchical top-down approach can be very useful in the design of a SoS; however, this method does not include an adequate process for identifying or quantifying SoS uncertainty.

5.2.7. <u>Joint Probabilistic Decision Making (JPDM)</u>

This method which is described in Reference 21 is a probabilistic multi-criteria decision making technique that was developed specifically for conceptual and preliminary aerospace systems design. [21] It was specifically designed to address the problem of identifying appropriate evaluation criteria for a system concept where there are multiple objectives. Instead of summing the criteria for all of the objectives, as is done in techniques such as the Overall Evaluation Criterion (OEC) technique, this method captures the probability of satisfying all of the criteria simultaneously. This probability is then used to select or optimize the system concept and can be used as both a Multi-Attribute Decision Making (MADM) tool and a Multi-Objective Decision Making (MODM) tool. [21] While this method alone cannot be used to design a SoS, it does provide useful techniques which can be implemented into the conceptual design process.

5.3. System-of-Systems Design Methods

A much smaller set of methods has been developed specifically for System-of-Systems design problems. This section reviews two representative methods that are used to design this class of system. Additional methods have been proposed, such as those by Biltgen in

Reference 28 and Fritz in Reference 80, but are not as applicable to the general conceptual design process of a SoS.

5.3.1. <u>PrOabilistic System-of-Systems Effectiveness Methodology (POSSEM)</u>

This methodology developed by Soban in Reference 179 is one of the first design processes to be developed specifically for a SoS in the aerospace industry. It discusses the creation of an integrated modeling environment incorporating models from the engineering level up to the campaign level (OES Level). It also requires the development of a full probabilistic environment where uncertainty pertaining to selected input variables throughout the entire SoS design problem is evaluated. This environment allows the user to explore the impacts of various inputs on the measures of effectiveness from one level of the SoS to the next. [179]

5.3.2. <u>Top-Down, Hierarchical, System-of-Systems Design</u>

This methodology by Ender from Reference 71 uses a Monte Carlo based design space exploration technique in conjunction with surrogate models to design a SoS from a “bottom-up” or a “top-down” perspective. The bottom-up approach encompasses the traditional multi-disciplinary optimization where inputs and outputs from different codes or disciplines are linked together in a hierarchical manner. Often subsystems must be designed first before the main system can be developed because initially there is not enough information about the subsystems to make design decisions about the overall system. While codes and inputs/outputs from different disciplines usually need to be performed or analyzed in a specific and hierarchical order, the “top-down” approach suggested by Ender circumvents this issue by first creating surrogate models of the design space and then explores this design space through a Monte Carlo design space exploration technique. [71]

Top-down technique is very useful for understanding SoS design problems. Both this methodology and the one proposed by Soban focus upon designing for variation within the system. Additional techniques will be needed to account for epistemic uncertainty. As discussed in Chapter 3, a significant portion of the uncertainty for the conceptual design of a SoS is epistemic which means that additional techniques and processes will be required.

CHAPTER 6: TECHNICAL CHALLENGES, RESEARCH QUESTIONS, AND HYPOTHESES

In Chapter 1, a number of technical gaps were observed to exist in existing conceptual design methods for SoS.

- Existing SoS Design Methods are incapable of modeling all of the different types of relevant uncertainty
- Existing SoS Design Methods do not specifically address the fact that there can be propitious effects from uncertainty as well as pernicious.
- Existing SoS Design Methods focus on identifying the most effective design alternative with respect to the relevant uncertainty. However, no method focuses on determining if the uncertainty should be reduced before making the final design decision.

These gaps result from various related technical challenges, and the purpose of this research is to address these challenges. In order to do this, a number of research questions and hypotheses are proposed to address the identified challenges. The primary research questions to be specificly addressed by the new techniques presented in this document are also identified. This chapter summarizes the identified technical challenges, relevant research questions, and resulting hypotheses.

6.1. Observation / Technical Gap A: Existing SoS Design Methods are incapable of modeling all of the different types of relevant uncertainty

Challenge A.1: There are a number of disparate types of uncertainty which involve different types of ignorance.

Challenge A.2: No existing uncertainty modeling technique is capable of incorporating all of the existing types of uncertainty

Challenge A.3: There are multiple levels of knowledge/uncertainty that must be considered in a design process

Based on these challenges there are a number of resulting research questions:

Research Question A.1: Is there a combination of uncertainty modeling theories / techniques that can model all of the different types of uncertainty?

If all seven of the different factors of uncertainty are considered at once, there are over 26,800 possible ways to model the uncertainty in this problem. This is because, as discussed in Chapter 4, each of the different types of uncertainty can be modeled with several different types of theories. Based upon this information a possible matrix of alternatives for selecting a potential uncertainty modeling theory is shown in Figure 6-1.

Uncertainty Factor	Potential Theory							
Randomness and Sampling	Probability	Statistics	Bayesian	Info-Gap				
Confusion and Conflict	Possibility	Evidence	Interval Probabilities	Interval Analysis				
Inaccuracy	Probability	Possibility	Interval Probabilities	Interval Analysis	Info-gap			
Ambiguity	Classical Sets	Probability	Statistics	Bayesian	Evidence	Interval Probabilities	Interval Analysis	Info-gap
Vagueness	Fuzzy Sets	Possibility	Interval Probabilities					
Coarseness	Fuzzy Sets	Rough Sets						
Simplification	Probability	Bayesian	Fuzzy Sets	Rough Sets	Interval Probabilities	Interval Analysis	Info-gap	

Figure 6-1: Uncertainty Modeling Matrix of Alternatives

Research Question A.2: Is there a combination of uncertainty modeling theories / techniques that can model different levels of knowledge?

As discussed in Chapter 4, Info-Gap Theory, Evidence Theory, and Probability Theory are each capable of modeling a particular level of knowledge. A technique that incorporates all three theories will be capable of modeling a range of levels of knowledge.

Research Question A.3: What set of theories/techniques, if somehow combined, could be used to model all of the different types of uncertainty for varying levels of knowledge?

As illustrated in Figure 6-2, and based on the discussions from Chapter 4, if the following four theories are combined the hybrid technique would be capable of modeling all of the different uncertainty factors at varying levels of knowledge: Probability Theory, Fuzzy Set Theory, Evidence Theory, and Info-Gap Theory.

Theory and Methodologies	Types of Uncertainty						
	Aleatory	Epistemic					
	Randomness	Confusion and Conflict	Inaccuracy	Ambiguity	Vagueness	Coarseness	Simplification
Classical Sets	Poor	Poor	Poor	Good	Fair	Poor	Fair
Probability	Good	Fair	Good	Good	Fair	Fair	Good
Statistics	Good	Fair	Fair	Good	Fair	Fair	Fair
Bayesian	Good	Poor	Fair	Good	Fair	Fair	Good
Fuzzy Sets	Fair	Fair	Fair	Fair	Good	Good	Good
Rough Sets	Poor	Poor	Fair	Fair	Fair	Good	Good
Possibility	Fair	Good	Good	Fair	Good	Fair	Fair
Evidence	Fair	Good	Fair	Good	Fair	Fair	Fair
Interval probabilities	Fair	Good	Good	Good	Good	Fair	Good
Interval analysis	Fair	Good	Good	Good	Fair	Fair	Good
Info-gap	Good	Fair	Good	Good	Fair	Fair	Good

Poor Fair Good

Figure 6-2: Appropriateness of a Theory in Modeling different Uncertainty Factors

Research Question A.4 (Primary Research Question 1): Is it possible to create a hybrid uncertainty modeling technique that can combine Probability Theory, Evidence Theory, Info-Gap Theory, and Fuzzy Set Theory?

Hypothesis 1: A hybrid uncertainty modeling technique effectively combining Probability Theory, Evidence Theory, Info-Gap Theory, and Fuzzy Set Theory can be created by utilizing a full factorial DOE to model all of the possible uncertainty combinations and by transferring relevant information about the uncertainty between theories.

6.2. Observation / Technical Gap B: Existing SoS Design Methods do not specifically address the fact that there can be propitious effects from uncertainty as well as pernicious.

The following challenges are based on this identified gap.

Challenge B.1: There are two sides to uncertainty

Challenge B.2: Existing design methods and techniques typically only address the negative side of uncertainty

Research Question B.1 (Primary Research Question 2): Is there any benefit to considering both the pernicious and propitious qualities of uncertainty in a design process?

The original hypothesis in response to this research question was that there was a benefit to considering both the negative and positive characteristics of uncertainty. It was thought that by including additional information in the design process that a better and more effective solution would be the result. However as discussed in Chapter 8, when there is a purely complementary relationship between the constraint and the desirement there is no benefit to considering both sides of the uncertainty. While a design method that considers both aspects to uncertainty will identify the “best” solution, with respect to the relevant uncertainty, this solution would also be obtained by only considering one of the aspects of the uncertainty.

To address this knowledge, the original hypothesis was modified to become as follows:

Hypothesis 2: For design problems characterized by competing constraint and desirement relationships, there is a benefit to considering both the pernicious and propitious qualities of uncertainty in a design process.

Research Question B.2 (Primary Research Question 3): How can both of the pernicious and propitious qualities of uncertainty be incorporated in a design process?

Two potential options were identified for incorporating both the positive and the negative aspects of the uncertainty in the design process. The first technique involves the careful setup of constraints and metrics so that a design will be penalized, in an analysis that considers the uncertainty, if a constraint is violated but also such that the design will be rewarded the closer it gets to the ideal solution. This technique is demonstrated in Reference 188.

The second option is by loosely reinterpreting the concept of the Robustness Function (α) and Opportunity function (β) from Info-Gap Theory.

Hypothesis 3: The pernicious and propitious qualities of uncertainty can be incorporated in a design process by maximizing the Robustness Function (α), defined as the expected difference between the value of the design metric and the respective constraint, and minimizing the Opportunity Function (β), defined as the expected difference between the value of the design metric and the respective desirement, for a given design alternative.

6.3. Observation / Technical Gap C: Existing SoS Design Methods focus on identifying the most effective design alternative with respect to the relevant uncertainty. But, none of these methods focus on determining if the uncertainty should be reduced before making the final design decision.

Challenge C.1: Typically, a design method will have already incorporated all available information in the design process. The uncertainty cannot be reduced without incurring some cost associated with this process.
Challenge C.2: The actual benefit to reducing the uncertainty and cost associated with reducing the uncertainty are unknown.

Research Question C.1: When should a design decision for a SoS be made with the available information and when is it more beneficial (cost effective) to reduce the uncertainty first?

The answer to this research question is discussed in Chapter 9. To summarize, a design decision should be made with existing information if the expected cost to reduce the uncertainty is greater than the expected benefit associated with reducing the uncertainty. The expected cost is estimated by the Expected Cost to Reduce Uncertainty (ECRU) and the expected benefit is estimated by the Expected Value of Reducing Uncertainty (EVRU).

Research Question C.2 (Primary Research Question 4): Is it possible, with the available information and analysis tools, to estimate the benefit associated with reducing the relevant uncertainty in the design process before a final design decision is made?

Hypothesis 4: The benefit associated with reducing the relevant uncertainty in the design process before a decision is finalized can be estimated by comparing the Expected Value of Reducing Uncertainty (EVRU) with the Expected Cost to Reduce Uncertainty (ECRU).

CHAPTER 7: HYBRID UNCERTAINTY MODELING METHOD

It is evident that there is a need for a technique that can successfully combine multiple uncertainty modeling techniques together for design problems with different types of uncertainty and different levels of knowledge. A number of hybrid techniques exist today that have been developed to handle a variety of different types of problems; however no existing technique can model: randomness, confusion/conflict, inaccuracy, ambiguity, vagueness, coarseness, and simplification simultaneously. As discussed in Chapter 4 Probability Theory, Evidence Theory, Fuzzy Set Theory, and Info-Gap Theory all can model some of these different types of uncertainty for varying levels of knowledge. This chapter discusses a technique specifically developed to combine the main principles of these theories together so that the different types of uncertainty that may be present in a design problem can be considered without incorporating unnecessary assumptions while still including all relevant information in the design process.

7.1. Technique Requirements

After reviewing the different types of uncertainty and different levels of knowledge that are possible within a SoS design problem (Chapter 3) and determining how different uncertainty modeling theories account for the different types of uncertainty (Chapter 4) it is possible to identify requirements for a Hybrid Uncertainty Modeling Method (HUMM) that is capable of modeling uncertainty for a SoS conceptual design problem.

First, there is a need for a technique that can model all of the different types of uncertainty, and second, there is a need for a technique that can model different levels of knowledge from the situation where the uncertainty is very well defined to the situation when very little information is known about the uncertainty. Additionally the technique needs to be capable of incorporating all available knowledge and minimizing the use of assumptions about the uncertainty within the problem. The approach should also consider both the positive and the negative aspects of uncertainty. This requirement will both lead toward a solution that is not overly conservative and toward the possibility of success while under uncertainty. Finally, the technique needs to be flexible to where only the applicable techniques are utilized within the process. If a problem only involves one type of uncertainty and one level of knowledge, then in all likelihood there is no need to use more than one uncertainty modeling theory.

To summarize, the requirements for a Hybrid Uncertainty Modeling Method (HUMM) are:

- Capable of modeling all types of uncertainty (randomness, confusion/conflict, inaccuracy, ambiguity, vagueness, coarseness, and simplification)
- Capable of modeling different levels of knowledge
- Capable of incorporating all available information
- Minimize use of assumptions about uncertainty variables and the system
- Capable of considering both the positive and negative aspects of uncertainty
- Capable of being modular so that only the applicable techniques/uncertainty modeling theories are used

7.2. Existing Hybrid Techniques

Most of the hybrid techniques in the literature involve combining some aspect of Probability Theory with another theory. For instance, Singpurwalla and Booker in Reference 174 discuss how fuzzy sets can be integrated into the basic framework of Probability Theory. Pedrycz in Reference 145 briefly discusses probability-based extensions of fuzzy sets. Guyonnet, et al. in Reference 86 proposes a technique that combines a Monte Carlo analysis with fuzzy calculus.

Oberkampf, et al in Reference 138 develop a new modeling and simulation framework that accounts for variability, uncertainty, and error (all which can be considered different types of uncertainty). In this framework they discuss how the difference between the different types of uncertainty results in a need for multiple mathematical representations for the uncertainty including Probability Theory, Evidence Theory, Possibility Theory, Fuzzy Set Theory, and Imprecise Probability Theory. Another reference that discusses the hybridization of Evidence Theory is Reference 141. Oblow in Reference 141 discusses Operator-Belief Theory, or O-Theory, which links Fuzzy Set Theory to Evidence Theory.

There are also several techniques linking Info-Gap Theory with various probabilistic techniques. Ben-Haim in Reference 25 discusses how Info-Gap Theory can be combined in a Poisson Process. He also discusses how it is possible to combine probability density functions into an Info-Gap model. Another possibility is to model the uncertainty parameters, as discussed in Chapter 4, with a probability distribution. [25]

While each of these techniques is capable of modeling different types of uncertainty, none of the existing hybrid techniques is capable of modeling all of the types of uncertainty for a varying range of levels of knowledge. The new modeling and simulation framework discussed in Reference 141 does discuss incorporating relevant uncertainty modeling techniques into the process, but it does not go into specifics for how to accomplish this for most of the techniques suggested and it does not discuss the additional complexity of considering different levels of knowledge.

7.3. Development of New Hybrid Technique

As briefly discussed in Chapter 4 all types of uncertainty can be modeled if a technique can be developed that will combine Probability Theory, Evidence Theory, and Fuzzy Set Theory. [18] Probability Theory can be considered a specific case of Evidence Theory which can assist in combing these two techniques, and there is a number of existing hybrid techniques that incorporate Fuzzy Set theory.

However, these three techniques, even if combined, cannot satisfy fully HUMM Requirement 2. Probability Theory and Evidence Theory can model different levels of knowledge, but neither technique can account for severe uncertainty. For this reason it is necessary to incorporate Info-Gap theory.

When combined, Probability Theory, Evidence Theory, Info-Gap Theory, and Fuzzy Set Theory, model all of the types of uncertainty (randomness and sampling, confusion and conflict, inaccuracy, ambiguity, vagueness, coarseness, and simplification) for different levels of knowledge. By incorporating these four techniques into one hybrid uncertainty modeling method it is possible to not only model different types of uncertainty but also to model different levels of knowledge.

Despite the potential advantage of such a technique, there are a number of technical challenges that must first be overcome before this can become applicable. The first challenge is being about to account for all of the different types of uncertainty. The big challenge here is how to model vagueness and coarseness along with other types of uncertainty such as randomness, ambiguity, inaccuracy, confusion and conflict, etc. These are very different types of uncertainty and require a different uncertainty modeling technique (Fuzzy Set Theory) to account for this. [18]

The second challenge is combining the different levels of knowledge within the design problem. The clearest way to realize the underlying challenge is to consider the design metrics associated with each technique. Probability Theory calculates the original design metric. For instance, for the example problem used throughout Chapter 4, the design metric was the aircraft cost. When using Evidence Theory the result is to bound the solution space. The designer now has a plausible and a believable value for the metric.

The metrics from Evidence Theory and the metrics from Probability Theory are very similar. The decrease in information bounded the space but did not change the final metric drastically. The main challenge is incorporating Info-Gap Theory into the technique. As discussed in Chapter 4, Info-Gap Theory considers extreme cases of uncertainty and in effect takes the inverse approach to modeling uncertainty. Instead of trying to identify the final value of the metric, this theory estimates the amount of variation in the uncertainty variables before either a constraint or desirement is reached. Because of this approach, the final metric is now related to the potential variance of the uncertainty variable, and it becomes much more problematic to combine this technique with other uncertainty modeling theories.

Another challenge is to incorporate all of the available information without making unnecessary assumptions. Each uncertainty modeling technique requires different types of information. Probability Theory uses density functions, Evidence Theory uses a ranges and basic probability assignments, Fuzzy Set Theory uses membership functions, and the modified Info-Gap Theory discussed in Chapter 4 uses an estimated nominal value for each of the uncertain variables and an estimated maximum/minimum possible range for the variables.

In summary the main technical challenges are:

- Combining different types of uncertainty
- Combining different levels of knowledge relating to uncertainty
- Including all available information
- Not incorporating unnecessary assumptions

The rest of this chapter proposes a new hybrid uncertainty modeling technique that can overcome these technical challenges and satisfy the HUMM requirements identified at the beginning of this chapter.

7.4. New Hybrid Technique

The foundation for this technique lies within each of the four uncertainty modeling techniques. The main objective for this process is to find a way to transfer information between the very dissimilar techniques.

The first step to determining how to combine these techniques is to consider the characteristics of the uncertainty modeling theories. Probability Theory can be considered a specific case of Evidence Theory, and Info-Gap was originally developed to model similar types of uncertainty to Probability Theory for scenarios where there is a significant amount of uncertainty. [140, 25] Fuzzy Set Theory, on the other hand, is based upon membership functions and is not trying to determine the likelihood of a value occurring so much as determining the likelihood of a variable belonging to a specific group. [203]

Fuzzy Set Theory is very dissimilar from the other techniques, it makes sense to either model this aspect of uncertainty in the beginning of the design process or at the end of the process. Additionally, this theory does not change the type of resulting metric. Recall that for the aircraft acquisition cost example in Chapter 4, that when Fuzzy Set Theory was used, the metric remained the aircraft cost. Once Evidence Theory and Info-Gap Theory are incorporated into the technique the metric will be transformed into multiple metrics. In order to simplify the process, it was determined that Fuzzy Set Theory should be applied at the beginning of the uncertainty modeling technique to minimize the number of metrics that will need to be considered in this technique.

The question now becomes what is the most appropriate way to combine Probability Theory, Evidence Theory, and Info-Gap Theory. Considering one of the main differences between these techniques is their utility for modeling different levels of knowledge, it was determined to arrange the transfer of knowledge based upon this characteristic. The flow of knowledge then should either flow from the technique that requires the least amount of knowledge (Info-Gap Theory) to the technique that requires the most amount of knowledge (Probability Theory). Or, the information should flow from the technique requiring the most knowledge down to the technique requiring the least amount of knowledge.

Of the three techniques, Info-Gap Theory is the most computationally expensive. In order to determine the Robustness and Opportunity Function, a number of runs are conducted in order to determine the relative location of the nominal value of the uncertainty variable to the constraint or desirement. It was found to be fairly easy to parallelize this aspect of the process if Info-Gap Theory is used before Probability Theory or Evidence Theory. For this reason it was determined to take the bottom up approach to information transfer where an Info-Gap analysis will be done first, followed by an Evidence Theory analysis, and then Probability Theory would be used to complete the process.

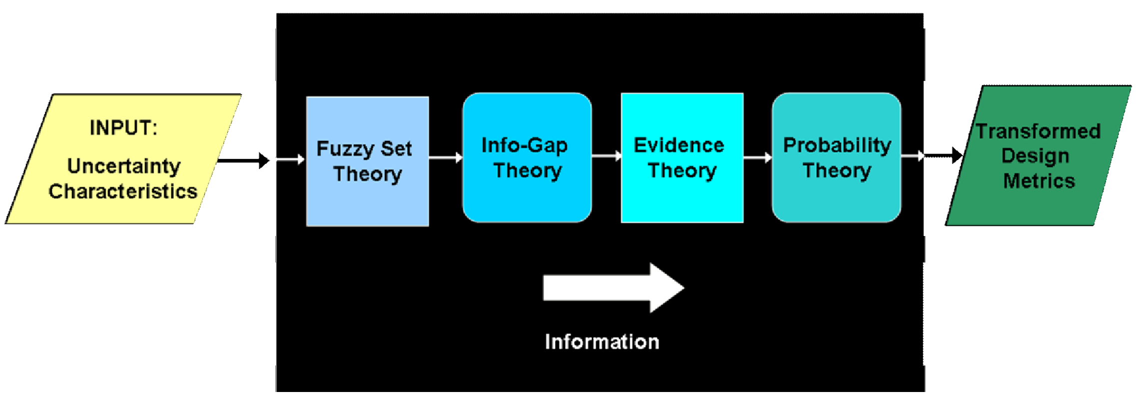

Figure 7-1: Overview of Hybrid Uncertainty Modeling Method (HUMM)

Because of the way that the information is transferred between techniques, the process illustrated in Figure 7-1 is modular. Because of this characteristic, it is possible to only include the relevant uncertainty modeling techniques. For instance, for a design problem that involves an uncertainty variable with a known distribution and another variable with an associated membership function, it is likely that the only required uncertainty modeling techniques are Probability Theory and Fuzzy Set Theory.

While the appropriate uncertainty modeling techniques should be considered carefully by the designer, Figure 7-2 is presented as a general guide to the appropriate modeling technique for each uncertainty level. [18, 25]

Theory and Methodologies	Types of Uncertainty						
	Aleatory	Epistemic					
	Randomness	Confusion and Conflict	Inaccuracy	Ambiguity	Vagueness	Coarseness	Simplification
Classical Sets	Poor	Poor	Poor	Good	Fair	Poor	Fair
Probability	Good	Fair	Good	Good	Fair	Fair	Good
Statistics	Good	Fair	Fair	Good	Fair	Fair	Fair
Bayesian	Good	Poor	Fair	Good	Fair	Fair	Good
Fuzzy Sets	Fair	Fair	Fair	Fair	Good	Good	Good
Rough Sets	Poor	Poor	Fair	Fair	Fair	Good	Good
Possibility	Fair	Good	Good	Fair	Good	Fair	Fair
Evidence	Fair	Good	Fair	Good	Fair	Fair	Fair
Interval probabilities	Fair	Good	Good	Good	Good	Fair	Good
Interval analysis	Fair	Good	Good	Good	Fair	Fair	Good
Info-gap	Good	Fair	Good	Good	Fair	Fair	Good

Poor Fair Good

Figure 7-2: Appropriate Uncertainty Modeling Techniques based on Uncertainty Factors and Level of Knowledge

As discussed in Chapter 4 there are different metric transformations associated with each of the uncertainty modeling techniques. Neither Probability Theory nor Fuzzy Set Theory affect the type of metric. However, if Evidence Theory is used there is now a plausible value for the metric and a believable Value. If Info-Gap Theory is used there is now a Robustness Function (α) or a Opportunity Function (β) for each metric. If Evidence Theory and Info-Gap Theory are both utilized, there are now four metrics: a Plausible Robustness Function ($\alpha_{Plausible}$), a Believable Robustness Function ($\alpha_{Believable}$), a Plausible Opportunity Function ($\beta_{Plausible}$), and a Believable Opportunity Function ($\beta_{Believable}$). Several possible combinations are mathematically presented in Equations 7-1 through 7-7. Equation 7-1 is for the scenario when both Probability Theory and Fuzzy Set Theory are used. The case when Probability Theory, Fuzzy Set Theory, and Evidence Theory are required to model the uncertainty in the design problem can be represented by Equations 7-2 and 7-3. If all four uncertainty modeling theories are utilized, then Equations 7-4 through 7-7 describe this scenario.

For Probability Theory and Fuzzy Set Theory

$$Metric = \sum_{i=1}^{NR}\left(\left[\sum_{h=1}^{NFS} Metric_h \cdot \mathrm{M}_h\right] \cdot \Pi_i\right) \qquad \textbf{Equation 7-1}$$

For Probability Theory, Fuzzy Set Theory, and Evidence Theory

$$Belief_{Total} = \sum_{i=1}^{NR}\left(\left[\sum_{h=1}^{NFS}\left(\sum_{k=1}^{NS} Belief_k \cdot BPA_k\right) \cdot \mathrm{M}_h\right] \cdot \Pi_i\right) \qquad \textbf{Equation 7-2}$$

$$Plausible_{Total} = \sum_{i=1}^{NR}\left(\left[\sum_{h=1}^{NFS}\left(\sum_{k=1}^{NS} Plausible_k \cdot BPA_k\right) \cdot \mathrm{M}_h\right] \cdot \Pi_i\right) \qquad \textbf{Equation 7-3}$$

For Probability Theory, Fuzzy Set Theory, Evidence Theory, and Info-Gap Theory

$$\alpha Belief_{Total,c,v} = \sum_{i=1}^{NR}\left(\left[\sum_{h=1}^{NFS}\left(\sum_{k=1}^{NS} \alpha Belief_k \cdot BPA_k\right) \cdot \mathrm{M}_h\right] \cdot \Pi_i\right) \qquad \textbf{Equation 7-4}$$

$$\alpha Plausible_{Total,c,v} = \sum_{i=1}^{NR}\left(\left[\sum_{h=1}^{NFS}\left(\sum_{k=1}^{NS}\alpha Plausible_k \cdot BPA_k\right)\cdot \mathrm{M}_h\right]\cdot \Pi_i\right)$$ **Equation 7-5**

$$\beta Belief_{Total,c,v} = \sum_{i=1}^{NR}\left(\left[\sum_{h=1}^{NFS}\left(\sum_{k=1}^{NS}\beta Belief_k \cdot BPA_k\right)\cdot \mathrm{M}_h\right]\cdot \Pi_i\right)$$ **Equation 7-6**

$$\beta Plausible_{Total,c,v} = \sum_{i=1}^{NR}\left(\left[\sum_{h=1}^{NFS}\left(\sum_{k=1}^{NS}\beta Plausible_k \cdot BPA_k\right)\cdot \mathrm{M}_h\right]\cdot \Pi_i\right)$$ **Equation 7-7**

For equations 7-1 through 7-7:

NFS: Number of runs from Full Factorial DOE for Fuzzy Set Theory

NR: Number of runs from Full Factorial DOE for Probability Theory Uncertainty Variables

NS**:** Number of runs from Full Factorial DOE for Evidence Theory Uncertainty Variables

7.5. Hybrid Uncertainty Example Problem

To demonstrate this concept consider the aircraft acquistion cost example from Chapter 4. For completeness, the relevant Tables and Figures from Chapter 4 are repeated in this chapter. The objective is to determine the aircraft acquisition cost for a persistent strike UAV, and the cost is calculated based upon Equations 7-8 through 7-10. [153] The Aeronautical Manufacturers Planning Report (AMPR) Weight is the product of the AMPR weight factor and the empty weight of the aircraft, and the cost is calculated to be the product of this weight and the cost per pound of the aircraft. [153]

$$ACCost = Costperpound \cdot W_{AMPR}$$ **Equation 7-8**

$$W_{AMPR} = AMPR\_Weight\_Factor * W_{empty}$$ **Equation 7-9**

$$W_{empty} = We/W0*TOGW$$ **Equation 7-10**

Hybrid Uncertainty Modeling Method (HUMM) Process Task 1 – Define the Design Metrics, Constraints, and Desirements

The metric for this example problem is the aircraft acquisition cost, as defined by Equation 7-8. The problem is constrained by a cost of $30 Million, but it is desired to have an aircraft that costs under $10 Million.

HUMM Process Task 2 – Define the uncertainty characteristics

For each possible design alternative, the uncertain variables need to be identified and their characteristics determined.

Determining AMPR Weight Factor

Reference 153 defines the range of the AMPR weight factor ($AMPR\_Weight\_Factor$) to be between 60-70%. For the example problem, the designer is also given the knowledge that the AMPR weight factor can be modeled by a uniform distribution.[20] There is ambiguity about the actual AMPR weight factor that will be used. Considering the level of information about the problem and the type of uncertainty being considered, it is appropriate to model this variable with Probability Theory.

[20] The reference for this variable does not provide a probability distribution for this variable. This distribution was assumed for the problem so that it would be appropriate to model the uncertainty variable with Probability Theory. In actuality, given the knowledge about the uncertainty variable (AMPR Weight Factor) it would be most appropriate to model it with Evidence Theory.

Determining Cost per Pound

The cost per pound was estimated from a linear regression of the data in Table 7-1. Figure 7-3 illustrates the resulting line. As evident from the table, the available cost information for this type of aircraft was limited. To determine the cost per pound for this simplified problem, the AMPR weight was estimated to be 65% of empty weight.

Table 7-1: Cost per Pound Data [40,149,162]

	Empty Weight (lb)	AMPR Weight Factor	Cost ($Million)
Predator A	1150	747.5	4.5
Heron TP	1764	1146.6	6.5
Predator B	3700	2405	8.3
Global Hawk	9200	5980	~30

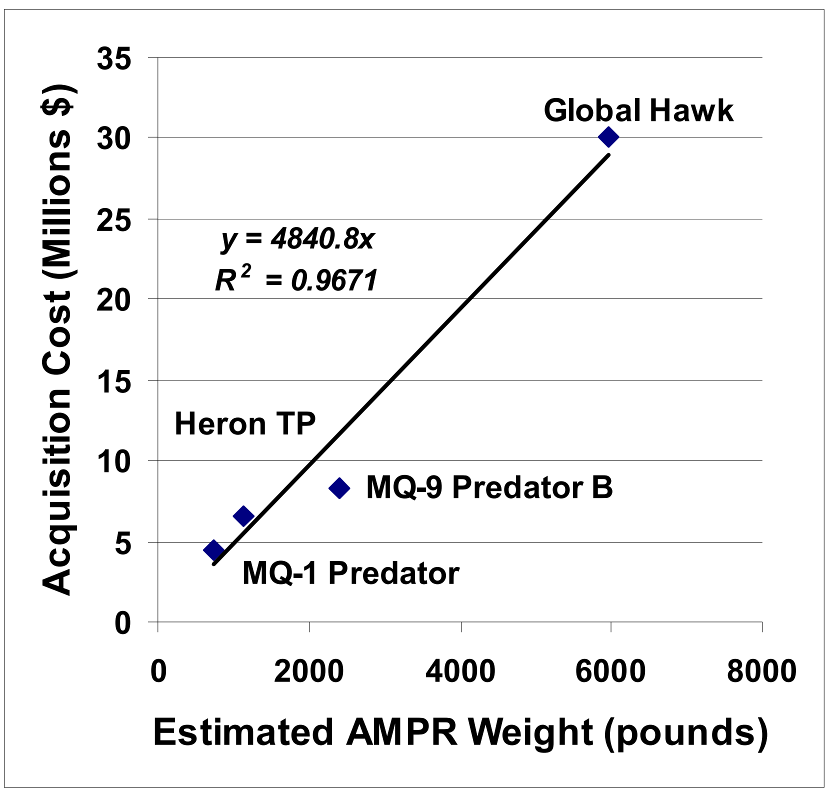

Figure 7-3: Cost Per Pound Determination

Considering the lack of data and the knowledge that the cost per pound will vary per aircraft based upon its system packages (weapon, sensors, etc), for this problem it is reasonable for the cost per pound to be between $4500-5000. There is not enough data to create a pdf for this uncertainty variable. This uncertainty variable is characterized by ambiguity, since there are multiple possible values for the cost per pound that could be used. Based upon the type of uncertainty that is characterized by this variable and the level of information about the variable, it is most appropriate to model it using Evidence Theory.

Determining Empty Weight Fraction of Aircraft

Existing aircraft data can be used to estimate the empty weight fraction for this class of aircraft. Data was collected from Reference 40 and organized by engine type for three different potential engine types: piston, turboprop, and jet. As illustrated by Figures 7-4 and 7-5, an empty weight fraction was determined based on linear regression of the empty weight data to the TOGW for each engine category.[21] Table 7-2 provides a summary of the empty weight fraction values for each engine type.

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[21] Typically as discussed in Reference 159 there is a linear relationship between the $\log_{10}$ (TOGW) and the $\log_{10}$ (W_{empty}). However, the best fit for the UAV data was found from the linear relationship between the TOGW and the W_{empty}.

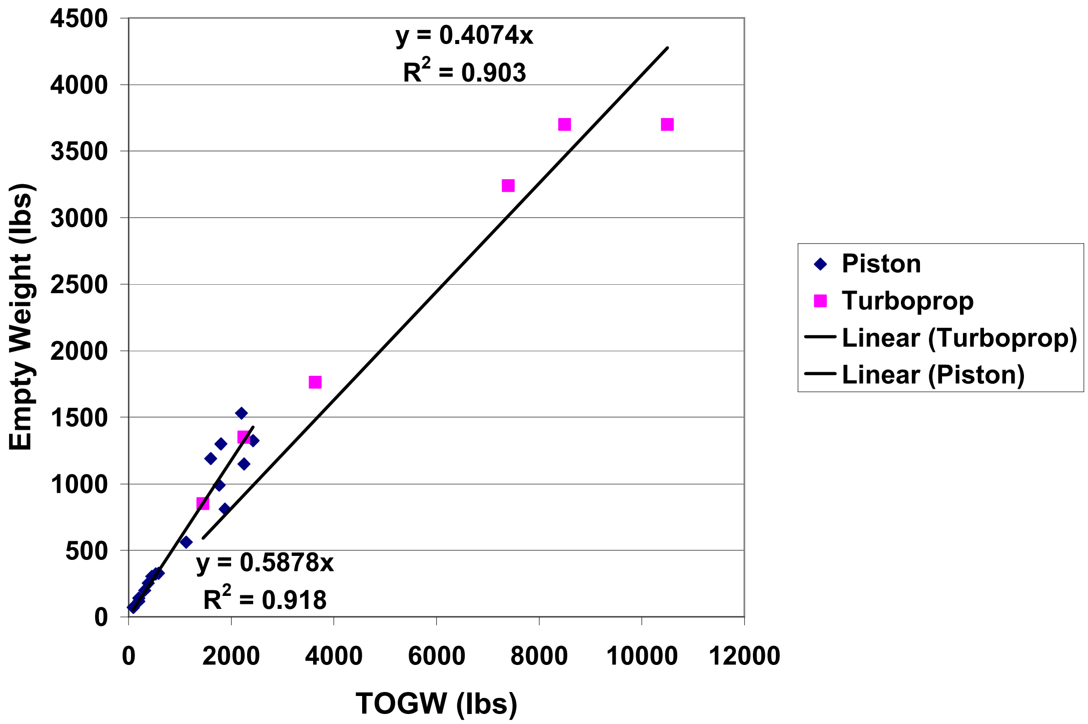

Figure 7-4: Empty Weight versus TOGW for Unmanned Aircraft (Piston/Turboprop)

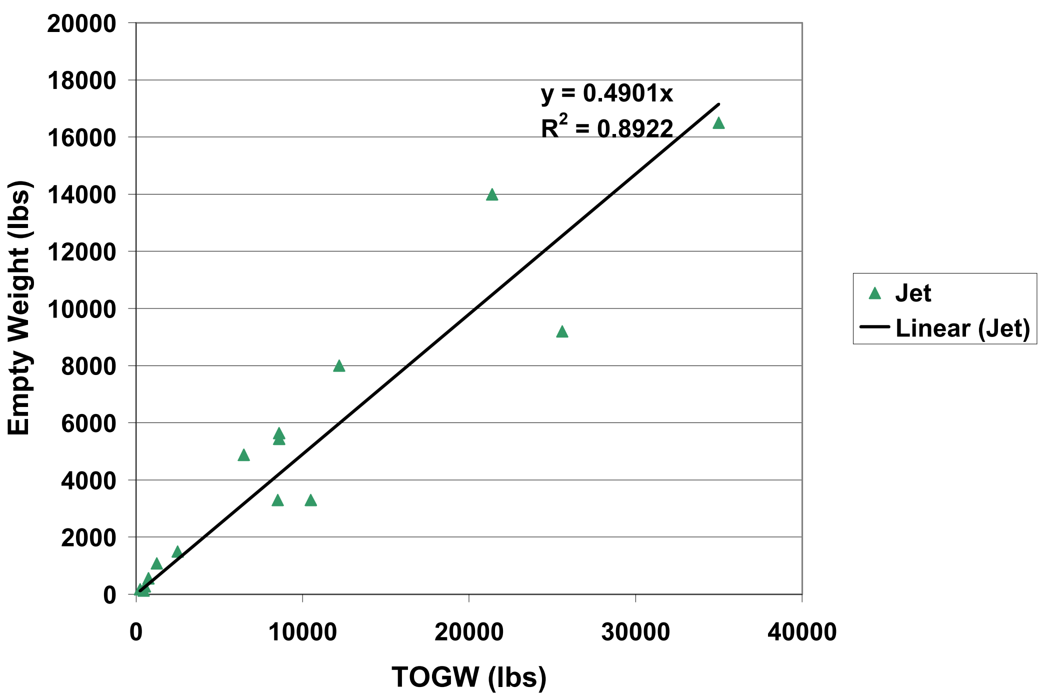

Figure 7-5: Empty Weight versus TOGW for Unmanned Aircraft (Jet)

Table 7-2: Empty Weight Fraction for Unmanned Aircraft

Engine Type	Empty Weight Fraction
Piston	0.5878
Turboprop	0.4074
Jet	0.4901

Determining Engine Type

For this example problem there are multiple types of engines that could be utilized on this aircraft. For this simplified problem the types of engine are only based upon the TOGW of the aircraft. Because of this relationship it is vague as to which type of engine will be used in this example for aircraft. This problem is characterized by the vagueness as opposed to ambiguity because the engine is determined, for this simplified example, by its relationship to TOGW.

For this problem, it is possible to determine the likelihood of the engine to be used by considering the historical relationship between the TOGW of this type of aircraft and its propulsion source. Based upon historical data from Reference 40 for unmanned aircraft, the relationship between the TOGW of the aircraft and the propulsion source can be specified using a membership function for each of the three propulsion sources. The membership functions, designated by μ, are specified in Equations 7-11 through 7-22 and are shown in Figure 7-6.

$TOGW <= 2500$ lbs **Equation 7-11**

$\mu_{Piston} = -0.000267*TOGW+0.66675$ **Equation 7-12**

$\mu_{Turboprop} = 0.000196*TOGW+0.20275$ **Equation 7-13**

μ_{Jet} = 0.000071*TOGW+0.1305 **Equation 7-14**

2500 lbs <= TOGW <= 12,250 lbs **Equation 7-15**

μ_{Piston} = 0 **Equation 7-16**

$\mu_{Turboprop}$ = -0.000071*TOGW+0.8695 **Equation 7-17**

μ_{Jet} = 0.000071*TOGW+0.1305 **Equation 7-18**

TOGW >= 12,250lbs **Equation 7-19**

μ_{Piston} = 0 **Equation 7-20**

$\mu_{Turboprop}$ = 0 **Equation 7-21**

μ_{Jet} = 1 **Equation 7-22**

For example, if the TOGW is 8,000 lb:

μ_{Piston} = 0 **Equation 7-23**

$\mu_{Turboprop}$ = 0.3015 **Equation 7-24**

μ_{Jet} = 0.6985 **Equation 7-25**

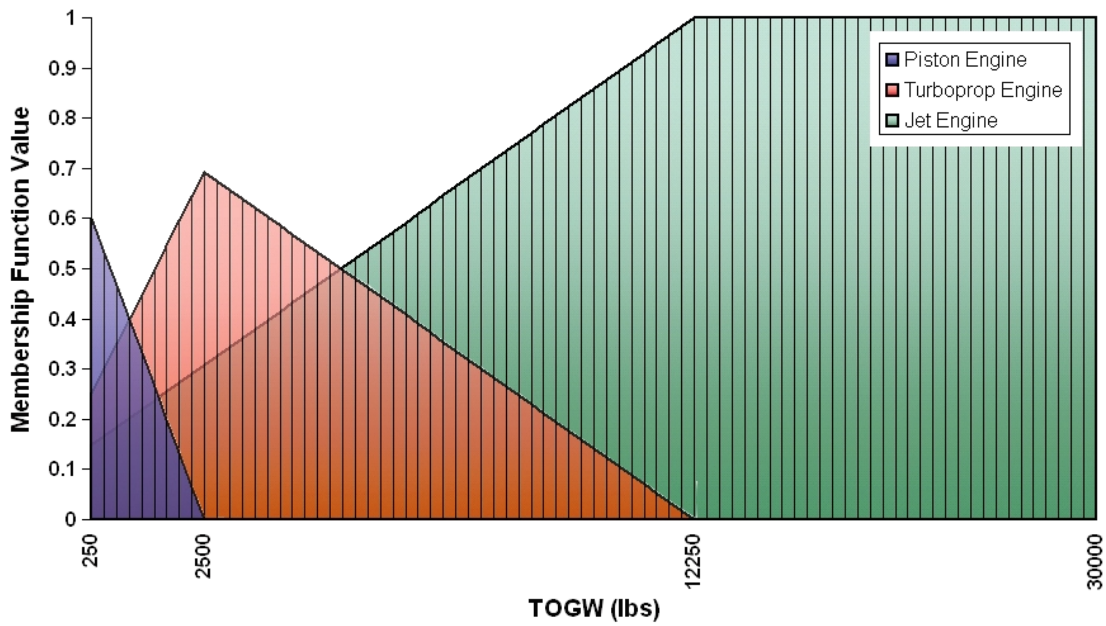

Figure 7-6: Fuzzy Membership Functions for HUMM Example Problem

Determining TOGW

It is uncertain what the value of the TOGW will be for this aircraft, or even the specific boundary values for the range of this variable. As with the AMPR Weight factor and the cost per pound this variable is characterized by ambiguity. There are a number of possible values that the TOGW could end up being. Considering that very little is known about the likely value of this variable and the fact that it is characterized by ambiguity, this variable is best modeled by Info-Gap Theory.

To utilize Info-Gap Theory as described in Chapter 4 it is necessary to estimated a nominal value for the TOGW and to estimate a maximum possible range of values for this aircraft. While very little is known about this specific aircraft, it is assumed that the aircraft will have increased performance over the MQ-9 Predator B, which has a TOGW of 10,500 lbs, and it is also assumed that its performance is under that of conventional fighters such as the F-16 (TOGW ~35,000 lbs). [149,75] Based upon this information a range of 10,000-30,000 lbs for the TOGW will be assumed for this problem. Based upon the TOGW for the MQ-9 Predator B and the X45C the nominal value is estimated to be 20,000 lbs.

HUMM Process Task 3 – Determine the number of analysis runs for each Uncertainty Variable

AMPR Weight Factor

As discussed in chapter 4, the number of intervals used to model the uncertainty should be based on the qualities of uncertainty variable, the sensitivity of the metric to the uncertainty variable, and the computational resources available for the analysis. However, for this problem the minimal number of reasonable runs was used so that it would be possible to include the full calculations in the text. For this simplified problem the number of intervals for the AMPR Weight Factor was set to three.

Cost per Pound

Evidence Theory uses the maximum and minimum values provided for the uncertainty variable. For this problem because there is only one source for the uncertainty variable and one range provided for the uncertainty variables (meaning that there is a 100% likelihood that the cost per pound is within this range), there will be two values for the cost per pound that are considered. It is not possible, without making additional assumptions, to determine any values between these bounds.

TOGW

The number of intervals for a variable modeling by Info-Gap Theory is based upon the estimated sensitivity of the metric to the uncertainty variable and the computational resources available for the analysis. As discussed in Chapter 4, the technique was developed so that even if small number of intervals is selected the results will still be accurate, though conservative. For this simplified example problem, the minimal number of reasonable runs was used so that the full calculations could be included in the text. For this reason the TOGW was set to five interval values.

HUMM Process Task 4 – Setup a Full-Factorial DOE

In this step of the process a full factorial DOE is setup for each type of uncertainty modeling technique that is included. In actuality the result is that a full-factorial DOE is run for the entire problem. For modularity and to emphasize the differences between the different uncertainty modeling techniques, the total full factorial design is broken into smaller full factorial DOEs for each uncertainty modeling theory.

For this example problem, because there is only one type of each uncertainty modeling technique, the DOE for each type of technique is simply the number of intervals/runs for each variable.

Table 7-3: Number of DOE runs per Uncertainty Modeling Method

	Fuzzy Set	Info-Gap	Evidence Theory	Probability Theory
Number of DOE Runs per Theory	2	6	2	3

For this example problem, while the TOGW was specified to have five runs in the previous step, there are actually six runs that are conducted for every combination of the other uncertainty variables. This additional run is due to the analysis for the nominal values of the variables modeled by this Info-Gap Theory.

The total number of analysis runs from this example problem is, 72, which is the product of the values listed in Table 7-3.

HUMM Process Task 5 – Run Model and Simulation Environment for all of the DOE Runs and Calculate Final Design Metrics

Run cases for Fuzzy Set Theory

For the cases when it is appropriate to model an uncertainty variable with Fuzzy Set Theory, the emphasis is on the different options that are available and the likelihood of a certain option being used. This technique is often used to determine which category or values to use in the design problem based upon a separate value or setting. In the example problem the type of engine selected is based upon the TOGW. While the selection of a type of engine does not directly affect the final metric value (aircraft acqusition cost) through Equations 7-8 through 7-10, the type of engine selected determines the value of the empty weight fraction, which is linked to the aircraft cost through Equation 7-10.

For every set of values for Probability Theory, Evidence Theory, and Info-Gap Theory, the value of the metric from Fuzzy Set Theory will be calculated. The final metric value for this theory is calculated using Equation 7-26 where the metric value ($Metric_i(x)$) is calculated based upon it belonging to a particular option (fuzzy set). The option number is indicated by the letter "i".

$$Metric = \sum_{i=1}^{NO} Metric_i(x) \cdot \mu_i(x)$$ **Equation 7-26**

NO: The number of Options/ Fuzzy Sets in the analysis

For the example problem, there are two options: either the aircraft propulsion source will be a turboprop or a jet. While the historical data produced three fuzzy membership sets (one for a piston engine, one for a turboprop, and a fuzzy membership set for a jet engine), the range of values for the TOGW results in a membership value of 0 for all runs related to the piston engine. For this reason, it is acceptable to drop this membership set from the analysis.

The calculations for the Fuzzy Set Theory analysis are shown in Tables 7-4 through 7-6. Table 7-4 lists the calculated values for fuzzy membership set 1 (turboprop) and Table 7-5 lists the values for fuzzy membership set 2 (jet). Table 7-6 presents the final metric values for the Fuzzy Set Analysis. Column 6 was calculated by using Equation 7-10.

Table 7-4: Metric Values from Fuzzy Set Analysis for TurboProp

					TurboProp			
PT Run	ET Run	TOGW	AMPR Weight Factor	Cost/lb	Empty Weight	Membership (μ)	Cost	μ*Cost
1	1	10000	60	4500	4070	0.1595	10.989	1.753
1	1	15000	60	4500	6105	0	16.4835	0
1	1	20000	60	4500	8140	0	21.978	0
1	1	25000	60	4500	10175	0	27.4725	0
1	1	30000	60	4500	12210	0	32.967	0
1	2	10000	60	5000	4070	0.1595	12.21	1.947
1	2	15000	60	5000	6105	0	18.315	0
1	2	20000	60	5000	8140	0	24.42	0
1	2	25000	60	5000	10175	0	30.525	0
1	2	30000	60	5000	12210	0	36.63	0
2	1	10000	65	4500	4070	0.1595	11.90475	1.899
2	1	15000	65	4500	6105	0	17.85713	0
2	1	20000	65	4500	8140	0	23.8095	0
2	1	25000	65	4500	10175	0	29.76188	0
2	1	30000	65	4500	12210	0	35.71425	0
2	2	10000	65	5000	4070	0.1595	13.2275	2.110
2	2	15000	65	5000	6105	0	19.84125	0
2	2	20000	65	5000	8140	0	26.455	0
2	2	25000	65	5000	10175	0	33.06875	0
2	2	30000	65	5000	12210	0	39.6825	0
3	1	10000	70	4500	4070	0.1595	12.8205	2.045
3	1	15000	70	4500	6105	0	19.23075	0
3	1	20000	70	4500	8140	0	25.641	0
3	1	25000	70	4500	10175	0	32.05125	0
3	1	30000	70	4500	12210	0	38.4615	0
3	2	10000	70	5000	4070	0.1595	14.245	2.272
3	2	15000	70	5000	6105	0	21.3675	0
3	2	20000	70	5000	8140	0	28.49	0
3	2	25000	70	5000	10175	0	35.6125	0
3	2	30000	70	5000	12210	0	42.735	0

Table 7-5: Metric Values from Fuzzy Set Analysis for Jet

					Jet			
PT Run	**ET Run**	**TOGW**	**AMPR Weight Factor**	**Cost/lb**	**Empty Weight**	**Membership (μ)**	**Cost**	**μ*Cost**
1	1	10000	60	4500	4901	0.8405	13.233	11.12208
1	1	15000	60	4500	7351.5	1	19.849	19.84905
1	1	20000	60	4500	9802	1	26.465	26.4654
1	1	25000	60	4500	12252.5	1	33.082	33.08175
1	1	30000	60	4500	14703	1	39.698	39.6981
1	2	10000	60	5000	4901	0.8405	14.703	12.35787
1	2	15000	60	5000	7351.5	1	22.055	22.0545
1	2	20000	60	5000	9802	1	29.406	29.406
1	2	25000	60	5000	12252.5	1	36.758	36.7575
1	2	30000	60	5000	14703	1	44.109	44.109
2	1	10000	65	4500	4901	0.8405	14.335	12.04892
2	1	15000	65	4500	7351.5	1	21.503	21.50314
2	1	20000	65	4500	9802	1	28.671	28.67085
2	1	25000	65	4500	12252.5	1	35.839	35.83856
2	1	30000	65	4500	14703	1	43.006	43.00628
2	2	10000	65	5000	4901	0.8405	15.928	13.38769
2	2	15000	65	5000	7351.5	1	23.892	23.89238
2	2	20000	65	5000	9802	1	31.857	31.8565
2	2	25000	65	5000	12252.5	1	39.821	39.82063
2	2	30000	65	5000	14703	1	47.785	47.78475
3	1	10000	70	4500	4901	0.8405	15.438	12.97577
3	1	15000	70	4500	7351.5	1	23.157	23.15723
3	1	20000	70	4500	9802	1	30.876	30.8763
3	1	25000	70	4500	12252.5	1	38.595	38.59538
3	1	30000	70	4500	14703	1	46.314	46.31445
3	2	10000	70	5000	4901	0.8405	17.153	14.41752
3	2	15000	70	5000	7351.5	1	25.730	25.73025
3	2	20000	70	5000	9802	1	34.307	34.307
3	2	25000	70	5000	12252.5	1	42.884	42.88375
3	2	30000	70	5000	14703	1	51.460	51.4605

Table 7-6: Final Metric Values from Fuzzy Set Analysis

PT Run	ET Run	TOGW	AMPR Weight Factor	Cost/lb	TurboProp μ*Cost	Jet μ*Cost	Cost (Fuzzy)
1	1	10000	60	4500	1.7527455	11.12208	12.87483
1	1	15000	60	4500	0	19.84905	19.84905
1	1	20000	60	4500	0	26.4654	26.4654
1	1	25000	60	4500	0	33.08175	33.08175
1	1	30000	60	4500	0	39.6981	39.6981
1	2	10000	60	5000	1.947495	12.35787	14.30537
1	2	15000	60	5000	0	22.0545	22.0545
1	2	20000	60	5000	0	29.406	29.406
1	2	25000	60	5000	0	36.7575	36.7575
1	2	30000	60	5000	0	44.109	44.109
2	1	10000	65	4500	1.8988076	12.04892	13.94773
2	1	15000	65	4500	0	21.50314	21.50314
2	1	20000	65	4500	0	28.67085	28.67085
2	1	25000	65	4500	0	35.83856	35.83856
2	1	30000	65	4500	0	43.00628	43.00628
2	2	10000	65	5000	2.1097863	13.38769	15.49748
2	2	15000	65	5000	0	23.89238	23.89238
2	2	20000	65	5000	0	31.8565	31.8565
2	2	25000	65	5000	0	39.82063	39.82063
2	2	30000	65	5000	0	47.78475	47.78475
3	1	10000	70	4500	2.0448698	12.97577	15.02063
3	1	15000	70	4500	0	23.15723	23.15723
3	1	20000	70	4500	0	30.8763	30.8763
3	1	25000	70	4500	0	38.59538	38.59538
3	1	30000	70	4500	0	46.31445	46.31445
3	2	10000	70	5000	2.2720775	14.41752	16.68959
3	2	15000	70	5000	0	25.73025	25.73025
3	2	20000	70	5000	0	34.307	34.307
3	2	25000	70	5000	0	42.88375	42.88375
3	2	30000	70	5000	0	51.4605	51.4605

For this example, the final result from this analysis shown in column 8 of Table 7-6 is the result that would be passed on to the other uncertainty modeling theories.

Run cases for Info-Gap Theory

From process, Step 2 of the Hybrid Uncertainty Modeling Method, there is a nominal value for each uncertainty variable modeled by Info-Gap Theory, a maximum possible range of values, and a set number of intervals to be evaluated. For every analysis run, the difference between the constraint and the design metric (aircraft cost) is calculated (column 3, Table 7-7). These values are used to determine the approximate location of the constraint with respect to the values of the uncertainty variable. The approximate location of the constraint is identified by a sign change with the values for the difference between the constraint and the design metric (column 3, Table 7-7).

While this example problem is only considering the uncertainty analysis for one design alternative, in a design process the different alternatives would be compared by their Robustness Functions (α) and Opportunity Functions (β). The objective in the design process is to minimize β while maximizing α.

Because it is desired to find the maximize value for α, but it is understood that the constraint value could be located anywhere between the analysis runs before and after the sign change, the α that occurs before the constraint is violated is selected. As an example consider Figure 7-7. For this notional example, the value of A3 would be selected for the α value. For a more detailed description of this process, see the Info-Gap Section of Chapter 4.

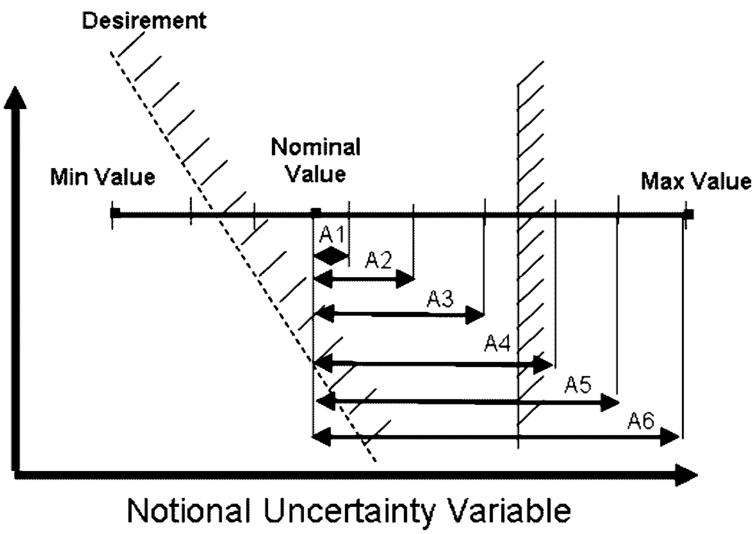

Figure 7-7: Illustration of potential α values

In Table 7-7, the information pertaining to the analysis for the nominal value for each combination Probability Theory DOE run and Evidence Theory DOE run is highlighted in either green or orange. The row is highlighted in green if the nominal value satisfies the constraint and orange if the nominal value fails the constraint. The identified value of α is highlighted in yellow for each combination Probability Theory DOE run and Evidence Theory DOE run.

For the case when the nominal value fails the constraint, it may be desired to apply a penalty function. To simplify the calculation for example purposes, for this problem, no penalty function is applied, but the value of α is given a negative sign to indicate that the constraint was failed. This sign change is applied in the next section. For more information on this technique, see Chapter 4.

Additionally for every analysis run, the difference between the desirement and the design metric is determined (column 3, Table 7-8). These values are used to determine the approximate location of the desirement with respect to the values of the uncertainty variable. The approximate location of the desirement is identified by a sign change with the values for the difference between the desirement and the design metric (column 3, Table 7-8).

Because it is desired to find the minimum value for β, but it is understood that the desirement value could be located anywhere between the analysis runs before and after the sign change, the β that occurs after the desirement is satisfied is selected. For example, consider Figure 7-8. For this notional example, the value of B2 would be selected for the β value. For a more detailed description of this process, see Chapter 4.

Table 7-7: Info-Gap Analysis Data for α

PT Run	ET Run	Δ Constraint	AMPR Factor	Cost/lb	TOGW	Potential α	Normalized Potential α
1	1	-9.6981	60	4500	30000	10000	50
1	1	-3.08175	60	4500	25000	5000	25
1	1	3.5346	60	4500	20000	0	0
1	1	10.15095	60	4500	15000	5000	25
1	1	17.12517	60	4500	10000	10000	50
1	1	3.5346	60	4500	20000	---	---
1	2	-14.109	60	5000	30000	10000	50
1	2	-6.7575	60	5000	25000	5000	25
1	2	0.594	60	5000	20000	0	0
1	2	7.9455	60	5000	15000	5000	25
1	2	15.69463	60	5000	10000	10000	50
1	2	0.594	60	5000	20000	---	---
2	1	-13.0063	65	4500	30000	10000	50
2	1	-5.83856	65	4500	25000	5000	25
2	1	1.32915	65	4500	20000	0	0
2	1	8.496863	65	4500	15000	5000	25
2	1	16.05227	65	4500	10000	10000	50
2	1	1.32915	65	4500	20000	---	---
2	2	-17.7848	65	5000	30000	10000	50
2	2	-9.82063	65	5000	25000	5000	25
2	2	-1.8565	65	5000	20000	0	0
2	2	6.107625	65	5000	15000	5000	25
2	2	14.50252	65	5000	10000	10000	50
2	2	-1.8565	65	5000	20000	---	---
3	1	-16.3145	70	4500	30000	10000	50
3	1	-8.59538	70	4500	25000	5000	25
3	1	-0.8763	70	4500	20000	0	0
3	1	6.842775	70	4500	15000	5000	25
3	1	14.97937	70	4500	10000	10000	50
3	1	-0.8763	70	4500	20000	---	---
3	2	-21.4605	70	5000	30000	10000	50
3	2	-12.8838	70	5000	25000	5000	25
3	2	-4.307	70	5000	20000	0	0
3	2	4.26975	70	5000	15000	5000	25
3	2	13.31041	70	5000	10000	10000	50
3	2	-4.307	70	5000	20000	---	---

Table 7-8: Info-Gap Analysis Data for β

PT Run	ET Run	Success	TOGW			β	Normalized β
1	1	-2.12517	10000			10000	50
1	1	4.84905	15000			5000	25
1	1	11.4654	20000			0	0
1	1	18.08175	25000			5000	25
1	1	24.6981	30000			10000	50
1	1	11.4654	20000			---	---
1	2	-0.69463	10000			10000	50
1	2	7.0545	15000			5000	25
1	2	14.406	20000			0	0
1	2	21.7575	25000			5000	25
1	2	29.109	30000			10000	50
1	2	14.406	20000			---	---
2	1	-1.05227	10000			10000	50
2	1	6.503138	15000			5000	25
2	1	13.67085	20000			0	0
2	1	20.83856	25000			5000	25
2	1	28.00628	30000			10000	50
2	1	13.67085	20000			---	---
2	2	0.49748	10000	**slope**	619.4392	10308.16	51.54
2	2	8.892375	15000	**Zero Int**	9691.841		
2	2	16.8565	20000	**TOGW**	9691.841		
2	2	24.82063	25000				
2	2	32.78475	30000				
2	2	16.8565	20000			---	---
3	1	0.020635	10000	**slope**	639.1039	10013.19	50.07
3	1	8.157225	15000	**Zero Int**	9986.812		
3	1	15.8763	20000	**TOGW**	9986.812		
3	1	23.59538	25000				
3	1	31.31445	30000				
3	1	15.8763	20000			---	---
3	2	1.689594	10000	**slope**	575.1935	10971.84	54.86
3	2	10.73025	15000	**Zero Int**	9028.156		
3	2	19.307	20000	**TOGW**	9028.156		
3	2	27.88375	25000				
3	2	36.4605	30000				
3	2	19.307	20000			---	---

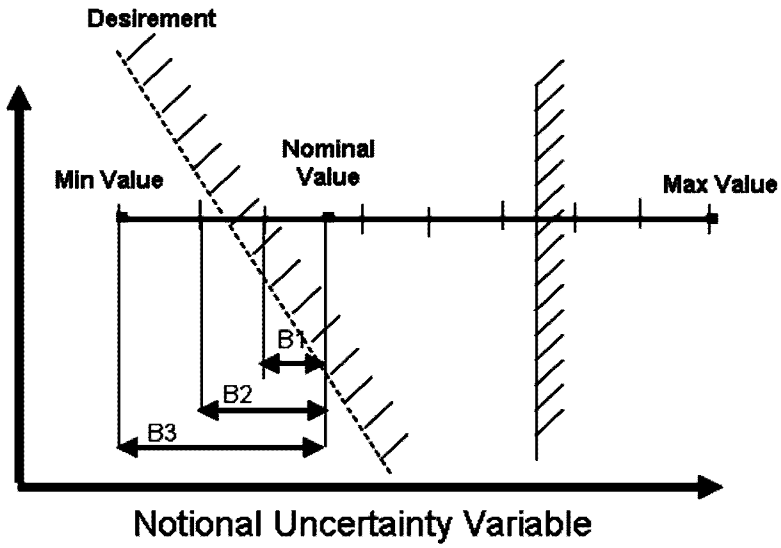

Figure 7-8: Illustration of potential β Values

In Table 7-8, the information pertaining to the analysis for the nominal value for each combination Probability Theory DOE run and Evidence Theory DOE run is highlighted in either green or blue. The row is highlighted in green if the nominal value satisfies the desirement and blue if the nominal value fails to meet the desirement. The identified value of β is highlighted in yellow for each combination Probability Theory DOE run and Evidence Theory DOE run.

For this example, no nominal values ever satisfied the constraint, as indicated by the lack of green rows. Additionally for some combinations of variables modeled by Probability Theory and variables modeled by Evidence Theory, the desirement is never satisfied within the proposed range of values for the TOGW. For the situations when this occurs, a line is generated between the maximum and minimum values of the distance between the desirement and the calculated design metric value (column 3, Table 7-8). This line is then used to extrapolate the location of the desirement with respect to the uncertainty variable modeled with Info-Gap Theory. The slope, zero intercept, and resulting TOGW value is recorded in Table 7-8 for these scenarios. Chapter 4 provides a more detailed description of this process. This section also discusses different options for penalizing the extrapolated value of β. For this problem, the extrapolated value of β is not penalized for simplification purposes.

While not applicable for this problem, for the situation when the nominal value satisfies the desirement it may be appropriate (as determined by the designer based on the problem) to apply a bonus function. Additional information on this technique is provided in Chapter 4.

7.5.1.1. Run cases for Evidence Theory

After the values of α and β for each Probability Theory DOE Run and Evidence Theory DOE Run have been determined from Info-Gap Theory, it is possible to use Evidence Theory to determine the values for the Plausible Robustness Function ($\alpha_{Plausible}$), a Believable Robustness Function ($\alpha_{Believable}$), a Plausible Opportunity Function ($\beta_{Plausible}$), and a Believable Opportunity Function ($\beta_{Believable}$).

For this example only one source is considered and one interval for the cost per pound. See Chapter 4 for an example where multiple sources and intervals are considered. The example problem discussed here, while it does not fully illustrate all of the aspects of how to utilize Evidence Theory, was setup to emphasize the basic difference between the plausible and believable values. For a more complete description the reader is directed to the discussion in Chapter 4.

Because the constraint and desirement information has already been included into the selection of the α and β values there is no need to incorporate it here. However, if Info-Gap Theory is not used in the process then it is necessary to determine the Believable and Plausible Values with respect to the constraint and desirement values. An example of how to determine the Believable and Plausible values with respect to constraint values is provided in Chapter 4.

Recall that it is desired to maximize the value of α and that it is desired to minimize the value of β. With this in mind, the plausible α will be the highest value within the set, and the believable α will be the minimum value of α within the set. So, the Plausible Robustness Function ($\alpha_{Plausible}$), will be the maximum identified α value, and the Believable Robustness Function ($\alpha_{Believable}$) will be the minimum value. Conversely, the plausible β will be the minimum value from the set of Evidence Theory results, but the believable β value will be the maximum value. For this reason the Plausible Opportunity Function ($\beta_{Plausible}$) is the minimum value of β and the Believable Opportunity Function ($\beta_{Believable}$) is the maximum value.

For this problem there are two Evidence Theory runs, one where the uncertainty variable modeled by Evidence Theory is set to its minimum value and another where the variable is set to its maximum value.

The normalized α and β values are listed in Column 3 of Tables 7-9 and 7-10 respectively. The identified Plausible Robustness Function ($\alpha_{Plausible}$) for each Evidence Theory set (which is associated with a specific run from the Probability Theory DOE.), is listed in Column 5 of Table 7-9. The Believable Robustness Function ($\alpha_{Believable}$) is presented in Column 4 of Table 7-9.

Table 7-9: Results from Evidence Theory Process related to α

PT Run	ET Run	Normalized α	α Believable	α Plausible
1	1	0		
1	2	0		
			0	0
2	1	0		
2	2	25		
			-25	0
3	1	25		
3	2	25		
			-25	-25

The Plausible Opportunity Function ($\beta_{Plausible}$) is listed in Column 5 of Table 7-10, and the Believable Opportunity Function ($\beta_{Believable}$) for each Evidence Theory set is listed in Column 4 of Table 7-10.

Table 7-10: Results from Evidence Theory Process related to β

PT Run	ET Run	Normalized β	β Believable	β Plausible
1 1	1 2	50 50		
			50	50
2 2	1 2	50 51.5407942		
			51.54079	50
3 3	1 2	50.065939 54.8592184		
			54.85922	50.06594

7.5.1.2. Run cases for Probability Theory

The final step in the uncertainty analysis is to pass the information from the previous uncertainty modeling analyses to be run in a probabilistic analysis. The equation for determining the final metric with this technique is shown in Equation 7-27. Equation 7-28 is used determine the combined probability when multiple variables are modeled with Probability Theory.

$$Metric = \sum_{i=1}^{NR} \left(Metric_i \cdot \Pi_i \right)$$ **Equation 7-27**

$$\Pi_i = \prod_{j=1}^{NPV} \pi_{j,i}$$ **Equation 7-28**

NR: Number of DOE runs for Probability Theory

NPV: Number of variables to be modeled with Probability Theory.

For the example problem the combined probability is equal to the probability of the AMPR Weight factor, because this is the only variable modeled by this theory.

The metric being determined depends on the uncertainty modeling techniques used previously in the design problem. For the example problem there are now four metrics for the original design metric (aircraft acquisition cost). Therefore, Equation 7-27 now becomes Equations 7-29 through 7-32.

$$\alpha_{Believable} = \sum_{i=1}^{NR} \left(\alpha_{Believable,i} \cdot \Pi_i \right)$$ **Equation 7-29**

$$\alpha_{Plausible} = \sum_{i=1}^{NR} \left(\alpha_{Plausible,i} \cdot \Pi_i \right)$$ **Equation 7-30**

$$\beta_{Believable} = \sum_{i=1}^{NR} \left(\beta_{Believable,i} \cdot \Pi_i \right)$$ **Equation 7-31**

$$\beta_{Plausible} = \sum_{i=1}^{NR} \left(\beta_{Plausible,i} \cdot \Pi_i \right)$$ **Equation 7-32**

The final results for this example problem are calculated and presented below by using the values from Tables 7-11 and 7-12. A summary of the results is presented in Table 7-13.

Table 7-11: Results from Probability Theory Process related to α

	Probability	Believable α	Plausible α
Probability Run 1	0.333	0	0
Probability Run 2	0.333	-25	0
Probability Run 3	0.333	-25	-25

$$\text{Final Believable } \alpha = (0*0.33)+(-25*0.33)+(-25*0.33) = -16.665 \quad \textbf{Equation 7-33}$$

$$\text{Final Plausible } \alpha = (0*0.33)+(0*0.33)+(-25*0.33) = -8.333 \quad \textbf{Equation 7-34}$$

Table 7-12: Results from Probability Theory Process related to β

	Probability	Believable β	Plausible β
Probability Run 1	0.333	50	50
Probability Run 2	0.333	51.5	50
Probability Run 3	0.333	54.9	50.1

$$\text{Final Believable } \beta = (50*0.33)+(51.5*0.33)+(54.9*0.33) = 52.1 \quad \textbf{Equation 7-35}$$

$$\text{Final Plausible } \beta = (50*0.33)+(50*0.33)+(50.1*0.33) = 50.0 \quad \textbf{Equation 7-36}$$

Table 7-13: Final Results for HUMM

Believable α	-16.7
Plausible α	-8.3
Believable β	52.1
Plausible β	50.0

The HUMM satisfys all of the requirements described at the beginning of this chapter. The main disadvantage to this technique is that without significant experience with this method and the problem of interest, the results are abstract and nonintutive. For instance, a designer may not know what a reasonable value of a Believable α or β should be for the design problem. However, when used to compare multiple alternatives, while considering the various kind of uncertainty, it is an extremely useful technique.

CHAPTER 8: ROBUST AND OPPORTUNISTIC DESIGN

Decision making is the essence of design. In order to make a well informed decision it is important to consider all of the available information. For this reason, when uncertainty is involved in the design process, it is important to consider this uncertainty in the decision making process. However, it can be difficult to analyze results when uncertainty is included. For instance consider Figure 8-1.

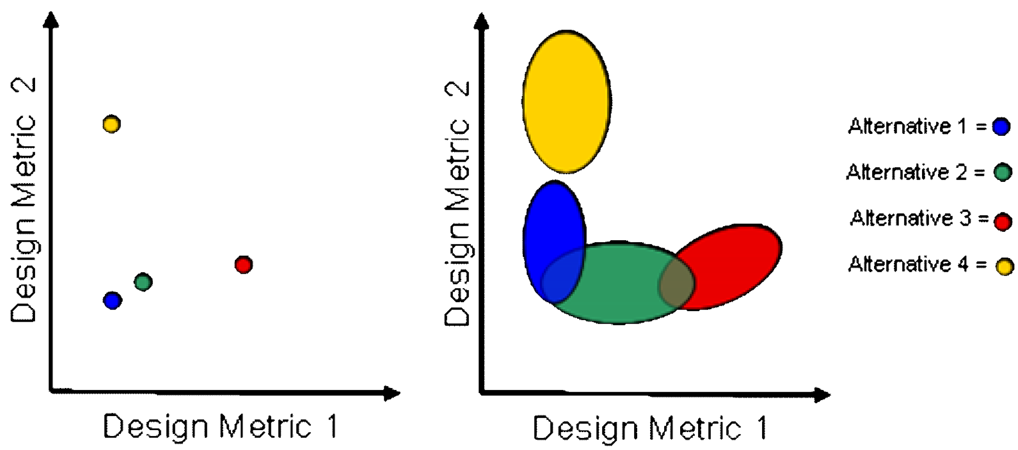

Figure 8-1: Illustration of the impact of including uncertainty

The left side of Figure 8-1 illustrates the scenario where design alternatives are modeled without considering the uncertainty, and the right side of Figure 8-1 illustrates the scenario where uncertainty is considered. In reality the resulting design for each alternative could end up being anywhere within the different ellipses shown in the right side of Figure 8-1. Even though the uncertainty is not illustrated in the left side of the figure, it exists. [188]

Consider the situation where the objective is to minimize both design metrics in Figure 8-1. If a designer did not consider the uncertainty when selecting between design alternatives, then alternative 1 would be selected. However, if uncertainty is considered by the designer then the design decision becomes more difficult. For some uncertainty values, Alternative 1 would be the best decision, but for other uncertainty values, Alternative 2 would be the best decision for minimizing the metrics. While the decision making process is likely to become more complicated, the designer is able to make a more informed decision by considering the uncertainty.

8.1. Positive and Negative Characteristics of Uncertainty

Often only the negative aspects of uncertainty are emphasized. For instance, it is unlikely to see a weather forecast that will predict an 80% chance of no rain. However, there is a positive and a negative side to uncertainty. [25] For instance consider Figure 8-2.

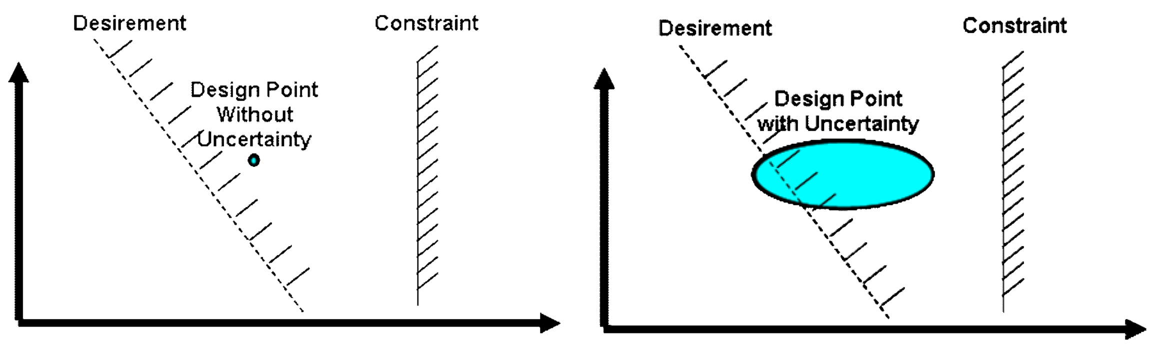

Figure 8-2: Illustration emphasizing the negative and positive aspects of uncertainty

In both Figures the objective is to identify an alternative that is as far from the constraint as possible and as close to (or surpassing) the desirement as possible. In the left side of Figure 8-2, without considering the uncertainty it would seem as though the design alternative illustrated satisfies the constraint, but fails to meet the desirement. When considering the uncertainty, as illustrated in the right side of the figure, it becomes apparent that for some values of the uncertainty it is possible for the identified design alternative to meet the desirement.

If only the negative side of uncertainty is considered, such as only focusing on the ability of a design alternative to satisfy a constraint, the final design selected will often be overly conservative. This is illustrated in several example problems throughout this chapter.
While it is possible to model both the negative and positive aspects to uncertainty in various uncertainty modeling techniques, Info-Gap Theory specifically includes both the positive and negative aspects of uncertainty concept through the use of the Robustness and Opportunity Functions. [25] Because the basic concept behind these functions from Info-Gap Theory can be utilized with other theories, and because it is likely that it will be appropriate to model uncertainty within a SoS conceptual design process with Info-Gap theory, this chapter will focus upon the robustness and the opportunity functions as the primary design metrics.
For a detailed description of how the Robustness and Opportunity Functions are used to model uncertainty within a design problem using Info-Gap Theory, see Chapter 4.

8.2. Different Approaches to Incorporating Uncertainty in Design

There are three different approaches to considering uncertainty in a design process: Robust Design, Opportunistic Design, as well as Robust and Opportunistic (RandO) Design.

8.2.1. Robust Design

Robust Design is the most common approach to incorporating uncertainty in a design process. There are a number of robust design methods within the literature such as: Taguchi Robust Design Method, Suh’s Axiomatic Design, Robust Concept Exploration

Method (RCEM), and Robust Optimization Incorporating Worst Case Uncertainty Propagation. [186,187,185,44] These methods and others are discussed in Chapter 5.

As with many common concepts, the definition of what constitutes Robust Design or robustness differs from source to source. [44, 25, 148] For this research, Robust Design is defined as follows:

Robust Design is a technique that identifies the design alternative that satisfies design constraints for a range of uncertainty values.

Based upon this definition for Robust Design, it is evident that this approach to design focuses on satisfying the negative aspects of uncertainty. In other words the focus of Robust Design is satisfying the design constraints. For this reason Robust Design only uses the metrics based upon the Robustness Function (α). If Info-Gap Theory and Evidence Theory are used in the design process, as is the case for the example problems in this chapter, the design metrics for Robust Design are the Plausible α and the Believable α.

8.2.2. Opportunistic Design

Opportunistic Design is not traditionally used in design from the sense that constraints are not considered. Instead the emphasis of this approach is based upon the ability of a design to achieve the desirements, or designated level of success, for the design problem. For this research Opportunistic Design is defined as:

Opportunistic Design is a technique that identifies the design alternative that achieves the design desirements for a range of uncertainty values.

This approach specifically focuses on the positive aspects of uncertainty. The main metric for this technique is based upon the Opportunity Function (β) from Info-Gap Theory. If both Info-Gap theory and Evidence Theory are used in the design process, the design metrics for this approach are the Plausible β and the Believable β.

8.2.3. Robust and Opportunistic Design

As indicated by the name of this design approach, this technique is a combination of Robust Design and Opportunistic Design. This technique strives to identify the design alternative that both satisfies the design requires and achieves the design desirements. [25] Robust and Opportunistic (RandO) Design is defined for this research as:

Robust and Opportunistic Design is a technique that identifies the design alternative that both satisfies the design constraints and achieves the design desirements for a range of uncertainty values.

Because RandO Design is a combination of Robust Design and Opportunistic Design and this approach focuses on both constraints and desirements, the main metrics for this design process are both α and β. If Evidence Theory and Info-Gap Theory are included in the design process, there will be four design metrics. These metrics are: the Plausible α, the Believable α, the Plausible β and the Believable β. See Chapter 7 for a discussion considering how these metrics can be calculated for a design problem.

8.3. Design Approach Comparison

Six example problems have been developed to illustrate different types of design problems and to emphasize the differences between the different approaches. Figure 8-3 illustrates the general design process for all three design approaches. In each example problem there is a set of 100 design alternatives that is to be compared. Each approach analyzes the design and the relevant uncertainty. A top design alternative is then selected through the Technique for Order Preference by Similarity to Ideal Solution (TOPSIS). TOPSIS is a Multi-Attribute Decision Making (MADM) tool where all of the potential designs are compared and ranked based upon which solution is closest to the positive solution and furthest from the negative solution. [204]

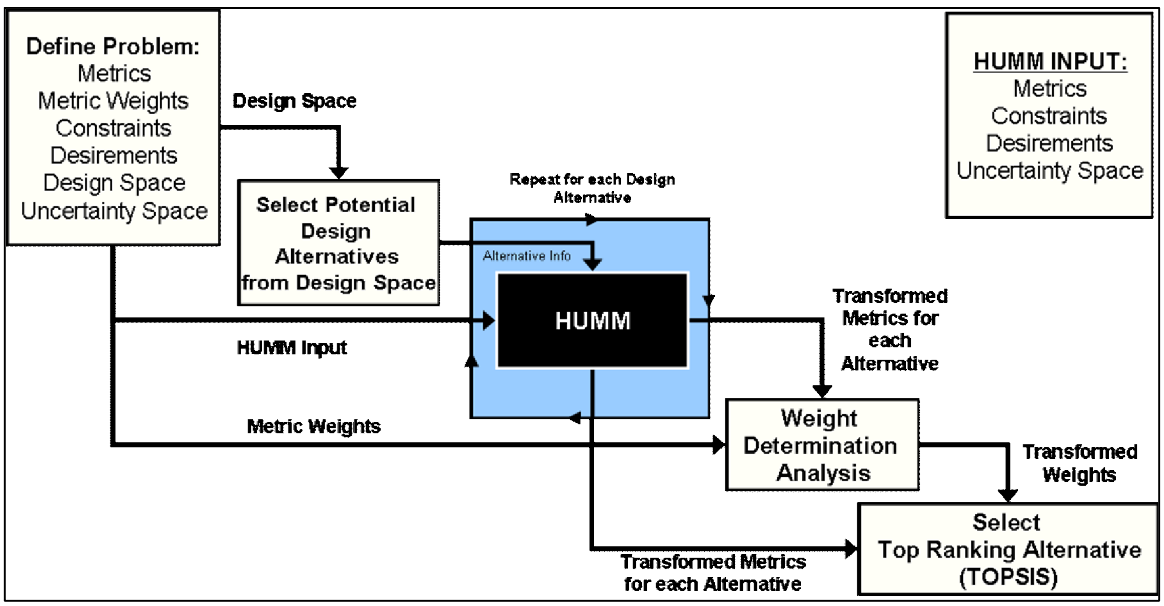

Figure 8-3: General Design Process for Robust, Opportunistic, and RandO Design Approaches

The top alternatives for each technique are then compared for three different uncertainty scenarios. For each scenario a range and distribution for the uncertainty variables is assumed for the problem. The first scenario is based upon average uncertainty values, the second scenario is based upon ideal uncertainty values (or less challenging uncertainty values as the case may be), and the third uncertainty scenario is based upon challenging uncertainty values. For each of the top alternatives and for each scenario, a Monte Carlo

Analysis consisting of 1000 runs was conducted using the provided range and distributions for the uncertainty variables.

The data for each of the alternatives from the Monte Carlo (MC) analysis was then compared using the distance based weight modeling method discussed in Reference 188. In this technique, the distance between the resulting design metric calculated by the MC analysis, which is the traditional metric and not one based on α or β, and the desirement is calculated. If a constraint is violated, the distance is increased through the use of a penalty function. The average resulting distance for each alternative is then compared and evaluated with TOPSIS. The final Overall Evaluation Criterion calculated by TOPSIS and the Monte Carlo data is presented for each of the example problems.

The example problems are as follows:

- Example A: Simple equation, 1 metric, constant constraint and desirement, complementary constraint and desirement
- Example B: Simple equation, 1 metric, constant constraint and desirement, competing constraint and desirement
- Example C: Simple equation, 1 metric, variable constraint and desirement, complementary constraint and desirement
- Example D: Simple equation, 1 metric, variable constraint and desirement, competing constraint and desirement
- Example E: Aircraft Design example
- Example F: Fleet Design example

Purpose of Examples A and B is to illustrate the differences between the different techniques for the two different types of constraints/desirement relationships (complementary and competing) as illustrated in Figure 8-4.

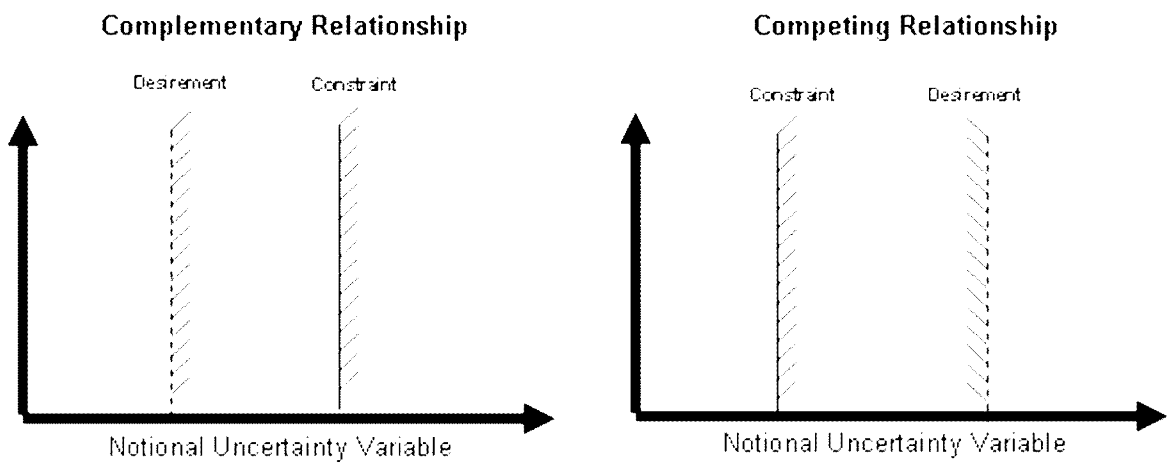

Figure 8-4: Illustration of the difference between complementary and competing constraint/desirement relationships

Examples C-D have a similar purpose as A and B, but adding in the complexity of variable constraints as is often the case in design problems.

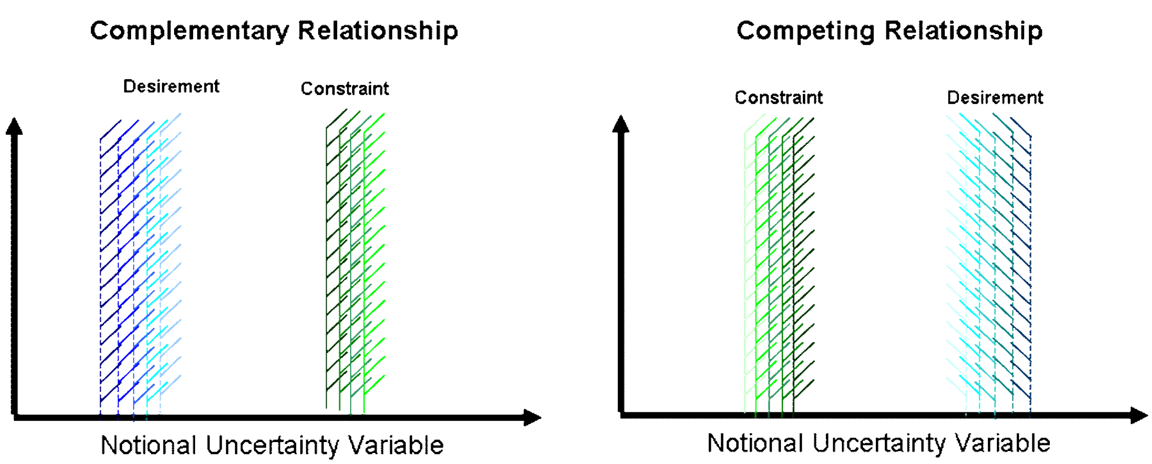

Figure 8-5: Illustration of the difference between complementary and competing constraint/desirement relationships for variable constraint and desirements

Example E was developed to show how these techniques could be used for a design problem with multiple metrics, constraints, and desirements, and finally Example F is used to show these techniques as applied to a SoS design problem. All of these examples build upon the previous in order to clearly emphasize the differences between Robust Design, Opportunistic Design, and Robust and Opportunistic design.

When evaluating the different design alternatives within a MADM technique an important aspect is the weight associated with the different metrics. In the design world with traditional metrics, this is often the subject of much debate since the weights significantly affects the final ranking of results. With the hybrid uncertainty modeling technique the traditional metrics have now become more abstract and more difficult for a designer to intuitively determine the appropriate weights. For this reason, for each of these example problems, a weight determination study is conducted as part of the analysis.

The main objective of this study is to determine if general weights can be used for the different metrics or if it is necessary to conduct a separate weight determination exercise for every design problem.

8.3.1. Weight Determination Analysis Overview

A set of 100 alternatives was analyzed for each approach. The alternatives were first generated using a latin hypercube sampling technique.[22] The HUMM was then performed on each of the identified alternatives from the design space. Each approach determined the respective metrics for each design alternative. In other words, the Robust Design approach determined the Plausible α and the Believable α, the Opportunistic Design approach determined the Plausible β and the Believable β, and the RandO Design calculated all four metrics for each design alternative.

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[22] This technique was originally presented in Reference 128 and was generated using the "lhsdesign" function within MATLAB. In addition to the standard space filling sampling technique, the algorithm iteratively created multiple latin hypercube samples to determine the set of values that would maximize the minimum distance between points in the space.

The next step was to determine the weights which resulted in the best performing values for the metrics from each approach. For this process the number of required weights was determined, which is equal to the number of metrics. Examples A-D have one traditional design metric (y), Example E has four traditional metrics, and Example F has three traditional metrics. Recall that each approach transforms the traditional design metric into multiple metrics. For example, Example F for the RandO Design approach now has 12 design metrics. Column 3 of Table lists the number of metrics for each design analysis.

Table 8-1: Weight Determination Test Matrix

Example	**Design Approach**	**Number of Metrics**	**Number of Weight Combinations Considered**	**Number of Monte Carlo Runs used in Weight Determination Analysis**
A	Robust	2	11	33,000
	Opportunistic	2	11	33,000
	RandO	4	1296	3,888,000
B	Robust	2	11	33,000
	Opportunistic	2	11	33,000
	RandO	4	1296	3,888,000
C	Robust	2	11	33,000
	Opportunistic	2	11	33,000
	RandO	4	1296	3,888,000
D	Robust	2	11	33,000
	Opportunistic	2	11	33,000
	RandO	4	1296	3,888,000
E	Robust	8	1000	3,000,000
	Opportunistic	8	1000	3,000,000
	RandO	16	1500	4,500,000
F	Robust	6	1000	3,000,000
	Opportunistic	6	1000	3,000,000
	RandO	12	1500	4,500,000

Based upon the number of metrics the number of weight combinations can be determined. For two metrics it was a simple matter of finding every combination of values from 0 to 1 summing to a value of one. The interval 0 to 1 was divided into tenths, resulting in eleven possible weight combinations.

When four metrics were involved, a full factorial DOE considering six levels of values for each of the four metrics was considered resulting in 1296 weight combination cases. The sum of each weight combination for each run in full factorial DOE was calculated, and then each of the four individual weight combinations was divided by this value to ensure that the summation of the weights would equal one.

For example problems E and F, the design analysis was too computationally expensive to run a Full Factorial DOE for every weight combination. Instead for each example, and each approach, the same latin hypercube sampling technique that was used to create a DOE to identify different potential alternative values was used to determine potential weight combinations. The number of weight combinations was set to 1000 for the Robust and Opportunistic Design in both example problems, and 1500 weight combinations were determined for the RandO design. In all cases the weight value was divided by the sum of the weight values for the individual DOE run to ensure that the total weight value for each run was equal to one.

Example problems E and F, each involved multiple traditional metrics and each of these traditional metrics was assigned a weight indicating its performance as part of the problem definition process. The weights associated with each of these traditional metrics are multiplied by the original weight in order to take into account the preference of the designer.

For each example problem and each design approach, a separate TOPSIS analysis was run for every weight combination in order to determine the “best” alternative. Each of these alternatives was then compared for three different uncertainty scenarios. For each scenario a range and distribution for the uncertainty variables is assumed for the problem. The first scenario is based upon average uncertainty values, the second scenario is based upon ideal uncertainty values (or less challenging uncertainty values as the case may be), and the third uncertainty scenario is based upon challenging uncertainty values. For each of the alternatives selected based upon the potential weight combination and for each scenario, a Monte Carlo Analysis consisting of 1000 runs is conducted using the provided range and distributions for the uncertainty variables. The total number of Monte Carlo runs completed for each analysis is shown in Column 5 of Table 8-1.

The data for each of the alternatives from the Monte Carlo (MC) analysis is then compared using the distance based weight modeling method discussed in Reference 188. In this technique, the distance between the resulting design metric calculated by the MC analysis, which is the traditional metric and not one based on α or β, and the desirement is calculated. If a constraint is violated the distance is increased through the use of an exterior penalty function.[198] The average resulting distance for each alternative is then compared and evaluated with TOPSIS to effectively rank the effectiveness of each of the weight combinations for each of the uncertainty scenarios. The top ranking weight combinations are determined and averaged to calculate the estimated best weight combination for each set of metrics.

Research Question: Is it necessary to repeat a similar process for every design problem or is it possible that certain weight combinations can be used for multiple problems without requiring a weight determination analysis process?

To answer this question 2 or more different weight combinations were calculated for each approach in each of the six example problems. The first weight combination, designated W1, consists of average values for the weights. If only two metrics are used then the weights are 0.5 for both metrics. If four metrics are used for each problem then each of the metrics is 0.25.

The second weight combination consisted of calculating the average of the determined weight combination for all of the metrics in the design problem. For example problems

A-D where there is only one metric, this weight combination is the same weight combination that was selected from the weight determination process. This weight combination is designated W2.

The third weight combination consists of calculating the average of the determined weight combination for each type of metric. As discussed early in this section there are two types of relationships between constraints and desirements, complementary and competing. Assuming that the constraints and desirements are directly related to the traditional metrics, there are two types of metrics, complementary and competing. For this weight combination, the determined weights associated with the complementary metric are averaged and the weights associated with the competing metric are averaged seperately. This weight combination is designated W3. For example problems A-D, there is no need to consider this weight combination because there is only one type of constraint considered for each problem and the value of this weight combination would be equal to W2.

The forth weight combination consists of the determined weight values. This weight combination is expected to produce the most accurate results, since it is the actual values that were selected from the weight determination process. These values are designated by W4, and for examples A-D is equivalent to W2.

Each of these weight combinations was then used for each design approach in each example problem. However, for the examples where multiple traditional metrics are involved, these average weights are multiplied by the provided weight to account for the preference of the user.

8.3.2. Penalty and Bonus Functions

Penalty and bonus functions all serve a purpose within each of the design approaches. Robust Design incorporates a penalty if a constraint has been violated, Opportunistic

Design incorporates a bonus function if the desirement is satisfied, and RandO utilizes both penalty and bonus functions within its process.

A traditional penalty function that is used in design is the exterior penalty function as discussed in Reference 198 and as shown in Equation 8-1. The advantage of this penalty function is that the penalty increases as the distance from the constraint increases.

$$\alpha_{c,v} = -\left|x_{c,v} - \tilde{x}_v\right| - r_c \cdot \left(\left|x_{c,v} - \tilde{x}_v\right|\right)^P$$ **Equation 8-1**

c is for each metric constraint

v is for each Info-Gap uncertainty variable

~ indicates nominal value

r_c and P are penalty function parameters

The two penalty parameters r_c and P can be selected by the designer based upon the design problem. Reference 198 sets P equal to two. For comparative purposes, Figure 8-6 plots the penalty function values for a variety of r_c where P is equal to 2. Figure 8-7 plots the penalty function values for the situation where P is equal to 3. After comparing these plots it is evident that for most design problems a P value of 2 will be adequate and setting P to a value of 3 or higher is likely to over penalize the metric value.

There is also some question as to the most appropriate value for the penalty parameters r_c. While the value is dependent upon the design problem it is necessary to penalize a value for violating a constraint, but it is also desired not to over penalize the constraint. Based upon Figures 8-6 and 8-7 a value of r_c equal to 0.3 represents a fair compromise between not including a strong enough penalty and over penalizing a value.

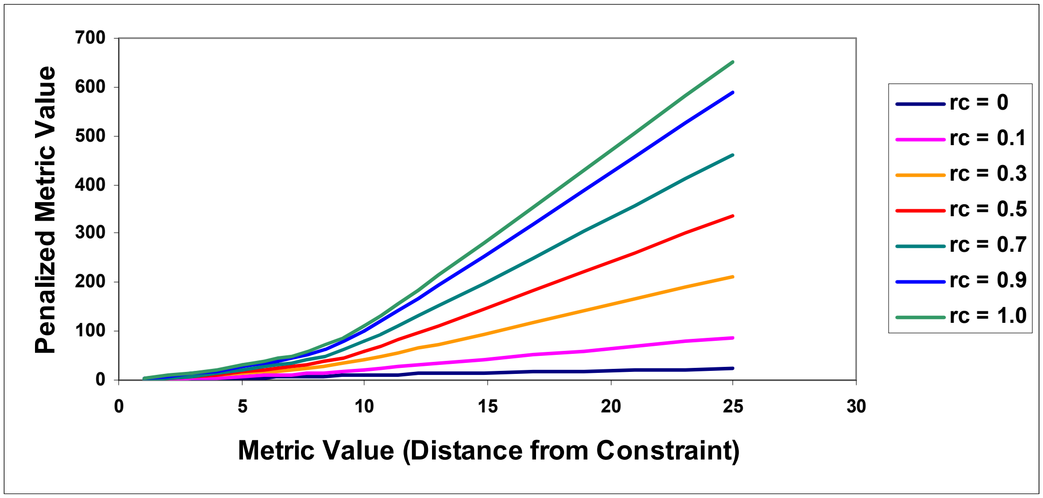

Figure 8-6: Penalty Function Values for P=2

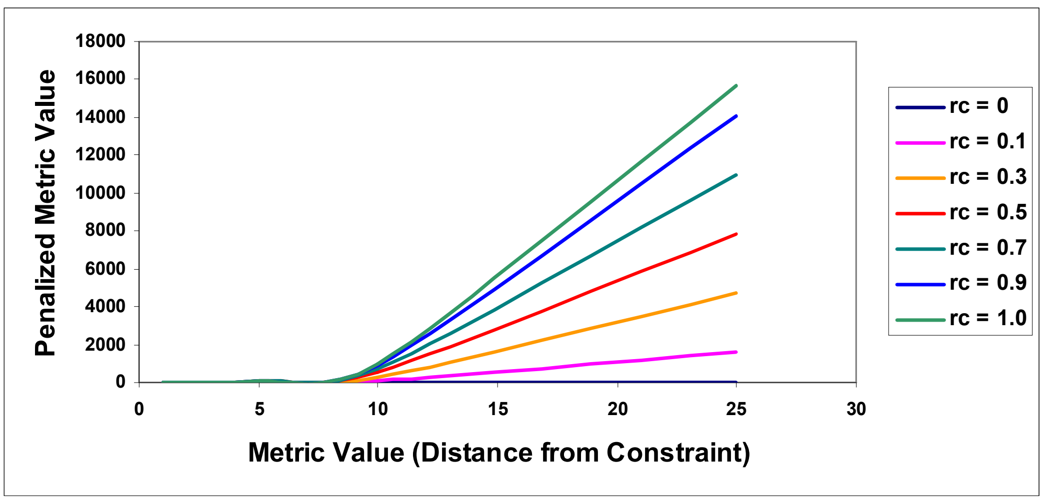

Figure 8-7: Penalty Function Values for P=3

As discussed in Chapter 4, in many cases the distance between the nominal value and the constraint may be such a small distance that the penalty is very minor. As shown in Figure 8-6, values that barely violate the constraint are hardly penalized. For cases where it is acceptable for the constraint to be slightly violated, this type of penalty function will be satisfactory. But, for situations where the constraint should not be violated, even by a small amount, it is suggested to incorporate an additional Penalty Factor (PF) to the penalty function. This penalty factor as shown in Equation 8-2 applies a set penalty for any constraint violation. Equation 8-3 provides an option for a general penalty factor, based on the maximum and minium values of the Robustness Function, where PF% is a penalty factor percentage provided by the designer.

$$\alpha_{c,v} = -\left|x_{c,v} - \tilde{x}_v\right| - PF - r_c \cdot \left(\left|x_{c,v} - \tilde{x}_v\right|\right)^P$$ **Equation 8-2**

$$PF = \left|\alpha_{\max} - \alpha_{\min}\right| \cdot PF\%$$ **Equation 8-3**

For the situation when it is appropriate to apply a bonus function, the exterior penalty function from Reference 198 can be modified to serve as a bonus function. This bonus function is shown in Equation 8-4. Because r_s and P are equivalent to the penalty function parameters previously discussed it is suggested to model these values as 0.3 and 2 respectively.

$$\beta_{s,v} = -\left|x_{s,v} - \tilde{x}_v\right| - r_s \cdot \left(\left|x_{s,v} - \tilde{x}_v\right|\right)^P$$ **Equation 8-4**

s is for each metric desirement

v is for each Info-Gap uncertainty variable

~ indicates nominal value

r_s and P are bonus function parameters

8.3.3. Example A: Simple equation, 1 metric, constant constraint and desirement, complementary constraint and desirement

This example is used to illustrate the performance of the different design approaches for the situation when there is a complementary constraint and desirement relationship. This problem, as well as Examples B-D, consists of simple equations for both its design metric and for its constraint and desirement so that the emphasis is on the design approaches and not on the example problem itself.

The design alternatives are modeled by ellipse by the equations below.

$$y = k + a \cdot \sin(\theta)$$ **Equation 8-5**

$$x = h + b \cdot \cos(\theta)$$ **Equation 8-6**

Constraint and success modeled by curves based upon the following equations.

$$y > y_{Constraint} = d \cdot x^{e} + f$$ **Equation 8-7**

$$y > y_{Desirement} = d \cdot x^{e} + f + g$$ **Equation 8-8**

For this problem the constraint and desirement are constant.

d = 0.3 **Equation 8-9**

e = 2 **Equation 8-10**

f = 0.5 **Equation 8-11**

$g = 20$ **Equation 8-12**

For this problem there are two design variables, h and k, and the uncertainty variables are a,b, and θ. Table 8-2 lists the design variable ranges that bounded design space. The design space was explored through the latin hypercube sampling technique discussed earlier in this chapter, and 100 design alternatives were selected. The selected design points (design alternatives) are illustrated in Figure 8-8. These design alternatives were also used for Example B, C, and D.

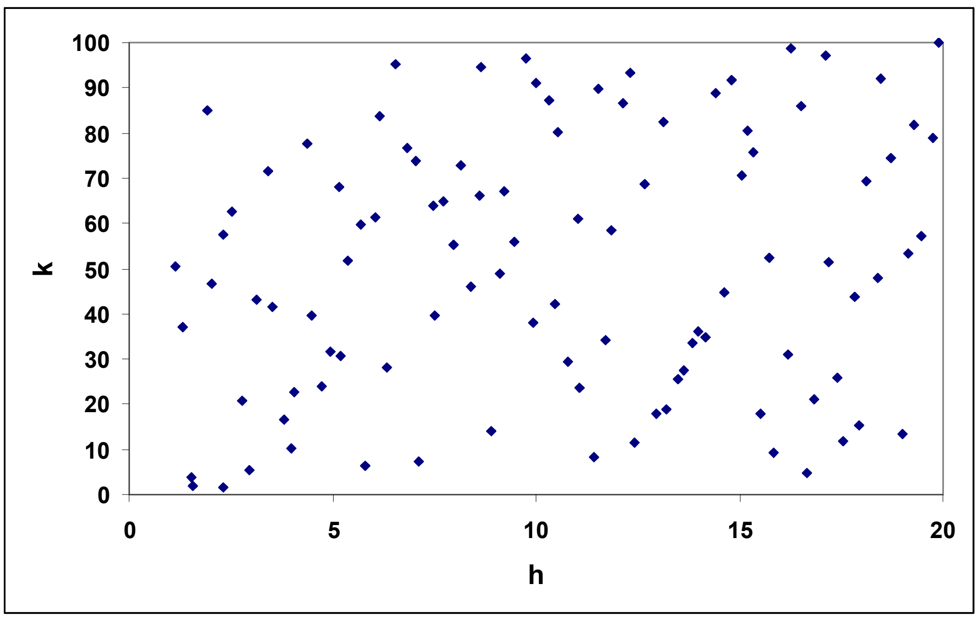

Figure 8-8: Selected design alternatives for Examples A-D

The uncertainty variable characteristics are listed in Table 8-3. Based upon the characteristics in columns 2-5 of Table 8-3, the uncertainty modeling technique was selected.

Table 8-2: Design variable ranges

Design Variable	Minimum Value	Maximum Value
h	1	20
k	1	100

Table 8-3: Uncertainty Variable Parameters

Uncertainty Variable	Uncertainty Type	Distribution	Minimum Value	Maximum Value	Uncertainty Modeling Technique
a	Ambiguity	unknown	0.5	4	Evidence Theory
b	Ambiguity	unknown	unknown	unknown	Info-Gap Theory
θ	Ambiguity	uniform	-π	π	Probability Theory

The nominal value of b to be used in Info-Gap Theory is 10 and the maximum expected range was set from 1 to 30. These values are used in the HUMM described in Chapter 7 and the metric values for each of the design approaches for each design alternative is calculated. In this problem, it is not appropriate to model any of the uncertainty with Fuzzy Set Theory, so this technique has been omitted from the process.

The results from the weight determination process for each of the approaches as described earlier in this chapter are presented in Table 8-4. The "best" alternative from each approach and each weight is presented in Table 8-5.

Table 8-4: Weight Results from Weight Determination Study (Example A)

		α Plausible	α Believable	β Plausible	β Believable
Robust Design	W1	0.5	0.5	--	--
	W2	0.5	0.5	--	--
Opportunistic Design	W1	--	--	0.5	0.5
	W2	--	--	0.5	0.5
RandO Design	W1	0.25	0.25	0.25	0.25
	W2	0.25	0.25	0.25	0.25

Table 8-5: Selected Alternatives from Each Design Approach (Example A)

		h	k
Robust Design	W1	1.9108	85.098
	W2	1.9108	85.098
Opportunistic Design	W1	1.9108	85.098
	W2	1.9108	85.098
RandO Design	W1	1.9108	85.098
	W2	1.9108	85.098

As shown in Table 8-5. All of the design approaches selected the same design alternative. This is because the constraint and desirement are complementary and by satisfying the desirement the constraint is automatically satisfied.

While it was not necessary to compare these different alternatives in TOPSIS the 3 Monte Carlo (MC) Scenarios were conducted to illustrate how the different uncertainty scenarios affected the selected alternative. The uncertainty parameters for the MC analyses are provided in Table 8-6 . The results from these analyses are presented in Figure 8-9 and Appendix B. Each blue dot represents the results from a specific MC run. The constraint is plotted as the solid line and the desirement is plotted by the dashed line.

Table 8-6: Uncertainty variable ranges for Monte Carlo Analyses (Example A)

Uncertainty Variable	Minimum Value	Maximum Value	MC Group 1 Beta Distribution Parameters	MC Group 2 Beta Distribution Parameters	MC Group 3 Beta Distribution Parameters
a	0.5	4	P1: 4, P2: 4	P1: 2, P2: 8	P1: 8, P2: 2
b	5	20	P1: 4, P2: 4	P1: 2, P2: 8	P1: 8, P2: 2
θ	-π	π	P1: 4, P2: 4	P1: 4, P2: 4	P1: 4, P2: 4

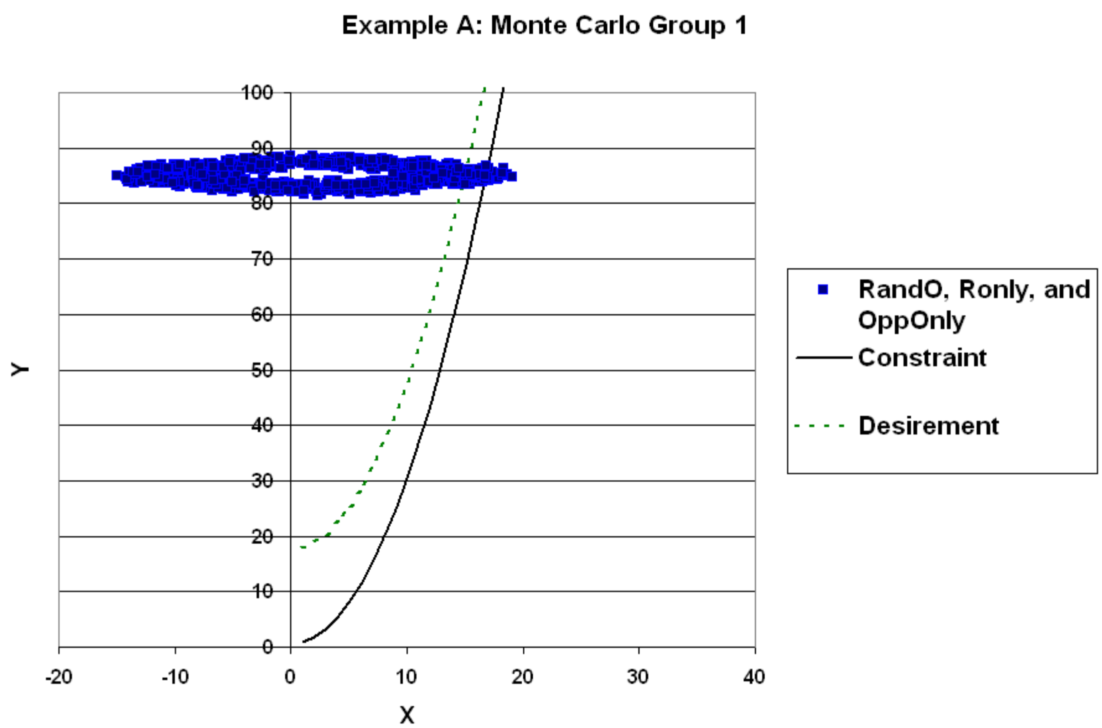

Figure 8-9: Monte Carlo Analysis Results from Uncertainty Scenario 1 (Example A)

<u>Example B: Simple equation, 1 metric, constant constraint and desirement, competing constraint and desirement</u>

This example is very similar to Example A, with one significant change. Now the constraint and the desirement have a competing relationship. As with Example A, the design alternatives are modeled by ellipse through the equations listed below.

$$y = k + a \cdot \sin(\theta) \qquad \textbf{Equation 8-13}$$

$$x = h + b \cdot \cos(\theta) \qquad \textbf{Equation 8-14}$$

The constraint and desirement are modeled by the following equations.

$$y > y_{Constraint} = d \cdot x^{e} + f \qquad \textbf{Equation 8-15}$$

$$y < y_{Desirement} = 0 \qquad \textbf{Equation 8-16}$$

For this example, the constraint requires that the value for y be above a specific value at every x location, but to satisfy the desirement, the y value would need to be less than 0. In actuality the objective of this problem is to minimize y, but it is constrained to be greater than the specified constraint value. Within the design analysis the highest priority typically relates to meeting the constraint requirements and then the next priority is to satisfying the desirement. For this reason, it is suggested to model the desirement with one of two options.

Option 1:

$$y < y_{Desirement} = d \cdot x^{e} + f \qquad \textbf{Equation 8-17}$$

Set the desirement to the constraint value. While it will not be possible to ever "succeed", the closer an alternative is to the constraint the better the alternative metric

Option 2:

$$y < y_{Desirement} = d \cdot x^e + f + 0.05 \cdot \left(d \cdot x^e + f\right) \qquad \textbf{Equation 8-18}$$

This option is where the desirement is pushed away from the constraint by a small value (5% in this example). The purpose of pushing the desirement from the constraint is to create a region in the design next to the constraint where an alternative is rewarded (through the bonus function) if it is near the constraint. The purpose of this is to encourage designs very near the constraint without violating the constraint value.

As with Example A, the constraint and desirement are constant and are set to the following values.

d = 0.3 **Equation 8-19**

e = 2 **Equation 8-20**

f = 0.5 **Equation 8-21**

For this problem there are two design variables h and k, and the uncertainty variables are a,b, and θ. Table 8-7 lists the design variable ranges that bounded the design space. The same design points that were selected for Example A through the latin hypercube sampling technique were used in this example.

The uncertainty variable characteristics are listed in Table 8-8. Based upon the characteristics in columns 2-5 of Table 8-8, the uncertainty modeling technique was selected. Note, that the uncertainty is the same for this problem as it was for Example A. The only difference between this problem and Example A is the desirement.

Table 8-7: Design variable ranges (Example B)

Design Variable	Minimum Value	Maximum Value
h	1	20
k	1	100

Table 8-8: Uncertainty Variable Parameters (Example B)

Uncertainty Variable	Uncertainty Type	Distribution	Minimum Value	Maximum Value	Uncertainty Modeling Technique
a	Ambiguity	unknown	0.5	4	Evidence Theory
b	Ambiguity	unknown	unknown	unknown	Info-Gap Theory
θ	Ambiguity	uniform	-π	π	Probability Theory

The nominal value of b to be used in Info-Gap Theory is 10 and the maximum expected range was set from 1 to 30. These values are used in the process described in Chapter 7 and the metric values for each of the design approaches for each design alternative is calculated. The results from the weight determination process for each of the approaches as described earlier in this chapter are presented in Table 8-9. The “best” alternative from each approach and each weight is presented in Table 8-10.

Table 8-9: Weight Results from Weight Determination Study (Example B)

		α Plausible	α Believable	β Plausible	β Believable
Robust Design	W1	0.5	0.5	--	--
	W2	0.5	0.5	--	--
Opportunistic Design	W1	--	--	0.5	0.5
	W2	--	--	0.5	0.5
RandO Design	W1	0.25	0.25	0.25	0.25
	W2	0.32	0.22	0.26	0.2

Table 8-10: Alternatives selected from each design approach (Example B)

		h	k
Robust Design	W1	1.9108	85.098
	W2	1.9108	85.098
Opportunistic Design	W1	19.002	13.531
	W2	19.002	13.531
RandO Design	W1	1.9108	85.098
	W2	1.1415	50.343

Now for the same problem, with a competing constraint and desirement, each of the three approaches identifies a different alternative as the best solution to the problem while considering the uncertainty. These alternatives are all compared for the three uncertainty scenarios listed in Table 8-11. The range of the uncertainty variables for each of the scenarios is constant, but the distribution parameters change. The results are shown in Figure 8-10 and Appendix B. The results from the TOPSIS analyses for each uncertainty scenarios are listed in Table 8-12 and the average Overall Evaluation Criterion values from the analysis are plotted in Figure 8-11.

Table 8-11: Uncertainty Variable Ranges

Uncertainty Variable	Minimum Value	Maximum Value	MC Group 1 Beta Distribution Parameters	MC Group 2 Beta Distribution Parameters	MC Group 3 Beta Distribution Parameters
a	0.5	4	P1: 4, P2: 4	P1: 2, P2: 8	P1: 8, P2: 2
b	5	20	P1: 4, P2: 4	P1: 2, P2: 8	P1: 8, P2: 2
θ	-π	π	P1: 4, P2: 4	P1: 4, P2: 4	P1: 4, P2: 4

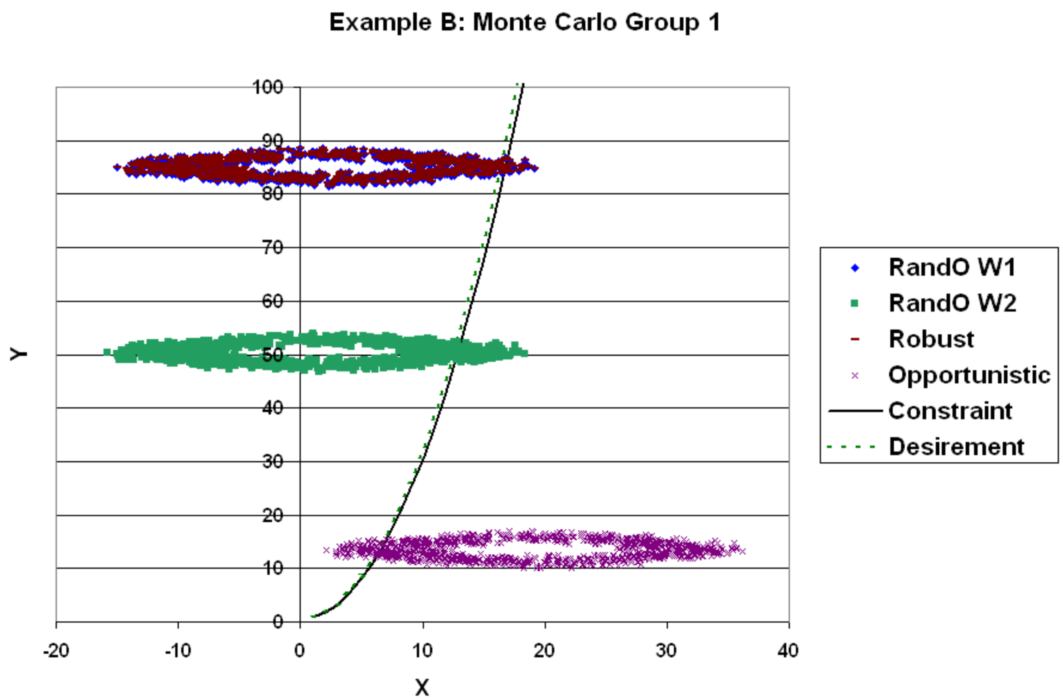

Figure 8-10: Monte Carlo Analysis Results for Uncertainty Scenario 1 for Example B

In this plot only three alternatives are visible because RandO Design (W1) and the Robust Design (W1 and W2) selected the same design alternative. The blue diamonds, representing the various RandO Design W1 Monte Carlo points, are obscured by the Robust Design data points.

Table 8-12: Overall Evaluation Criterion Values from MADM Analysis (Example B)

	Monte Carlo Group 1	Monte Carlo Group 2	Monte Carlo Group 3	Average
RandO W1	0.98792584	0.978789204	1	0.988905
RandO W2	1	1	0.994794537	0.998265
Robust W1	0.987925865	0.978789204	1	0.988905
Robust W2	0.987925865	0.978789204	1	0.988905
Opportunistic W1	0	0	0	0
Opportunistic W2	0	0	0	0

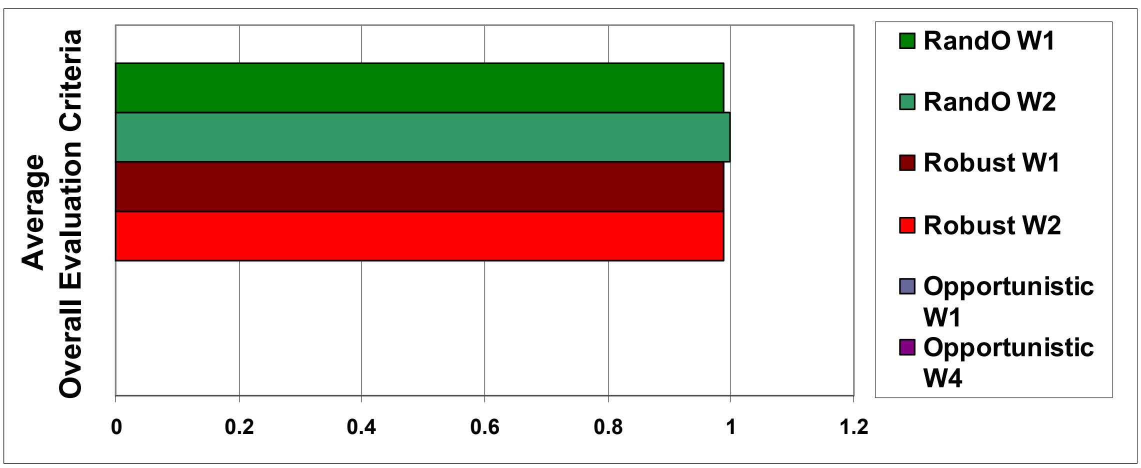

Figure 8-11: Average Overall Evaluation Criterion Values from MADM Analysis (Example B)

8.3.4. <u>Example C: Simple equation, 1 metric, variable constraint and desirement, complementary constraint and desirement</u>

Often in design, the constraints and desirements themselves will be functions of the uncertainty. The purpose of this example is to illustrate the concept of variable constraint and desirements, and is very similar to Example A. The difference is that variables d,e,f, and g are no longer constant.

The equations for the design alternatives, the constraints, and the desirements are shown below.

$$y = k + a \cdot \sin(\theta)$$ **Equation 8-22**

$$x = h + b \cdot \cos(\theta)$$ **Equation 8-23**

$$y > y_{Constraint} = d \cdot x^{e} + f$$ **Equation 8-24**

$$y > y_{Desirement} = d \cdot x^{e} + f + g$$ **Equation 8-25**

In general (for g>0) this is a complementary constraint and desirement. If the desirement is satisfied, the constraint is satisfied.

The design alternatives are the same as those used in both Example A and B. The uncertainty variables are a,b,θ,d,e,f, and g.

Table 8-13: Uncertainty Variable Parameters (Example C)

Uncertainty Variable	Uncertainty Type	Distribution	Minimum Value	Maximum Value	Uncertainty Modeling Technique
a	Ambiguity	unknown	0.5	4	Evidence Theory
b	Ambiguity	unknown	unknown	unknown	Info-Gap Theory
θ	Ambiguity	uniform	-π	π	Probability Theory
d	Ambiguity	unknown	unknown	unknown	Info-Gap Theory
e	Ambiguity	unknown	2	3	Evidence Theory
f	Ambiguity	unknown	unknown	unknown	Info-Gap Theory
g	Ambiguity	unknown	5	20	Evidence Theory

The nominal value of b to be used in Info-Gap theory is 10 and the maximum expected range was set from 1 to 30. For the variable d, the nominal value is 2 and the maximum range is from 0.1 to 4. The third variable to be modeled with Info-Gap Theory is f. The nominal value of this variable is 15 and it may potentially vary from 0 to 30.

These values are used in the HUMM described in Chapter 7 and the metric values for each of the design approaches for each design alternative is calculated. The results from the weight determination process for each of the approaches as described in the beginning of this chapter are presented in Table 8-14. The "best" alternative from each approach and each weight is presented in Table 8-15.

Table 8-14: Weight Results from Weight Determination Study (Example C)

		α Plausible	α Believable	β Plausible	β Believable
Robust Design	W1	0.5	0.5	--	--
	W2	0.5	0.5	--	--
Opportunistic Design	W1	--	--	0.5	0.5
	W2	--	--	0.5	0.5
RandO Design	W1	0.25	0.25	0.25	0.25
	W2	0.32	0.22	0.26	0.2

Table 8-15: Alternatives Selected from each Design Approach (Example C)

		h	k
Robust Design	W1	1.9108	85.098
	W2	1.9108	85.098
Opportunistic Design	W1	1.9108	85.098
	W2	1.9108	85.098
RandO Design	W1	1.9108	85.098
	W2	1.9108	85.098

As with Example A, all of the design approaches selected the same design alternative. This was expected for this example because the constraint and desirement are complementary and by satisfying the desirement the constraint is automatically satisfied. While it was not necessary to compare these different alternatives in TOPSIS the three Monte Carlo (MC) Scenarios were conducted to illustrate the variable constraint and desirement as well as to illustrate how the different uncertainty scenarios affected the selected alternative. The uncertainty parameters for the MC analyses are provided in Table 8-16 . The results from these analyses are presented in Figure 8-12 and Appendix B. Each blue dot represents the results from a specific MC run. The constraint is plotted as the solid line and the constraint is plotted by the dashed line.

Table 8-16: Uncertainty Variable Values (Example C)

Uncertainty Variable	Minimum Value	Maximum Value	MC Group 1 Beta Distribution Parameters	MC Group 2 Beta Distribution Parameters	MC Group 3 Beta Distribution Parameters
a	0.5	4	P1: 4, P2: 4	P1: 2, P2: 8	P1: 8, P2: 2
b	5	20	P1: 4, P2: 4	P1: 2, P2: 8	P1: 8, P2: 2
θ	-π	π	P1: 4, P2: 4	P1: 4, P2: 4	P1: 4, P2: 4
d	0.3	3	P1: 4, P2: 4	P1: 2, P2: 8	P1: 8, P2: 2
e	2	3	P1: 4, P2: 4	P1: 2, P2: 8	P1: 8, P2: 2
f	0	25	P1: 4, P2: 4	P1: 2, P2: 8	P1: 8, P2: 2
g	5	20	P1: 4, P2: 4	P1: 2, P2: 8	P1: 8, P2: 2

A number of representative constraint lines are plotted in Figure 8-12. Each of these constraint lines is applicable for a specific set of values for the uncertainty variables. In actuality at one time or another, the constraint line could fall anywhere within the bounded region. The desirement was not plotted in this figure to avoid cluttering the figure unnecessarily. For every x value, the desirement has the same value of y as the constraint plus the value of the uncertain variable g, which varies between values of 5 and 20.

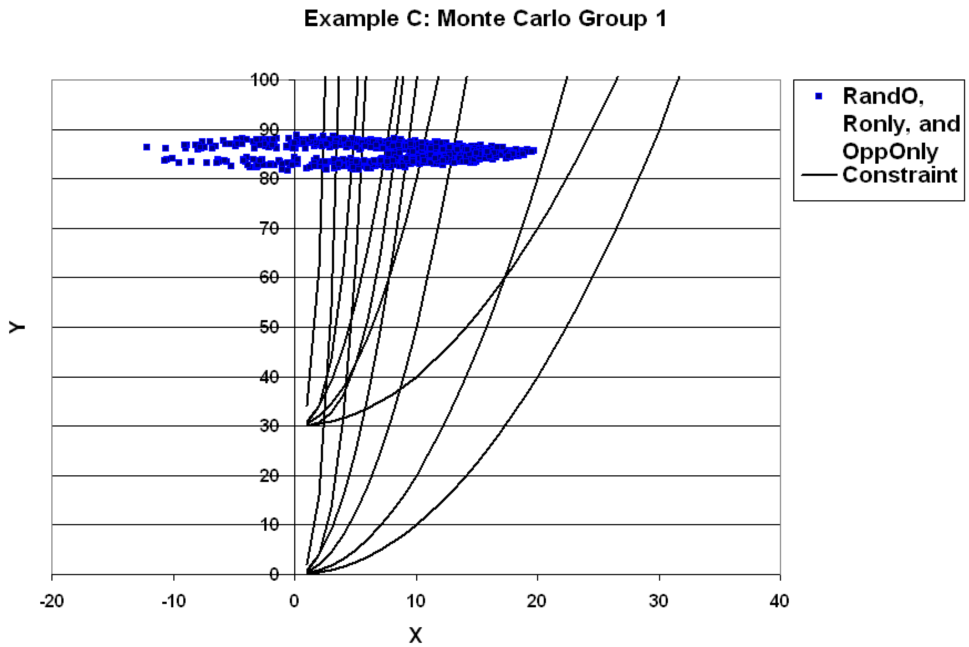

Figure 8-12: Monte Carlo Analysis Data for Uncertainty Scenario 1 (Example C)

8.3.5. <u>Example D: Simple equation, 1 metric, variable constraint and desirement, competing constraint and desirement</u>

Example D is very similar to example B, except now as with the previous example, the constraint and desirement are dependent on the uncertainty. The equations for the design variables, constraints, and desirements are listed below.

$$y = k + a \cdot \sin(\theta) \qquad \textbf{Equation 8-26}$$

$$x = h + b \cdot \cos(\theta) \qquad \textbf{Equation 8-27}$$

$$y > y_{Constraint} = d \cdot x^{e} + f \qquad \textbf{Equation 8-28}$$

$$y < y_{Desirement} = 0$$ **Equation 8-29**

Because the desirement violates the constraint, this equation is modified to meet the constraint and is shown below.

$$y > y_{Constraint} = d \cdot x^e + f$$ **Equation 8-30**

$$y < y_{Desirement} = d \cdot x^e + f + 0.05 \cdot \left(d \cdot x^e + f\right)$$ **Equation 8-31**

The design alternatives are the same as those used in both Example A and B. The uncertainty variables are a,b,θ,d,e, and f.

Table 8-17: Uncertainty Variable Parameters

Uncertainty Variable	Uncertainty Type	Distribution	Minimum Value	Maximum Value	Uncertainty Modeling Technique
a	Ambiguity	unknown	0.5	4	Evidence Theory
b	Ambiguity	unknown	unknown	Unknown	Info-Gap Theory
θ	Ambiguity	uniform	-π	π	Probability Theory
d	Ambiguity	unknown	unknown	Unknown	Info-Gap Theory
e	Ambiguity	unknown	2	3	Evidence Theory
f	Ambiguity	unknown	unknown	Unknown	Info-Gap Theory

The nominal value of b to be used in Info-Gap Theory is 10 and the maximum expected range was set from 1 to 30. For the variable d, the nominal value is 2 and the maximum range is from 0.1 to 4. The third variable to be modeled with Info-Gap theory is f. The nominal value of this variable is 15 and it may potentially vary from 0 to 30.

These values are used in the HUMM described in Chapter 7 and the metric values for each of the design approaches for each design alternative is calculated. The results from the weight determination process for each of the approaches are presented in Table 8-18. The "best" alternative from each approach and each weight is presented in Table 8-19.

Table 8-18: Weight Results from Weight Determination Study (Example D)

		α Plausible	**α Believable**	**β Plausible**	**β Believable**
Robust Design	W1	0.5	0.5	--	--
	W2	0.5	0.5	--	--
Opportunistic Design	W1	--	--	0.5	0.5
	W2	--	--	0.5	0.5
RandO Design	W1	0.25	0.25	0.25	0.25
	W2	0.44	0.31	0.13	0.12

Table 8-19: Alternatives selected for each Design Approach (Example D)

		h	**k**
Robust Design	W1	1.9108	85.098
	W2	1.9108	85.098
Opportunistic Design	W1	19.001	13.531
	W2	19.001	13.531
RandO Design	W1	12.957	17.918
	W2	1.9108	85.098

The Robust Design approach consistently throughout all four examples has selected the same point which maximizes the distance to the constraint. For Examples B and D, on the other hand, Opportunistic Design maximizes the distance by which the alternative satisfies the constraint. The RandO approach then combines these techniques to reach a design compromise that both strives to satisfy the constraints and the desirements simultaneously.

The selected alternatives are all compared for the three uncertainty scenarios listed in Table 8-20. The range of the uncertainty variables for each of the scenarios is constant, but the distribution parameters change. The results are shown in Figure 8-13 and Appendix B. The results from the TOPSIS analyses for each uncertainty scenarios are listed in Table 8-21 and the average Overall Evaluation Criterion (OEC) values from the analysis are plotted in Figure 8-14.

In Figure 8-13 a number of representative constraint lines are shown. Each of these constraint lines is applicable for a specific set of values for the uncertainty variables. In actuality at one time or another, the constraint line could fall anywhere within the bounded region. As with the figures for Example B, the desirement was not plotted in this figure to avoid cluttering the figure unnecessarily. Due to the competing relationship between the desirement and the constraint, the desirement was offset by +5% of the value of the constraint.

Table 8-20: Uncertainty variable ranges (Example D)

Uncertainty Variable	Minimum Value	Maximum Value	MC Group 1 Beta Distribution Parameters	MC Group 2 Beta Distribution Parameters	MC Group 3 Beta Distribution Parameters
a	0.5	4	P1: 4, P2: 4	P1: 2, P2: 8	P1: 8, P2: 2
b	5	20	P1: 4, P2: 4	P1: 2, P2: 8	P1: 8, P2: 2
θ	-π	π	P1: 4, P2: 4	P1: 4, P2: 4	P1: 4, P2: 4
d	0.3	3	P1: 4, P2: 4	P1: 2, P2: 8	P1: 8, P2: 2
e	2	3	P1: 4, P2: 4	P1: 2, P2: 8	P1: 8, P2: 2
f	0	25	P1: 4, P2: 4	P1: 2, P2: 8	P1: 8, P2: 2

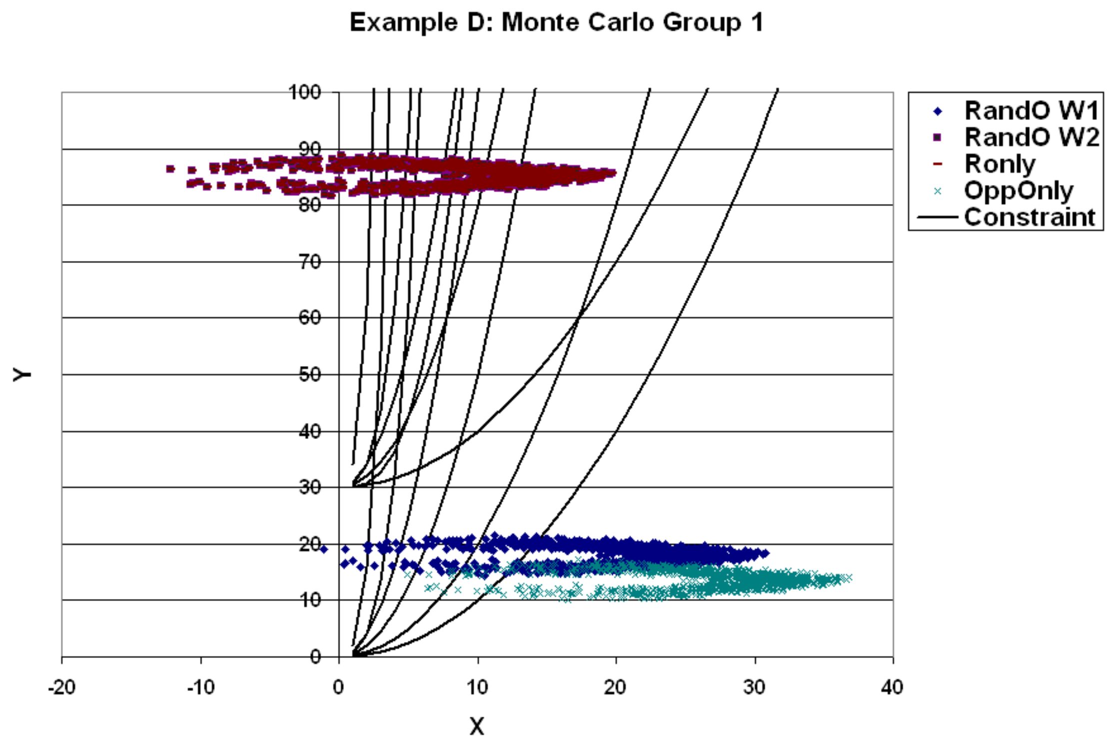

Figure 8-13: Monte Carlo Analysis Results for Uncertainty Scenario 1 (Example D)

Table 8-21: Overall Evaluation Criterion from MADM Analysis (Example D)

	Monte Carlo Group 1	**Monte Carlo Group 2**	**Monte Carlo Group 3**	**Average**
RandO W1	0.684	0.684	0.676	0.681333
RandO W2	1	1	1	1
Robust W1	1	1	1	1
Robust W2	1	1	1	1
Opportunistic W1	0	0	0	0
Opportunistic W2	0	0	0	0

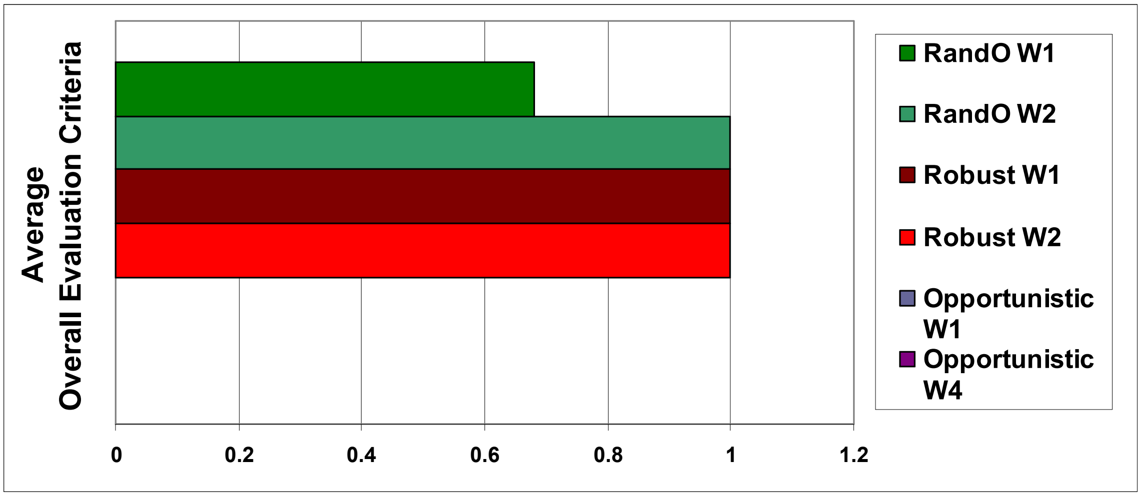

Figure 8-14: Average Overall Evaluation Criterion from MADM Analysis (Example D)

8.3.6. <u>Example E: Aircraft Design Example</u>

The three different approaches have been illustrated in simple examples for the different constraint/desirement relationships and for scenarios with variable constraints and desirements. In each of these cases there was only one metric and only one type of metric being considered. Additionally, while these examples illustrated differences between the different approaches, they are purely theoretical and have no connection to systems or SoS.

Example E was selected to illustrate the effects from multiple traditional metrics with both types of constraint/desirement relationships as well as to illustrate how these approaches relate to a simplified aircraft design problem. The aircraft design problem has been simplified to emphasize the differences between the approaches but still demonstrates how this process could be used for a system/SoS design process.

The traditional design metrics for this simplified problem were selected based upon metrics that were found to be important in historical persistent strike scenarios. For an actual design process for this type of aircraft it is likely that a number of additional design metrics would need to be considered. Because the purpose of this example is to illustrate the differences between the various design approaches and not to develop a design for an actual persistent strike aircraft, these simplifications are reasonable.

The objective of this example problem is to design a conceptual UAV for a general persistent strike scenario. The aircraft is to be designed for a mission where it takes off from a base, flies to a designated killbox (or search area), loiters in this killbox, and then returns to the base. From historical data[23] it is common that for most sorties the aircraft will not locate a target. [48] For this reason, in this example, the aircraft will be sized based upon the mission where the aircraft only searches for a target. The mission profile is illustrated in Figure 8-15.

Additionally, the aircraft design is to be constrained by takeoff performance requirements and maximum speed requirements. In many persistent strike scenarios, the number of nearby bases and runways will be limited. The aircraft needs to be capable of taking off from the available runways. Additionally, the aircraft needs to be capable of rapidly flying to a region where a target has been located.[24] For this reason, the aircraft is constrained by the need to be capable of flying at a certain maximum Mach number.

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[23] In the Scud Hunt in Desert Storm, it was common for the search/strike aircraft to return at the end of a mission without having located any mobile scud launchers (targets). [177]

[24] In Desert Storm, once a Scud missile was launched, nearby aircraft would fly to the estimated location of the launch in order to try and locate the mobile missile launcher. [177]

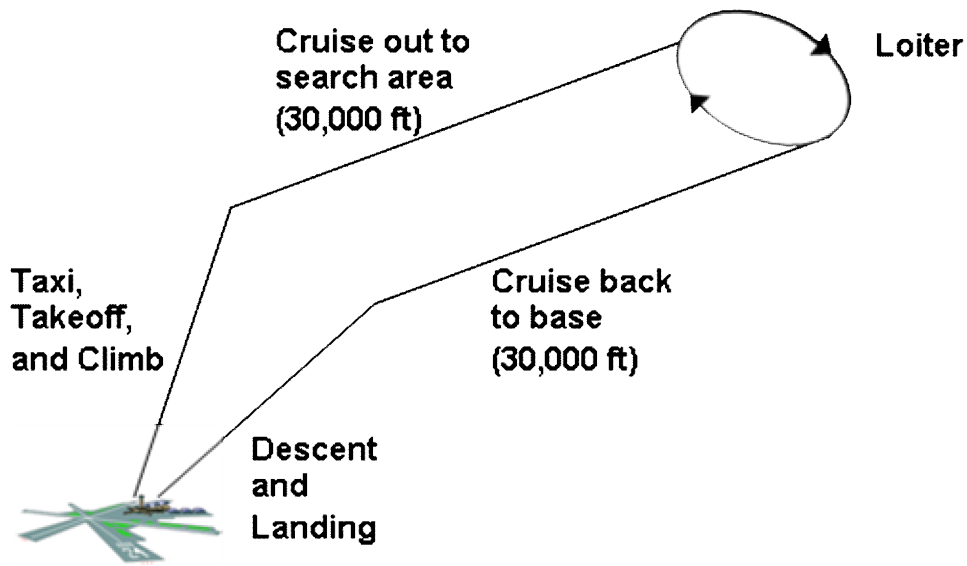

Figure 8-15: Mission Profile for Example E

As with the design of any system, economic metrics must be considered along with the performance metrics. Even if a system (or SoS) is capable of achieving, even excelling, its performance metrics, a system will not be produced if it is not economically viable.

The design metrics, constraints, and desirements are listed in Table 8-22. Both the constraint and desirements relating to the cost metric and the loiter time metric are complementary, but the constraint and desirement compete for the requirements relating to the aircrafts thrust to weight (T/W) metrics. For both of the T/W metrics it is necessary to have a higher available T/W ratio than is required to meet the performance requirement. However, the higher this ratio typically the larger the engine, which adds weight and cost to the aircraft. Therefore, it is desired to minimize the available T/W. So that the constraint is not violated by the desirement, the difference between the available T/W and the required T/W is set to a value of 0.05 so that design alternatives are rewarded the closer they are to the constraint.

The relative importance of each metric (Column 4, Table 8-22) were selected based upon a broad understanding of the historical events described in Reference 47. However, these weights should be considered as notional values used in this example for the purpose of illustrating that it is likely in a design problem for the design metrics to have different relative importances.

Table 8-22: Constraint / Desirement Definition (Example E)

Traditional Design Metric	**Design Constraint**	**Design Desirement**	**Relative Importance of Metric**
Aircraft Acquisition Cost	< \$30 Million	< \$10 Million	0.2
Loiter Time	> 10 hours	20 hours	0.3
$T/W_{Available}$-$T/W_{Required}$ for Takeoff Requirements	> 0	< 0.05	0.1
$T/W_{Available}$-$T/W_{Required}$ for Maximum Mach Requirements	> 0	< 0.05	0.4

The model for this problem considers three aspects of the design problem: the sizing of the aircraft based upon fuel requirements, performance constraint requirements, and the acquisition cost of the aircraft.

Aircraft Sizing

The TOGW (W_0) for an unmanned aircraft can be estimated from an initial guess for the TOGW, the weight of the payload, the empty weight fraction (W_e/W_0), and the total mission weight fraction (W_f/W_0). The equation for the TOGW is shown in Equation 8-32.

Equation 8-32

$$W_0 = W_{payload} + \frac{W_f}{W_0} W_{0,guess} + \frac{W_e}{W_0} W_{0,guess}$$

Empty Weight Fraction

The empty weight fraction can be estimated from historical trends based upon characteristics of the aircraft. Based upon historical data for the unmanned aircraft the empty weight fraction can be selected based upon the type of propulsion system used as illustrated in Figures 8-16 and 8-17.

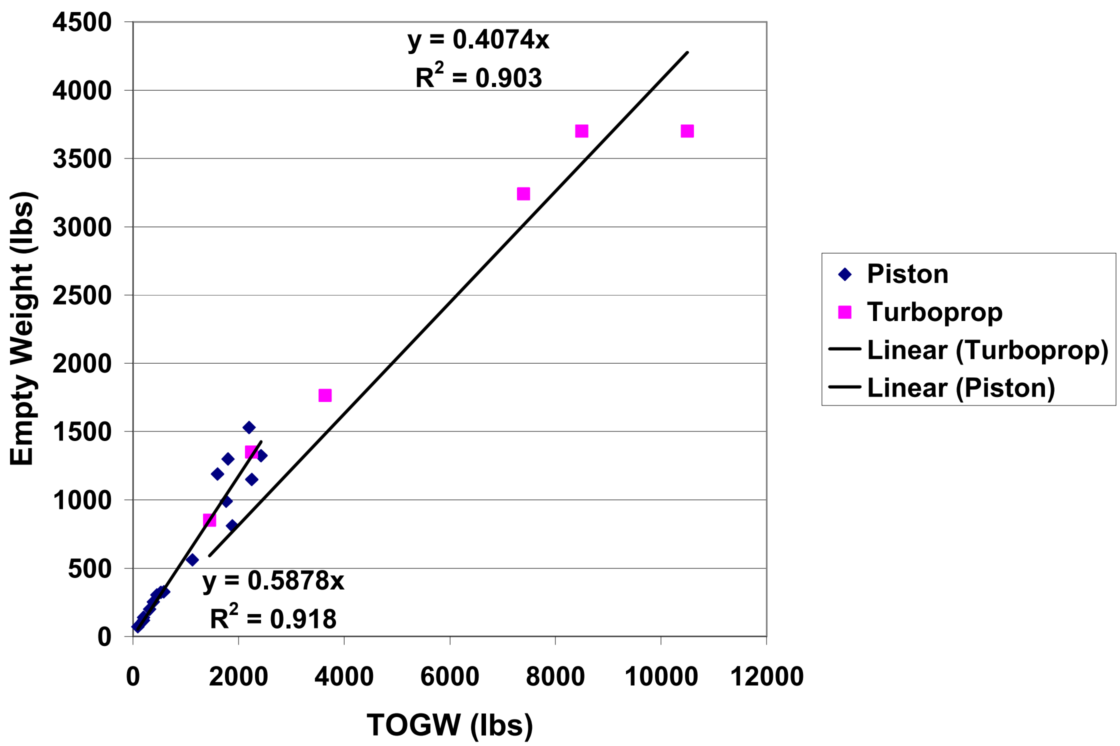

Figure 8-16: Empty Weight Fraction Determination (Piston / Turboprob)

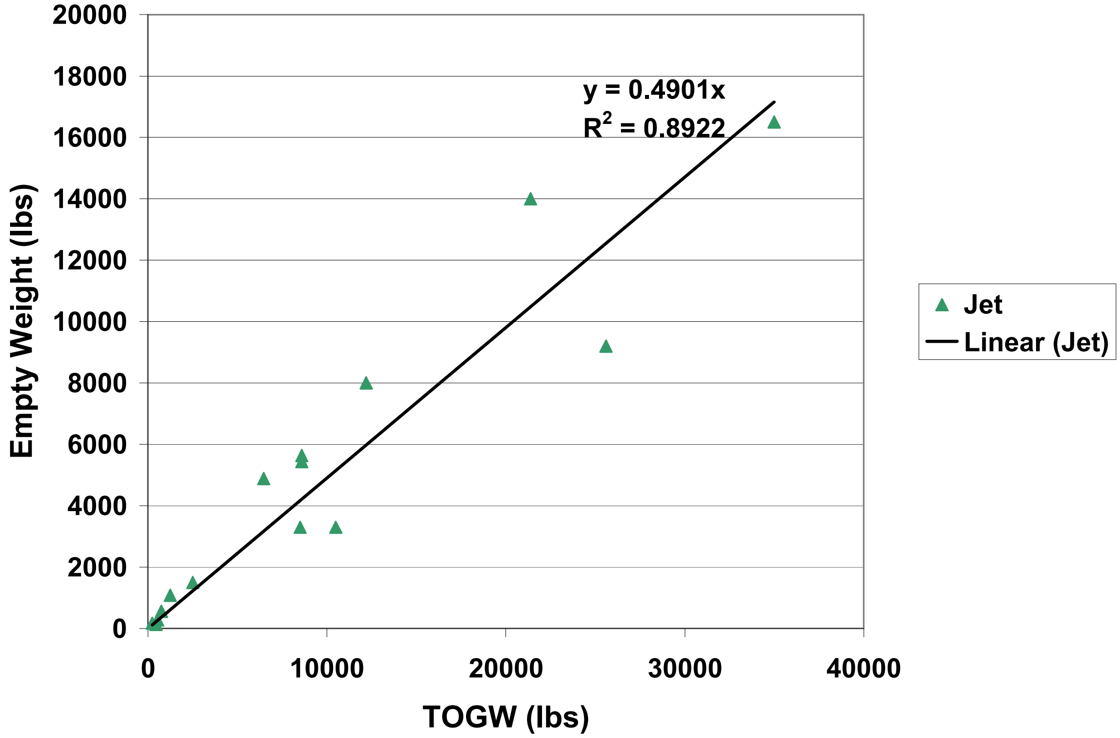

Figure 8-17: Empty Weight Fraction Determination (Piston / Turboprob)

The type of engine selected will be based on a number of factors. For this example problem, only the relationship between the maximum required Mach number and the engine type will be used in the selection process.

Based on historical data from Reference 40, the Fuzzy Set membership functions can be created to determine the relationship between the maximum required Mach number and the type of engine selected. The use of these membership functions will be discussed later in this section.

Total Mission Weight Fraction

The fuel used throughout each mission segment is a function of the aircraft weight. As discussed in Reference 153 as a first order estimate, it is possible to approximate the fuel required for each mission segment to be proportional to the weight of the aircraft. In other words, it is possible to estimate the fuel required for the mission through the use of the total mission weight fraction (W_f/W_0).

The weight fractions for the different segments of the mission were either historical weight fractions provided in Reference 153 or were estimated from the Brequet Range Equation. [153] The different values for the weight fractions, or the equations used for their estimation, are presented in Table 8-23.

Table 8-23: Weight Fraction values and Equations

Mission Segment	Weight Fraction
Warm up and takeoff	$\frac{W_1}{W_0} = 0.970$
Climb	$\frac{W_2}{W_1} = 0.985$
Cruise to killbox	$\frac{W_3}{W_2} = \exp\left(\frac{-R \cdot C}{V \cdot \frac{L}{D}}\right)$
Search in killbox	$\frac{W_4}{W_3} = \exp\left(\frac{-LT \cdot C}{V \cdot \frac{L}{D}}\right)$
Cruise to base	$\frac{W_5}{W_4} = \exp\left(\frac{-R \cdot C}{V \cdot \frac{L}{D}}\right)$
Landing	$\frac{W_6}{W_5} = 0.995$

The combined total mission weight fraction is calculated below. [153]

$$\frac{W_6}{W_0} = \frac{W_1}{W_0} \cdot \frac{W_2}{W_1} \cdot \frac{W_3}{W_2} \cdot \frac{W_4}{W_3} \cdot \frac{W_5}{W_4} \cdot \frac{W_6}{W_5}$$ **Equation 8-33**

To account for the typical 6% allowance reserve and trapped fuel, the following equation is used to calculation the total fuel fraction. [153]

$$\frac{W_f}{W_0} = 1.06\left(1 - \frac{W_6}{W_0}\right)$$ **Equation 8-34**

Take-off Weight Estimation

The TOGW (W_0) is calculated using an iterative procedure where an initial guess is used, which calculates a new value for the TOGW. This new value is then used to determine an updated TOGW value. This process is repeated until the TOGW guess and the final TOGW value converge. [153,125]

In some scenarios, depending on the loiter time (LT) and range (R) values required for the mission, it is not possible for an aircraft to complete the mission. While it is possible to increase the amount of fuel available, this increases the total weight of the aircraft, which then also increases the empty weight of the aircraft through the empty weight fraction.

In the iterative process for this example, the loiter time is set to the desired value of 20 hours. If the mission cannot be satisfied by the aircraft alternative under consideration, the loiter time is reduced by one hour. This process repeats until it is possible for the TOGW to converge, effectively sizing the aircraft.

The primary variables/parameters associated with this process are: $W_{payload}$, range (R), loiter time (LT), thrust specific full consumption (TSFC), velocity (V), and the lift to drag ration (L/D).

Performance Constraint Analysis

The first performance related constraint that was considered in this example problem relates to the required Takeoff Distance for the aircraft. The required thrust to weight (T/W) to meet a specific takeoff distance for a given wing loading (W/S) value is calculated below using the Take Off Parameter (TOP). [153] As shown in Reference 153 this parameter can be approximated by values of 100 through 250 for related aircraft for runways of length from ~1500ft to 5000 ft respectively.

$$\frac{T_{SL}}{W_{TO}} = \frac{\beta}{\alpha} \frac{\left(\frac{W_{TO}}{S} \cdot \beta \right)}{TOP \cdot \sigma \cdot C_{L_{TO}}}$$ **Equation 8-35**

The thrust at sea level (T_{SL})is related to the installed thrust (T) through the following equation where the installed full throttle thrust lapse, α[25], is calculated based on the type of engine selected. [124]

$T_{SL} = T/\alpha$ **Equation 8-36**

$$\alpha_{Turboprop} = \sqrt{\sigma} \text{ for } M \le 0.1$$ **Equation 8-37**

[25] The use of α to represent the installed full throttle thrust lapse is done to remain consistent with the literature. [124] This value is not to be confused with the Robustness Function from Info-Gap Theory.

$$\alpha_{Turboprop} = \frac{0.12}{M+0.02}\sqrt{\sigma} \ \text{ for } 0.1 < M < 0.1$$ **Equation 8-38**

$$\alpha_{LowBPR} = 0.72\left\{0.88 + 0.245\left(\left|M - 0.6\right|\right)^{1.4}\right\}\sigma^{0.7}$$ **Equation 8-39**

$$\alpha_{HighBPR} = \left\{0.568 + 0.25\left(1.2 - M\right)^{3}\right\}\sigma^{0.6} \ \text{ for } M < 0.9$$ **Equation 8-40**

The weight at takeoff (W_{TO}), which is also assumed to be equal to the TOGW determined in the previous section, is related to the instantaneous weight (W) through the Equation 8-41. The parameter β[26] is determined based upon the fuel consumed and payload expended up to that time in the mission. This value is calculated from the mission fuel fraction values for each mission segment. [125]

$$W_{TO}=W/\beta$$ **Equation 8-41**

The required wing loading for the maximum speed constraint can be calculated from the following equation. [125] The equation for the dynamic pressure (q) is also presented. [125]

$$\frac{T_{SL}}{W_{TO}} = \frac{\beta}{\alpha}\left\{K_1 \cdot \frac{\beta}{q_{MAX}} \cdot \frac{W_{TO}}{S} + K_2 + \frac{C_{D0}}{\frac{\beta}{q_{MAX}} \cdot \frac{W_{TO}}{S}}\right\}$$ **Equation 8-42**

$$q_{MAX} = \frac{1}{2} \cdot \rho \cdot V_{MAX}^2$$ **Equation 8-43**

[26] The use of β is done to remain consistent with the literature. [125] This value is not to be confused with the Opportunity Function from Info-Gap Theory.

$$V_{MAX} = M_{MAX} / a$$ **Equation 8-44**

The primary variables/parameters associated with this process are: weight at takeoff (W_{TO}) or TOGW, wing area (S), takeoff parameter (TOP), takeoff lift coefficient (C_{LTO}), zero lift drag coefficient (C_{D0}), and maximum Mach number (M_{MAX}).

Economic Constraint Analysis

The aircraft acquisition cost is estimated by the same approach utilized in Chapters 4. The equations are provided below for completeness.

$$ACCost = Costperpound \cdot W_{AMPR}$$ **Equation 8-45**

$$W_{AMPR} = AMPR\_Weight\_Factor * W_{empty}$$ **Equation 8-46**

W_{empty} = We/W0*TOGW **Equation 8-47**

The primary variables/parameters associated with this process are: TOGW, cost per pound (Costperpound), AMPR Weight Factor (AMPR_Weight_Factor), empty weight fraction (We/W0).

Design Parameter and Variable Selection

Now that the model has been determined it is possible to define the design variables and uncertainty variables.

Payload Weight ($W_{payload}$): For this example problem, the required payload (which consists of various potential sensor packages and weapon systems) is mission specific.

Because this example problem is for a general persistent strike aircraft and the exact mission parameters are not specified, this value is unknown.

Empty weight fraction (We/W0): The empty weight fraction can be determined from historical data based upon the type of engine selected.

Type of Engine: This parameter, while not present in any of the equations, determines which equations should be used for the installed full throttle thrust lapse (α) and thrust specific fuel consumption (TSFC). Additionally this parameter determines which value should be selected for the empty weight fraction.

Range (R): This variable is mission specific. Because this example problem is for a general persistent strike aircraft and the exact mission parameters are not specified, this value is unknown.

Loiter Time (LT): This value is determined through the sizing process. The original value of this variable was set to 20 hours within the sizing analysis and then decreased as necessary in the iterative process to determine the TOGW.

Thrust specific full consumption (TSFC): This parameter is calculated by the following equations. [124] The equation used depends upon the type of engine selected.

$$TSFC_{turboprop} = (0.18 + 0.8M)\sqrt{\theta}$$ **Equation 8-48**

$$TSFC_{LowBPR} = (0.9 + 0.3M)\sqrt{\theta}$$ **Equation 8-49**

$$TSFC_{HighBPR} = (0.45 + 0.54M)\sqrt{\theta}$$ **Equation 8-50**

Cruise velocity (V): This parameter affects the sizing of the aircraft and can be considered a general characteristic of the aircraft. This variable is selected in this problem to be one of the design variables.

Lift to drag ration (L/D): This parameter is used in the sizing of the aircraft and highly dependent on the aircraft configuration. For this reason it is appropriate to use the maximum L/D as a design variable. Based upon the information from Reference 153, as a first order approximation, the L/D ratio can be estimated as 0.866 L/D_{max} for cruise and L/D_{max} for loiter mission segments.

TOGW: This parameter is determined in the sizing analysis and used in the constraint and economic analysis.

Wing area (S): The wing area is based upon the aircraft configuration and is appropriate to be considered as a design variable in the problem.

Takeoff parameter (TOP): From Reference 153 the TOP can be estimated for an aircraft based upon the runway length. The runway length is mission dependent and is an uncertain parameter since it is not know where the aircraft will be required to operate from in the future. It is estimated that the aircraft may be required to takeoff from moderate to short runways and so the focus in this problem is on runways between 1500-5000ft.

Takeoff lift coefficient (C_{LTO}): As discussed in Reference 153, an aircraft can be assumed to takeoff at 1.1 times the stall speed of the aircraft. So, the takeoff lift coefficient is estimated for this example problem to be the maximum lift coefficient divided by 1.2 (1.1 squared). [153] At this stage in the design process the actual maximum lift coefficient (C_{Lmax}) is unknown.

Zero lift drag coefficient (C_{D0}): The actual value of the zero lift drag coefficient (C_{D0}) is unknown at this stage of the design process.

Maximum Mach number (M_{MAX}): The maximum required Mach number is mission dependent based upon the type of target to be located and the local terrain/environment. Because this example problem is for a general persistent strike aircraft and the exact mission parameters are not specified, this value is unknown.

Cost per pound (Costperpound): The uncertain characteristics of this variable have been discussed extensively in Chapters 4 and 7 . Because this example problem considers the same class of aircraft, the identified characteristics are still valid.

AMPR Weight Factor (AMPR_Weight_Factor): As with the cost per pound, the uncertain characteristics of this variable have been discussed extensively in Chapters 4 and 7. Similarly, because this example problem considers the same class of aircraft, the identified characteristics are still valid.

In summary, Table 8-24 lists the potential design variables and the associated design space for each variable. The design space was explored through the latin hypercube sampling technique discussed earlier in this chapter, and 100 design alternatives were selected.

Table 8-24: Design Variable Parameters (Example E)

Design Variable	Minimum Value	Maximum Value
Wing Area (ft^2)	200	600
L/Dmax	15	35
Thrust at sea level (lbs)	2,000	50,000
Cruise velocity (kts)	200	500

Table 8-24 lists the uncertain variables and the characteristics of these variables. The uncertainty modeling technique was selected for each of the uncertainty variables based on the variables characteristics.

While the mission radius is unknown because of the variety of future missions, the maximum potential range was set to be from 50 to 2500 nm, and the nominal value was estimated to be 500nm.

Additionally historical data from Reference 40 was used to determine fuzzy membership functions that relate the maximum Mach number to the type of propulsion system used. These membership functions are illustrated in Figure 8-18.

Table 8-25: Uncertain Variables (Example E)

Uncertainty Variable	Uncertainty Type	Distribution	Minimum Value	Maximum Value	Uncertainty Modeling Technique
AMPR Weight Factor (%)	Ambiguity	normal	60	70	Probability Theory
CD0	Ambiguity	unknown	0.01	0.019	Evidence Theory
TOP	Ambiguity	unknown	100	250	Evidence Theory
Payload (lbs)	Ambiguity	unknown	2,000	5,000	Evidence Theory
C_{Lmax}	Ambiguity	unknown	1.2	2	Evidence Theory
M_{MAX}	Ambiguity	unknown	0.6	0.9	Evidence Theory
Cost per pound ($/lb)	Ambiguity	unknown	4500	5000	Evidence Theory
Mission Radius (nm)	Ambiguity	unknown	unknown	unknown	Info-Gap Theory
Type of Engine	Vagueness	NA	NA	NA	Fuzzy Set Theory

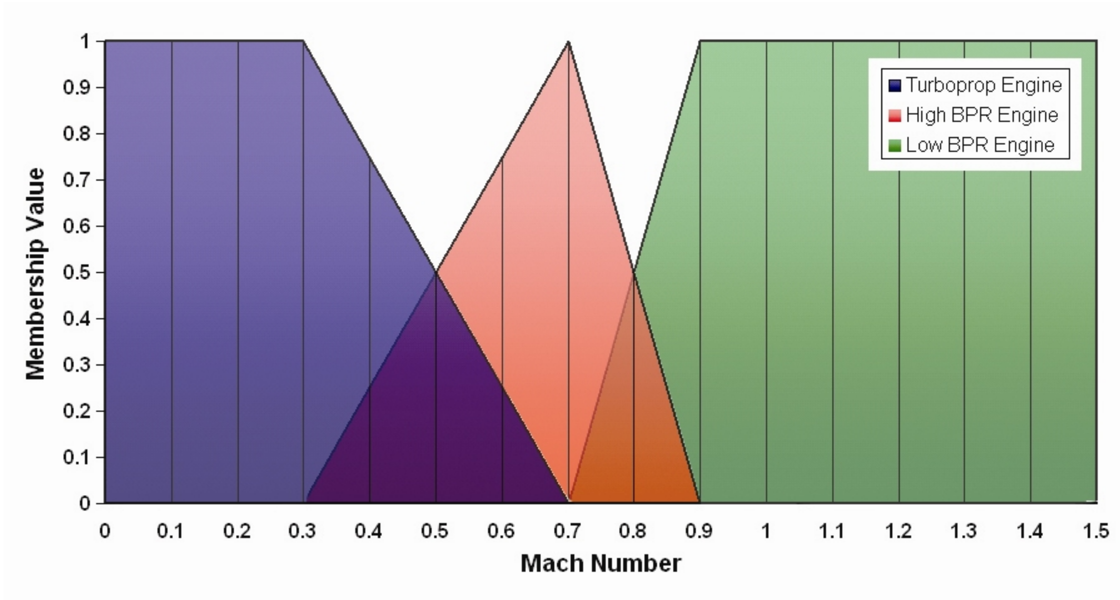

Figure 8-18: Fuzzy Set Membership Function for Example E

The information and equations discussed previously in this chapter were used in the process described in Chapter 7 to determine the metric values for each of the design approaches for each design alternative is calculated. The results from the weight determination process for each of the approaches is presented in Table 8-26 and 8-27. The "best" alternative from each approach and each weight is presented in Table 8-28.

Table 8-26: Weight Results from Weight Determination Study (Example E – Part 1)

		Metric 1: Aircraft Acquisition Cost				Metric 2: Loiter time			
		α Plausible	α Believable	β Plausible	β Believable	α Plausible	α Believable	β Plausible	β Believable
Robust Design	**W1**	0.5	0.5	--	--	0.5	0.5	--	--
	W2	0.33	0.67	--	--	0.33	0.67	--	--
	W3	0.42	0.58	--	--	0.42	0.58	--	--
	W4	0.49	0.51	--	--	0.34	0.66	--	--
Opp. Design	**W1**	--	--	0.5	0.5	--	--	0.5	0.5
	W2	--	--	0.5	0.5	--	--	0.5	0.5
	W3	--	--	0.5	0.5	--	--	0.5	0.5
	W4	--	--	0.5	0.5	--	--	0.5	0.5
RandO Design	**W1**	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25
	W2	0.24	0.24	0.27	0.25	0.24	0.24	0.27	0.25
	W3	0.27	0.25	0.24	0.24	0.27	0.25	0.24	0.24
	W4	0.25	0.31	0.22	0.22	0.28	0.19	0.27	0.26

Table 8-27: Weight Results from Weight Determination Study (Example E – Part 2)

		Metric 3: T/W Available – T/W Required (Takeoff)				Metric 4: T/W Available – T/W Required (Max Speed)			
		α Plausible	α Believable	β Plausible	β Believable	α Plausible	α Believable	β Plausible	β Believable
Robust Design	W1	0.5	0.5	--	--	0.5	0.5	--	--
	W2	0.33	0.67	--	--	0.33	0.67	--	--
	W3	0.24	0.76	--	--	0.24	0.76	--	--
	W4	0.36	0.64	--	--	0.12	0.88	--	--
Opp. Design	W1	--	--	0.5	0.5	--	--	0.5	0.5
	W2	--	--	0.5	0.5	--	--	0.5	0.5
	W3	--	--	0.5	0.5	--	--	0.5	0.5
	W4	--	--	0.5	0.5	--	--	0.5	0.5
RandO Design	W1	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25
	W2	0.24	0.24	0.27	0.25	0.24	0.24	0.27	0.25
	W3	0.22	0.24	0.29	0.25	0.22	0.24	0.29	0.25
	W4	0.33	0.21	0.23	0.23	0.1	0.26	0.36	0.28

Table 8-28: Selected Alternatives from each Design Approach (Example E)

		Wing Area (ft^2)	L/Dmax	Thrust at sea level (lbs)	Cruise velocity (kts)
Robust Design	W1	468.68	28.59	10785	319.71
	W2	468.68	28.59	10785	319.71
	W3	468.68	28.59	10785	319.71
	W4	583.17	31.86	29404	267.74
Opportunistic Design	W1	341.43	30.08	6417.29	457.68
	W2	341.43	30.08	6417.29	457.68
	W3	341.43	30.08	6417.29	457.68
	W4	341.43	30.08	6417.29	457.68
RandO Design	W1	468.68	28.59	10785.5	319.71
	W2	468.68	28.59	10785.5	319.71
	W3	468.68	28.59	10785.5	319.71
	W4	410.62	28.31	12587	439.19

The selected alternatives are all compared for the three uncertainty scenarios listed in Table 8-28. The range of the uncertainty variables and the distribution parameters both change for each uncertainty scenario. The results are shown in Figures 8-19 through 8-27 and Appendix B. Figures 8-20 and 8-22 show the actual traditional metric values from the Monte Carlo analysis. Figures 8-21 and 8-23 show the metrics after they have been normalized and penalized. The normalized and penalized values are then compared in TOPSIS for the three different uncertainty scenarios.

Table 8-29: Uncertainty variable ranges: Uncertainty Scenario 1

Uncertainty Variable	Minimum Value	Maximum Value	MC Group 1 Beta Distribution Parameters
AMPR Weight Factor	60	70	P1: 4, P2: 2
CD0	0.01	0.019	P1: 4, P2: 2
TOP	100	200	P1: 2, P2: 4
Wpayload (lb)	2500	5000	P1: 4, P2: 2
CLmax	1.2	1.6	P1: 2, P2: 4
MMax	0.75	0.9	P1: 4, P2: 2
Cost per pound ($/lb)	4750	5000	P1: 4, P2: 2
Mission radius (nm)	200	1000	P1: 1, P2: 1

Table 8-30: Uncertainty variable ranges: Uncertainty Scenario 2

Uncertainty Variable	Minimum Value	Maximum Value	MC Group 2 Beta Distribution Parameters
AMPR Weight Factor	60	65	P1: 2, P2: 4
CD0	0.01	0.015	P1: 2, P2: 4
TOP	150	250	P1: 4, P2: 2
Wpayload (lb)	2000	3500	P1: 2, P2: 4
CLmax	1.5	2	P1: 4, P2: 2
MMax	0.6	0.8	P1: 2, P2: 4
Cost per pound ($/lb)	4500	4750	P1: 2, P2: 4
Mission radius (nm)	200	600	P1: 2, P2: 4

Table 8-31: Uncertainty variable ranges: Uncertainty Scenario 3

Uncertainty Variable	Minimum Value	Maximum Value	MC Group 3 Beta Distribution Parameters
AMPR Weight Factor	65	70	P1: 4, P2: 2
CD0	0.014	0.019	P1: 4, P2: 2
TOP	100	200	P1: 2, P2: 4
Wpayload (lb)	3500	5000	P1: 4, P2: 2
CLmax	1.2	1.6	P1: 2, P2: 4
MMax	0.75	0.9	P1: 4, P2: 2
Cost per pound ($/lb)	4750	5000	P1: 4, P2: 2
Mission radius (nm)	700	1500	P1: 4, P2: 2

The Overall Evaluation Criterion values from the TOPSIS analyses are plotted in Figure 8-24 through 8-26. As an example of the normalization process consider Figure 8-19. The x axis indicates the original metric value and the y axis is the metric value after the normalization and penalization process. Both Metrics 2 and 3 are illustrated in the plot for the RandO (indicated by green) and the Opportunistic Design Approach (indicated by purple). The darker colors (dark purple and dark green) are for Metric 2 (Takeoff constraint) and Metric 3 (Max Speed) is shown in the lighter colors (light purple and light green).

For these metrics the alternatives selected by the RandO approach always satisfy the constraint and are not penalized, but nor are they ever rewarded for reaching the desirement. The Opportunistic approach, on the other hand, selected an alternative where it was possible to satisfy the desirement. These values were rewarded by the bonus function and are indicated by negative values on the y axis. However the Opportunistic approach also had a number of points from the Monte Carlo analysis that failed the constraint. Based upon the discussion earlier in this chapter, a penalty factor of 0.3 and a penalty push off factor of 20% was used to model the penalty function. This penalty is visible in Figure 8-19. The discontinuity in the values for the Opportunistic approach at the origin is due to the penalty push off factor and the exterior penalty function is clearly visible by the points that failed the constraint. All metrics after the normalization and penalty/bonus process are to be minimized.

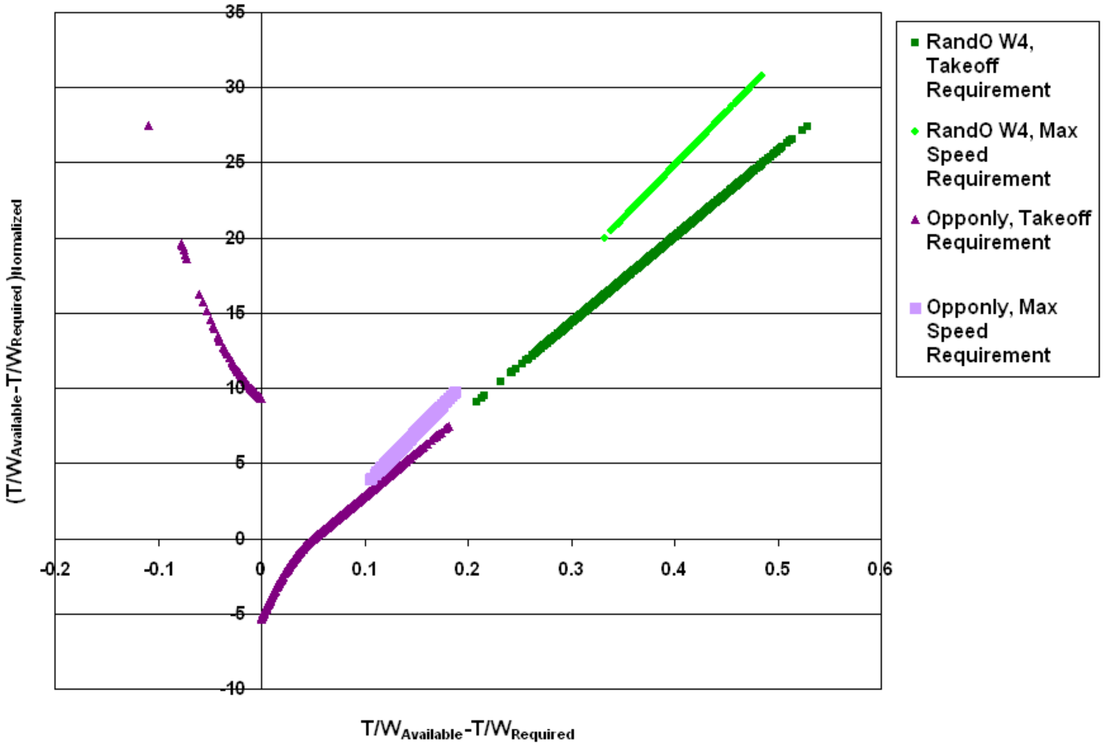

Figure 8-19: Monte Carlo Analysis Data comparing actual T/W Metric Values with normalized T/W Metric Values

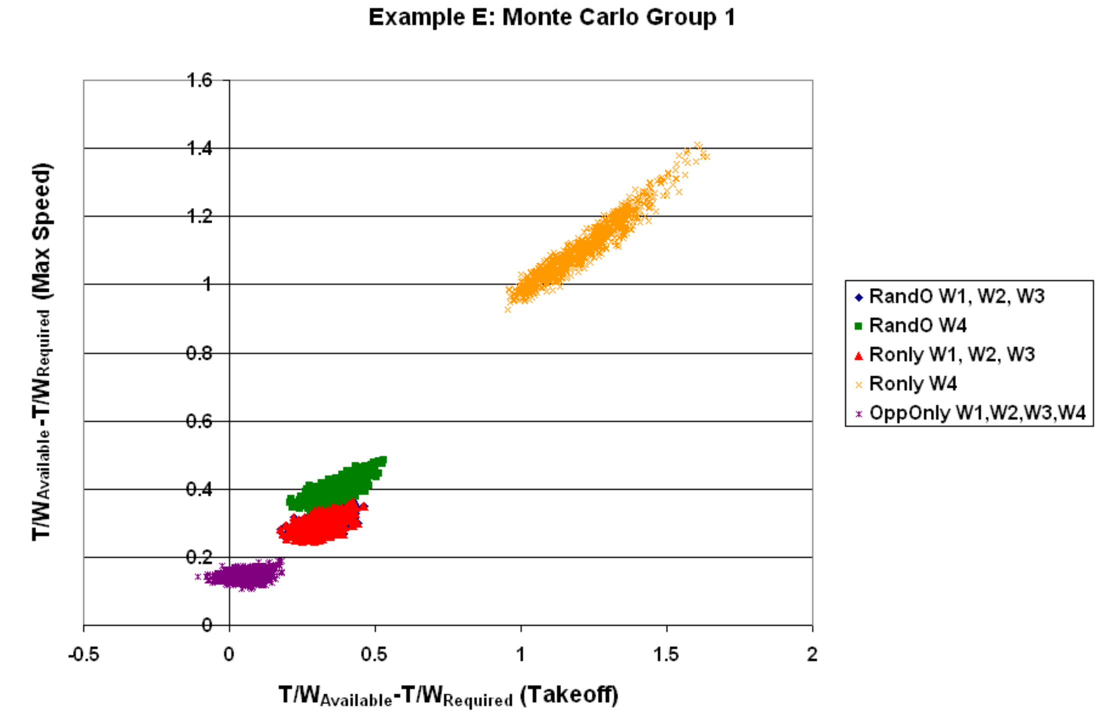

Figure 8-20: Monte Carlo Analysis Data from Uncertainty Scenario 1 for T/W Metrics

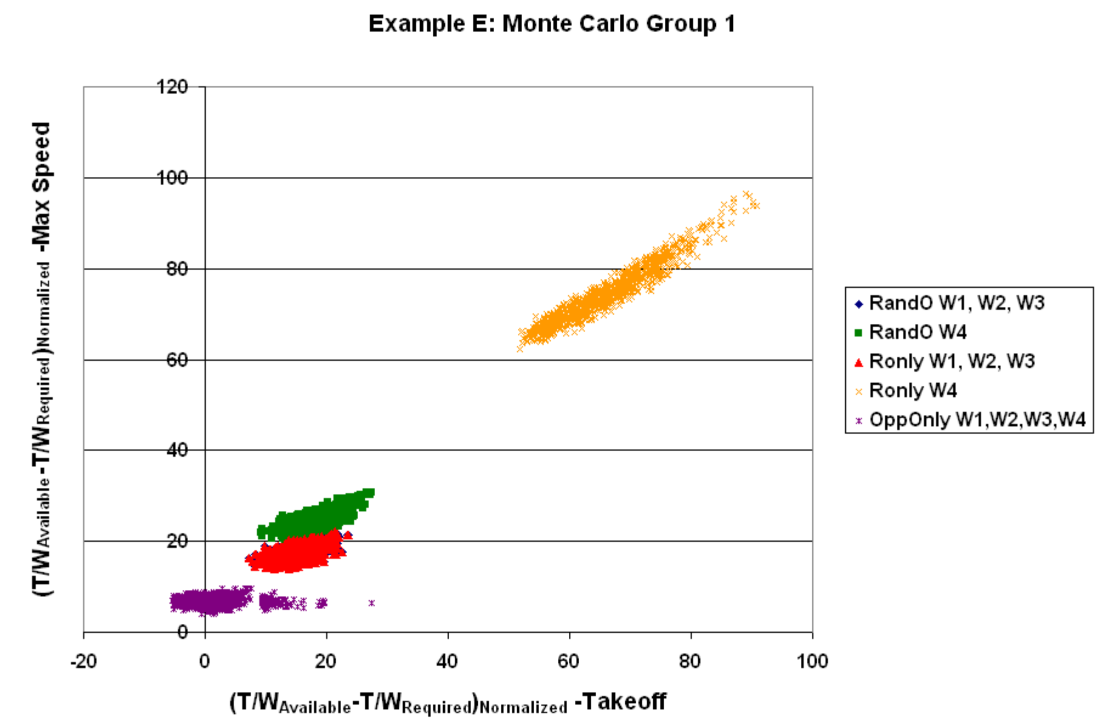

Figure 8-21: Monte Carlo Analysis Data from Uncertainty Scenario 1 for Normalized T/W Metrics

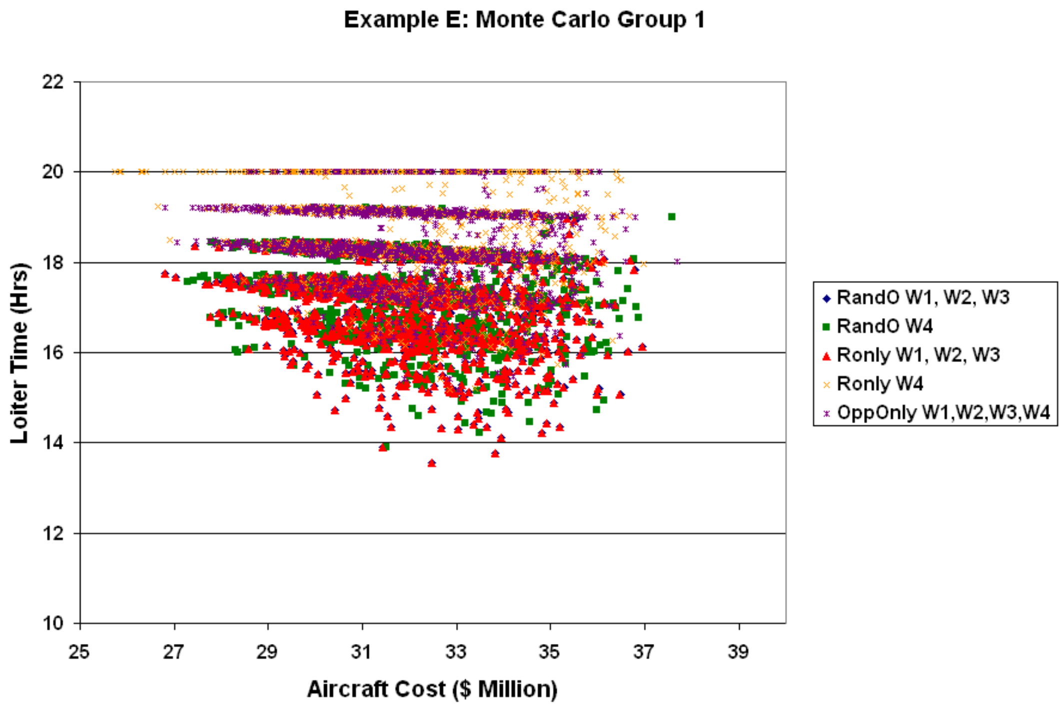

Figure 8-22: Monte Carlo Analysis Data from Uncertainty Scenario 1 for Loiter Time and Cost Metrics

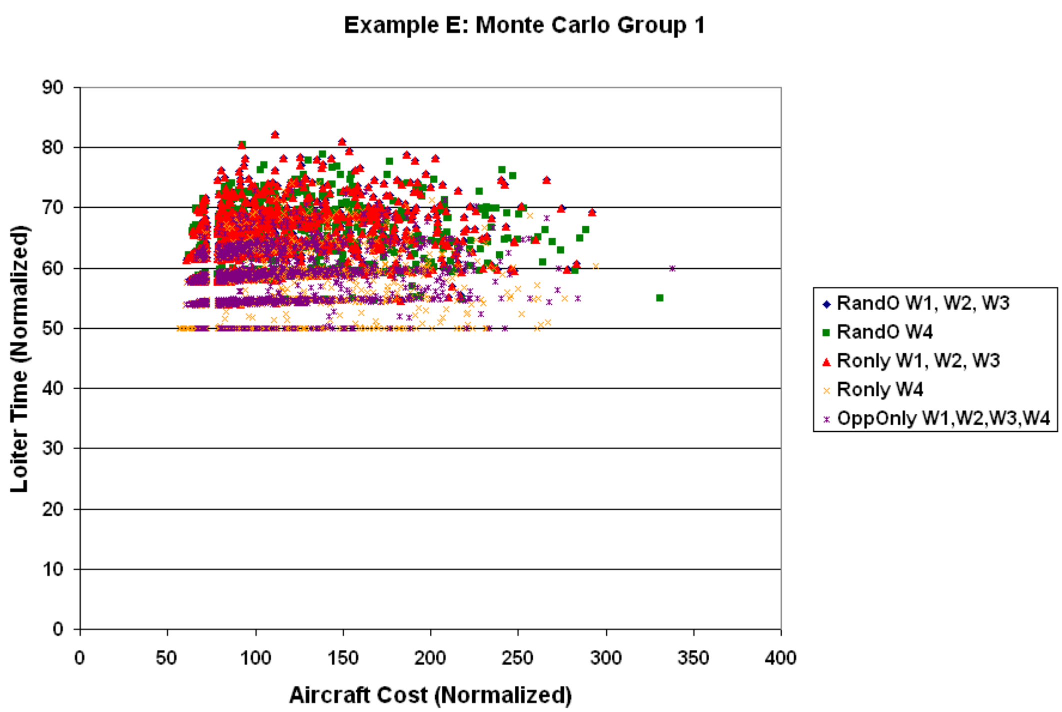

Figure 8-23: Monte Carlo Analysis Data from Uncertainty Scenario 1 for Normalized Loiter Time and Cost Metrics

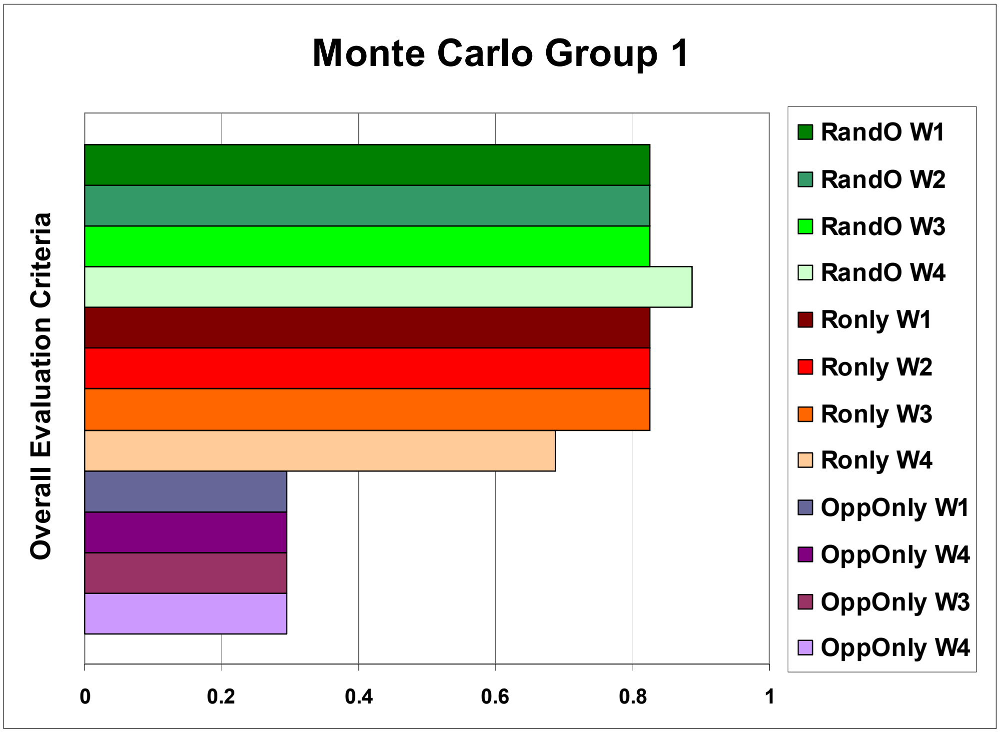

Figure 8-24: Overall Evaluation Criterion Results from MADM Analysis for Uncertainty Scenario 1 (Example E)

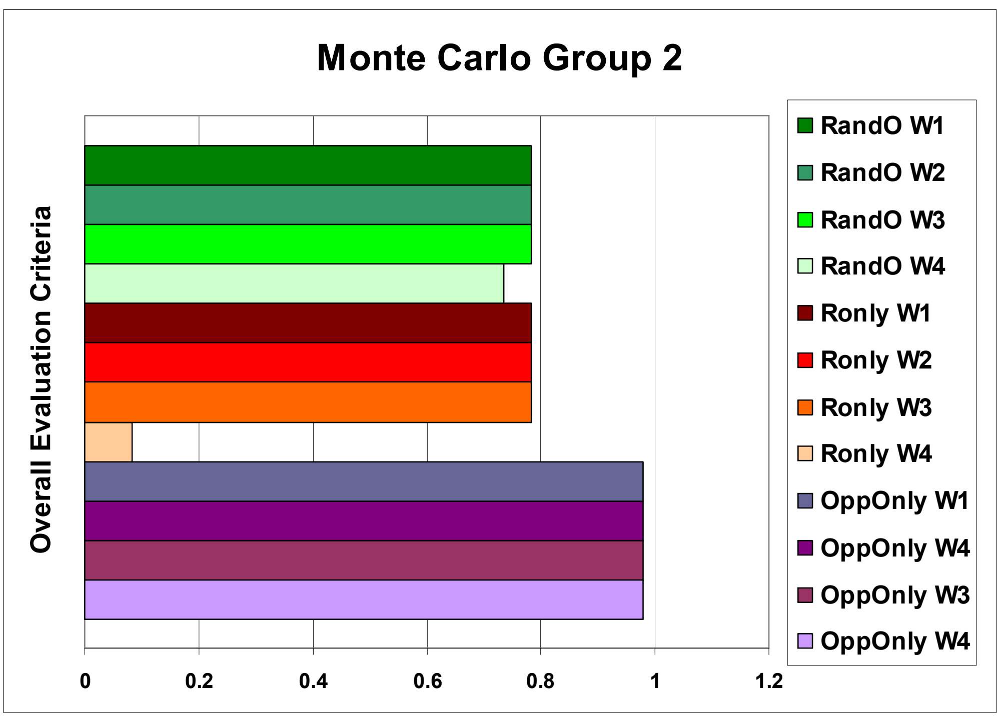

Figure 8-25: Overall Evaluation Criterion Results from MADM Analysis for Uncertainty Scenario 2 (Example E)

For this uncertainty scenario, it is interesting to note that now the alternative selected by the Opportunistic Design approach is ranked the highest by the TOPSIS analysis.

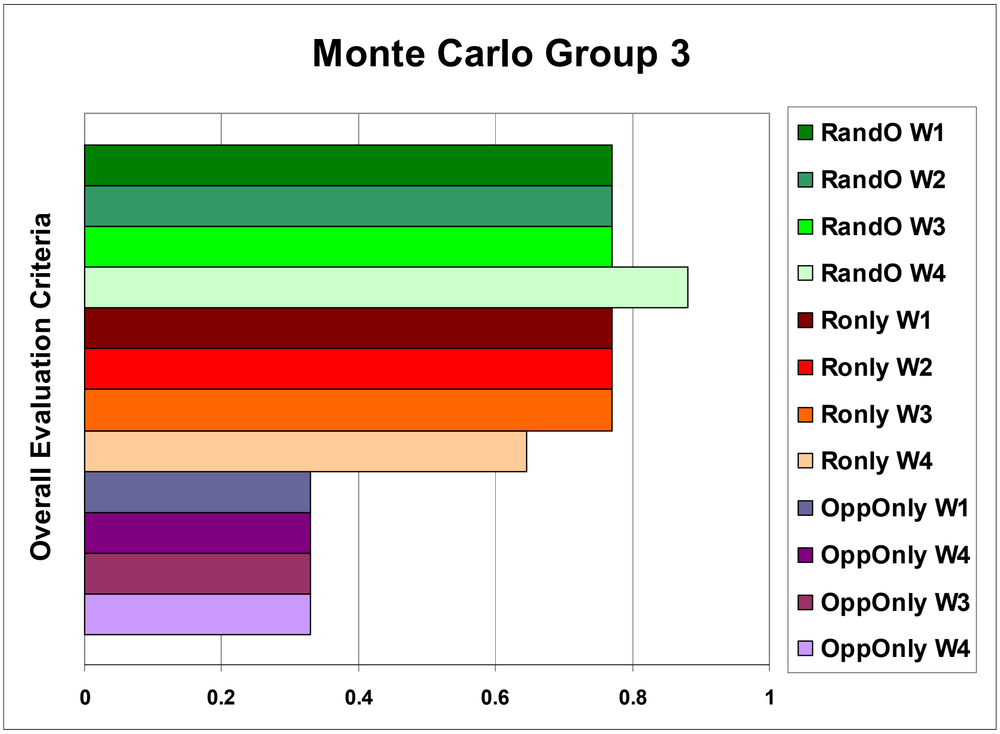

Figure 8-26: Overall Evaluation Criterion Results from MADM Analysis for Uncertainty Scenario 3 (Example E)

Figure 8-27 presents the average Overall Evaluation Criterion values determined from the TOPSIS process. Based on the different uncertainty scenarios, the alternative selected by the RandO Design approach with weight W4 proved to be ranked highest. However, the alternative selected by the RandO Design approach for weights W1, W2, and W3 and the Robust Design approach for weights W1, W2, and W3 resulted in a similar average Overall Evaluation Criteria value.

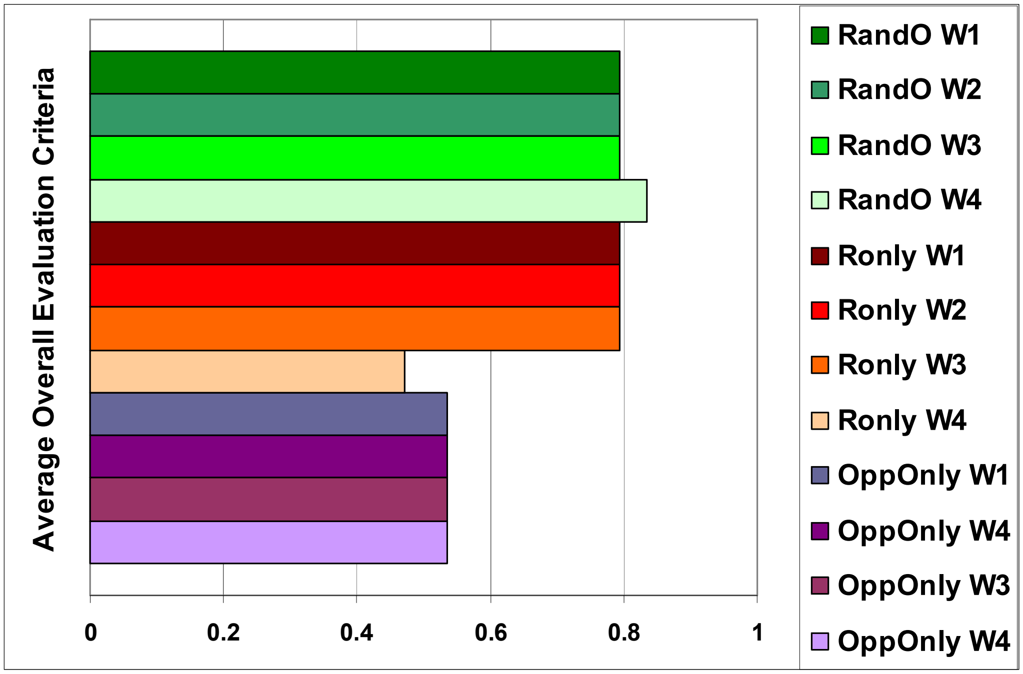

Figure 8-27: Average Overall Evaluation Criterion Results from MADM Analysis (Example E)

8.3.7. <u>Example F: Fleet Design example</u>

The initial motivation for this research was to develop a method for the conceptual design of a System-of-Systems (SoS). While an aircraft can be considered a system-of-systems as discussed in Chapter 2, it is important to recognize that these aircraft do not operate in isolation and in determining the most important characteristics of the aircraft it is necessary to consider the systems interaction in its operational environment. For a modern persistent strike concept, this is likely to be a fleet of the aircraft collaborating to search a designated area for an extended period of time.

Example E is now expanded to consider the problem where a single killbox (30nm x 30nm)[27] is to be continuously searched by an unmanned aircraft. The killbox is searched through a lawnmower (or switch-back) type search pattern. When an aircraft leaves the killbox to return to base (as determined by the mission radius range), another aircraft enters the killbox. An aircraft that returns to base must wait a designated period of time (ground time) before it can start another mission.

Additionally the revisit time is considered in the analysis variable. The revisit time is the amount of time that passes before an aircraft will cover the same point in the killbox. The revisit time, the sensor radius, and the loiter speed of the aircraft determine the number of aircraft that are needed to search the killbox for any given instant.

The traditional design metrics for this example are listed in Table 8-32. The T/W requirements from Example E are included, but now there is a new metric, the Fleet Cost. The Fleet Cost is composed of the product of the cost of the aircraft and the number of aircraft required for the mission scenario. The number of aircraft required is a function of the loiter time from Example E, and the fleet cost directly considers the cost of the aircraft, so there is no need to include either of these metrics from the previous example.

Table 8-32: Constraint / Desirement Definition (Example F)

Traditional Design Metric	Design Constraint	Design Desirement	Relative Importance of Metric
Fleet Cost	< $5000 Million	< $500 Million	0.5
$T/W_{Available}$-$T/W_{Required}$ for Takeoff Requirements	> 0	< 0.05	0.25
$T/W_{Available}$-$T/W_{Required}$ for Maximum Mach Requirements	> 0	< 0.05	0.25

[27] The size of the killbox was based upon the size of the killboxes used for the Scud Hunt in Desert Storm.

The design variables and the design alternatives selected from the design space by the latin hypercube sampling technique are the same as were used in Example E.

The same uncertainty variables from Example E are considered in this example, except now there are two additional uncertainty variables, the ground time for the vehicle and the revisit time required for the mission. Little is known about either of these variables either because the system is not yet well defined enough to estimate the required ground time or because, in the case of the revisit time, the time required is based upon the type of target which is unknown for future scenarios.

Because of the information available for the uncertainty variables, it was determined that Info-Gap Theory was the most appropriate modeling technique. The nominal value for the ground time was estimated to be 4 hours and the maximum possible range for the ground time was between 1 and 10 hours. The maximum range for the revisit time was estimated to be between 15 and 120 minutes, and the nominal value was estimated to be at 60 minutes.

HUMM, as described in Chapter 7, was used to determine the metric values for each of the design approaches for each design alternative. As with the other examples, the weight determination process was used to determine the weights, which selected the highest ranking design alternative for each approach. The results from the weight determination process for each of the approaches are presented in Table 8-33 and 8-34. The "best" alternative from each approach and each weight is presented in Table 8-35.

Table 8-33: Weight Results from Weight Determination Study (Example F Part 1)

		Metric 1: Fleet Cost			
		α Plausible	α Believable	β Plausible	β Believable
Robust Design	W1	0.5	0.5	--	--
	W2	0.5	0.5	--	--
	W3	0.5	0.5	--	--
	W4	0.5	0.5	--	--
Opp. Design	W1	--	--	0.5	0.5
	W2	--	--	0.38	0.62
	W3	--	--	0.48	0.52
	W4	--	--	0.48	0.52
RandO Design	W1	0.25	0.25	0.25	0.25
	W2	0.22	0.3	0.2	0.28
	W3	0.26	0.19	0.18	0.37
	W4	0.26	0.19	0.18	0.37

Table 8-34: Weight Results from Weight Determination Study (Example F Part 2)

		Metric 2: T/W Available – T/W Required (Takeoff)				Metric 3: T/W Available – T/W Required (Max Speed)			
		α Plausible	α Believable	β Plausible	β Believable	α Plausible	α Believable	β Plausible	β Believable
Robust Design	W1	0.5	0.5	--	--	0.5	0.5	--	--
	W2	0.5	0.5	--	--	0.5	0.5	--	--
	W3	0.5	0.5	--	--	0.5	0.5	--	--
	W4	0.5	0.5	--	--	0.5	0.5	--	--
Opp. Design	W1	--	--	0.5	0.5	--	--	0.5	0.5
	W2	--	--	0.38	0.62	--	--	0.38	0.62
	W3	--	--	0.33	0.67	--	--	0.33	0.67
	W4	--	--	0.62	0.38	--	--	0.05	0.95
RandO Design	W1	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25
	W2	0.22	0.3	0.2	0.28	0.22	0.3	0.2	0.28
	W3	0.19	0.36	0.21	0.24	0.19	0.36	0.21	0.24
	W4	0.19	0.24	0.4	0.17	0.2	0.48	0.02	0.3

Table 8-35: Alternatives Selected from each Design Approach (Example F)

		Wing Area (ft^2)	L/Dmax	Thrust at sea level (lbs)	Cruise velocity (kts)
Robust Design	W1	532.58	33.64	33002	443.74
	W2	532.58	33.64	33002	443.74
	W3	532.58	33.64	33002	443.74
	W4	532.58	33.64	33002	443.74
Opportunistic Design	W1	486.32	26.98	4770	381.02
	W2	486.32	26.98	4770	381.02
	W3	486.32	26.98	4770	381.02
	W4	227.61	30.63	2948.7	498.58
RandO Design	W1	486.32	26.98	4770	381.02
	W2	486.32	26.98	4770	381.02
	W3	486.32	26.98	4770	381.02
	W4	543.01	31.06	13850	346.64

The selected alternatives from each approach are all compared for the three uncertainty scenarios listed in Table 8-36 through 8-38. The range of the uncertainty variables and the distribution parameters both change for each uncertainty scenario. The results are shown in Figures 8-28 through 8-35. Figures 8-28 and 8-30 show the actual traditional metric values from the Monte Carlo analysis. Figures 8-29 and 8-31 show the metrics after they have been normalized and penalized. The normalized and penalized values are then compared in TOPSIS for the three different uncertainty scenarios.

Table 8-36: Uncertainty variable ranges: Uncertainty Scenario 1

Uncertainty Variable	Minimum Value	Maximum Value	MC Group 1 Beta Distribution Parameters
TOP	100	250	P1: 4, P2: 4
Wpayload (lb)	2000	5000	P1: 4, P2: 4
C_{Lmax}	1.2	2	P1: 4, P2: 4
M_{Max}	0.6	0.9	P1: 4, P2: 4
Cost per pound ($/lb)	4500	5000	P1: 2, P2: 4
Mission radius (nm)	50	2500	P1: 4, P2: 4
Ground Time (min)	60	600	P1: 4, P2: 4
Revisit Time (min)	15	120	P1: 1, P2: 1

Table 8-37: Uncertainty variable ranges: Uncertainty Scenario 2

Uncertainty Variable	Minimum Value	Maximum Value	MC Group 2 Beta Distribution Parameters
TOP	100	250	P1: 4, P2: 2
Wpayload (lb)	2000	5000	P1: 2, P2: 4
C_{Lmax}	1.2	2	P1: 4, P2: 2
M_{Max}	0.6	0.9	P1: 2, P2: 4
Cost per pound ($/lb)	4500	5000	P1: 2, P2: 4
Mission radius (nm)	50	2500	P1: 2, P2: 4
Ground Time (min)	60	600	P1: 2, P2: 4
Revisit Time (min)	15	120	P1: 4, P2: 2

Table 8-38: Uncertainty variable ranges: Uncertainty Scenario 3

Uncertainty Variable	Minimum Value	Maximum Value	MC Group 3 Beta Distribution Parameters
TOP	100	250	P1: 2, P2: 4
Wpayload (lb)	2000	5000	P1: 4, P2: 2
C_{Lmax}	1.2	2	P1: 2, P2: 4
M_{Max}	0.6	0.9	P1: 4, P2: 2
Cost per pound ($/lb)	4500	5000	P1: 4, P2: 2
Mission radius (nm)	50	2500	P1: 4, P2: 2
Ground Time (min)	60	600	P1: 4, P2: 2
Revisit Time (min)	15	120	P1: 2, P2: 4

The results from the TOPSIS analyses for each uncertainty scenarios are presented in Figures 8-32 through 8-34 and the average Overall Evaluation Criterion values from the analysis are plotted in Figure 8-35.

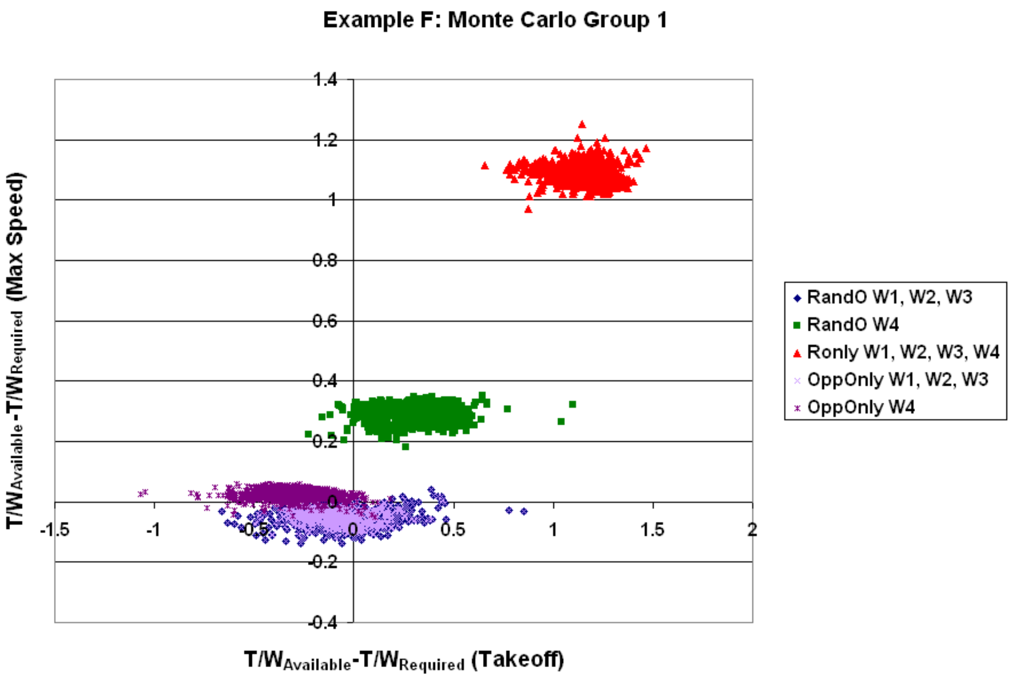

Figure 8-28: Monte Carlo Analysis Data from Uncertainty Scenario 1 for T/W Metrics (Example F)

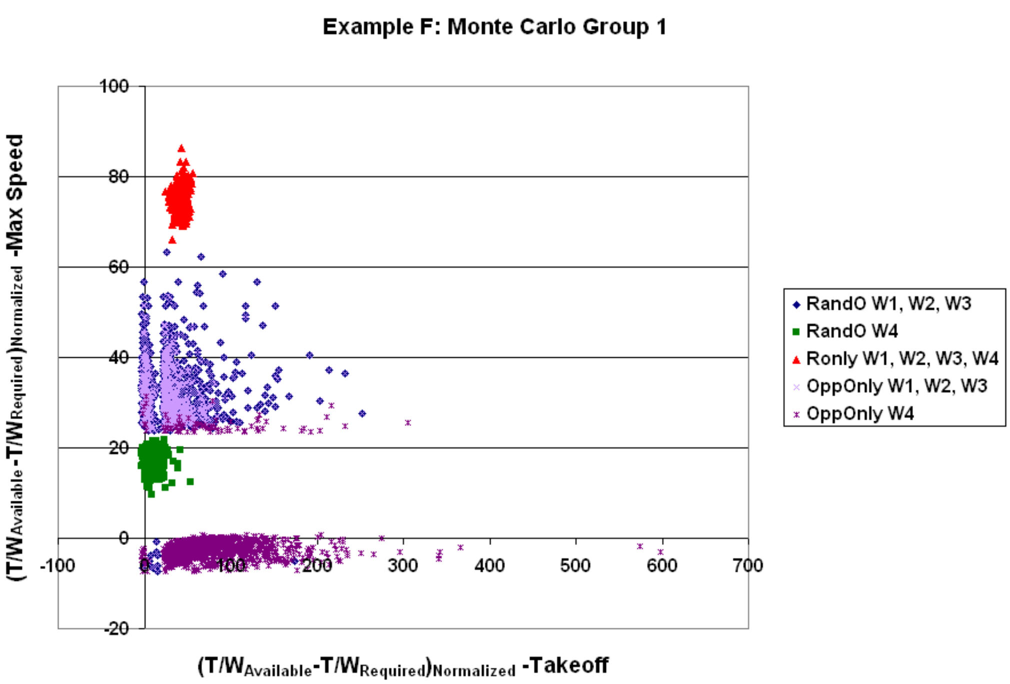

Figure 8-29: Monte Carlo Analysis Data from Uncertainty Scenario 1 for Normalized T/W Metrics (Example F)

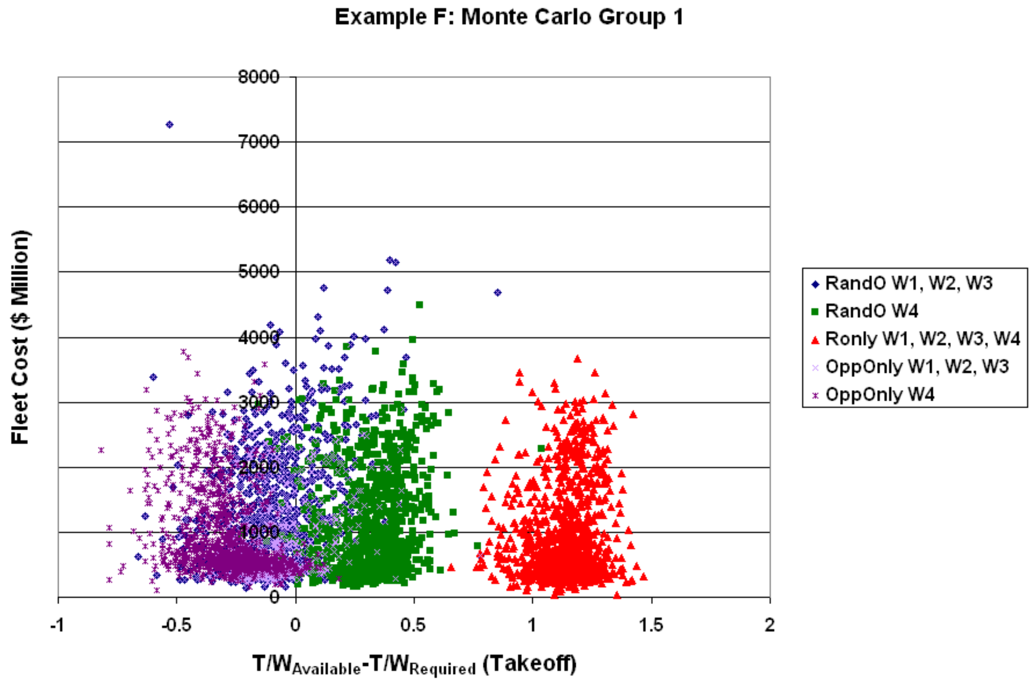

Figure 8-30: Monte Carlo Analysis Data from Uncertainty Scenario 1 for Takeoff and Fleet Cost Metrics (Example F)

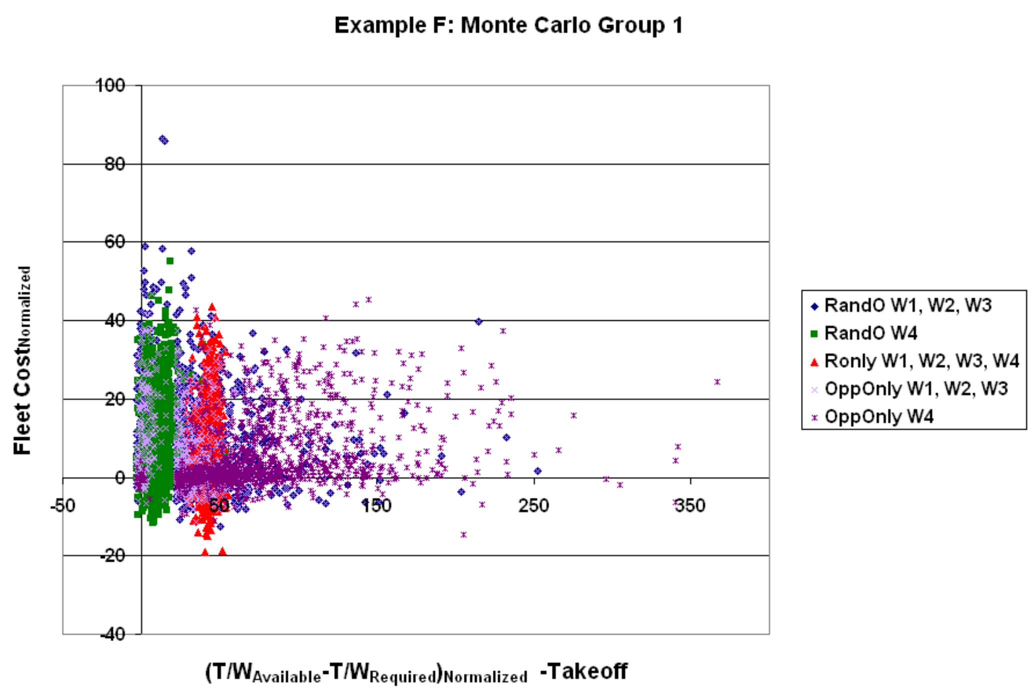

Figure 8-31: Monte Carlo Analysis Data from Uncertainty Scenario 1 for Normalized Takeoff and Fleet Cost Metrics (Example F)

From Figures 8-28 through 8-31 several trends are apparent. First, it is evident that the Robust Design approach selects the conservative alternative. This result is expected because the Robust Design approach is only focusing on satisfying the constraints, and is in fact trying to identify the alternative as far from the constraints as possible. On the other hand, the Opportunistic Design approach identifies the alternative that has the greatest likelihood of satisfying the desirement values. This particular design technique does not consider the constraints in the selection process, which explains why alternatives were selected that significantly violate the T/W constraints.

From these figures, it is also evident that the RandO Design approach selects the alternatives that compromise between these extremes. As a result, the alternative selected by the RandO Design approach with weight group 4 (W4) has the highest overall evaluation criterion for the three different uncertainty scenarios.

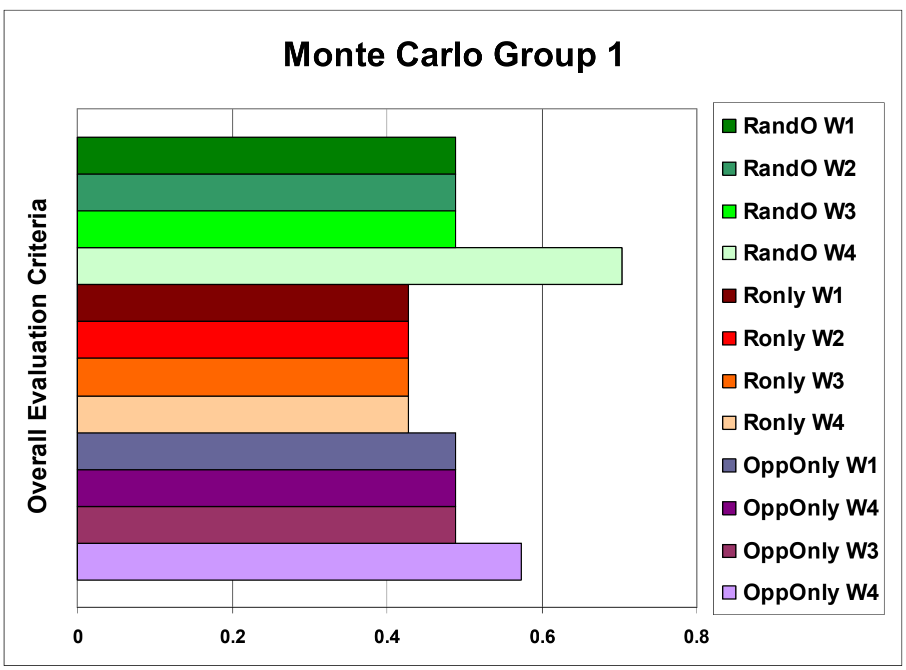

Figure 8-32: Overall Evaluation Criterion Results from MADM Analysis for Uncertainty Scenario 1 (Example F)

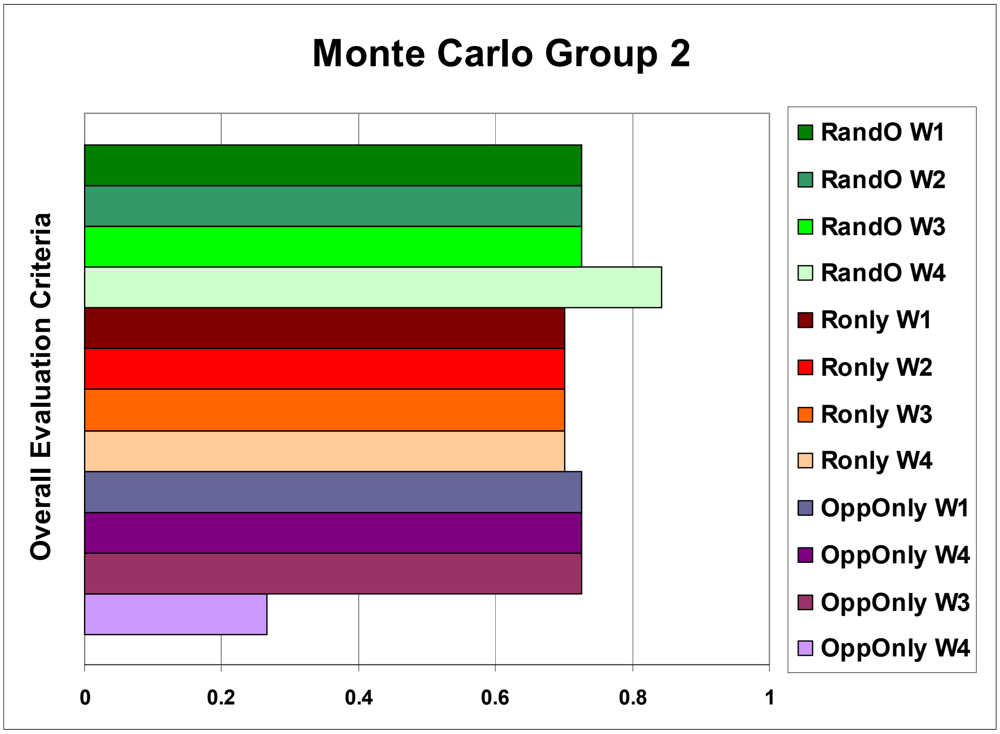

Figure 8-33: Overall Evaluation Criterion Results from MADM Analysis for Uncertainty Scenario 2 (Example F)

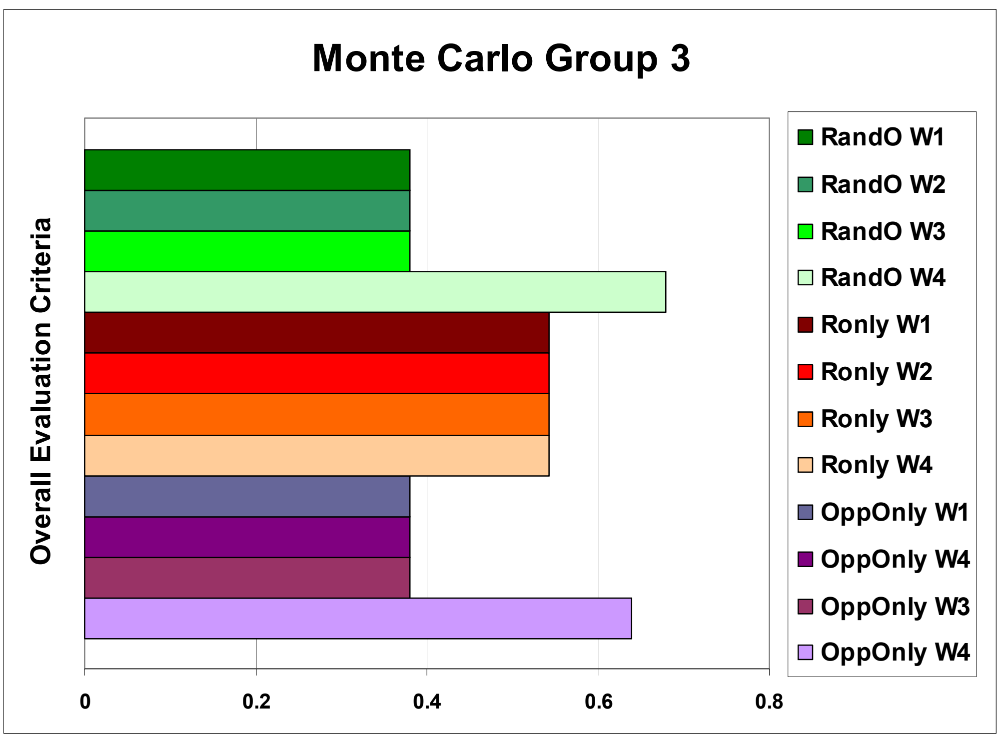

Figure 8-34: Overall Evaluation Criterion Results from MADM Analysis for Uncertainty Scenario 3 (Example F)

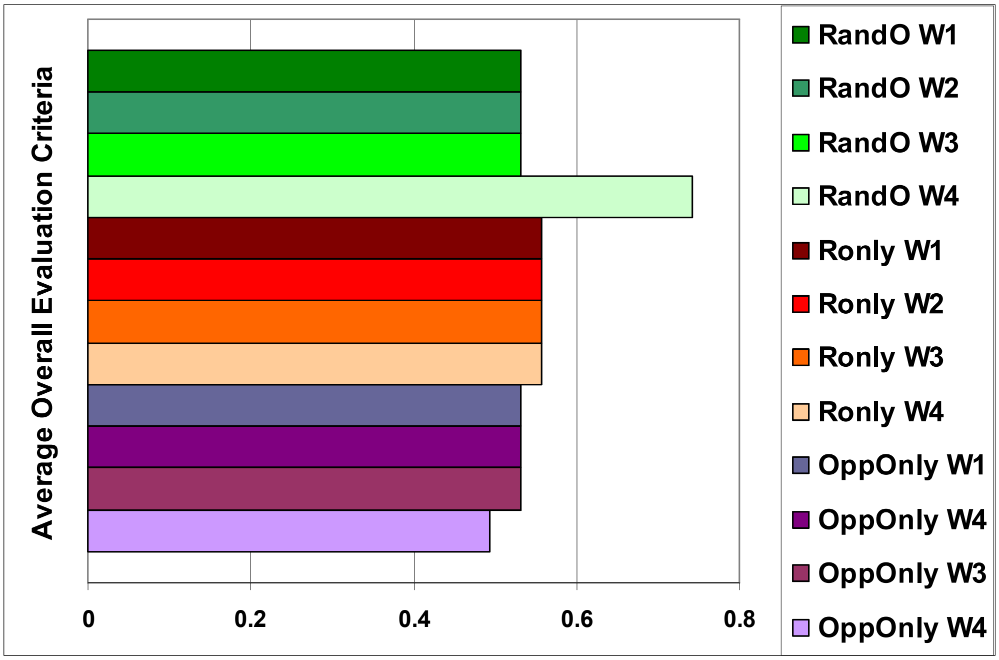

Figure 8-35: Average Overall Evaluation Criterion Results from MADM Analysis (Example F)

CHAPTER 9: THE VALUE OF REDUCING UNCERTAINTY

In design problems with uncertainty, it is likely that an alternative that was not selected as the final design may actually have been the best solution. The designer made the decision with limited information and it is possible that this was not the best design concept. If the designer was in a situation where all of the necessary information was available, the designer would be looking for the optimal design instead of the robust design.

The basic principle is that often in the design process the uncertainty leaves doubt as to which alternative or design decision should be made. If the uncertainty goes one way, then alternative A should be selected, and if the uncertainty goes another way, alternative B or C should be selected. Often it is possible for a designer to select an alternative based upon a worst case scenario situation or based upon the most likely conditions of the uncertainty, but the designer cannot be 100% certain that they are making the correct decision.

It is unlikely that all of the uncertainty can be removed from a SoS design process. However, it is possible to reduce some of the epistemic uncertainty by gaining additional information or running additional experiments. But, there is some cost associated with gaining these new sources of information. The cost may actually be a monetary value, it could be related to the expenditure of resources, or it could simply be the loss of time before the design decision is made. Early market entry can make or break a product, or delaying the design process can prevent a critical service from being available.

Part of valuing the act of obtaining additional information is determining the potential benefit that the designer will receive by having this information. To understand if the additional cost is worth the expense, it is necessary to know what the designer would be gaining. It is possible that the designer would make a different decision if they were able to obtain new knowledge from additional information. What would the designer lose if the decision was made now with no information? This relates to the risk associated with selecting the "wrong" alternative.

However in a couple of cases there is no need to gain additional information. In the first case, even if the uncertainty is reduced, the design decision would be the same. In this case there is no need for additional information unless it would benefit the design process in the future. In the second case the cost associated with gaining the new information/knowledge is greater than the likely potential savings by choosing the correct alternative.

In general the question becomes not should additional information be obtained, but is it worth the additional cost?

9.1. Literature Review

Based upon the existing literature, there is not an existing technique specifically dedicated to determining the value of reducing uncertainty in a design problem. However there are several techniques that address similar issues. Some of the most relevant are as follows: Expected Utility Theory, Cost/Benefit Analysis, Regret Theory, Expected Value of Information (EVI), Expected Value of Perfect Information (EVPI), and the Expected Value of Including Uncertainty (EVIU).

9.1.1. Expected Utility Theory

One of the most common techniques for decision making when there are multiple outcomes is the Expected Utility Theory. This technique is used to estimate outcomes based upon both an outcomes utility as well as its likelihood of occurrence. There are a number of variants of the expected utility model as discussed in Reference 166; however the most common model is shown in the following equation. [166,20]

$$E[u(x)] = \sum_{x} \pi(x)u(x)$$ **Equation 9-1**

This equation determines the expectation of the utility. In this equation, x represents a specific outcome, u(x) is the utility of that outcome and π(x) is the probability of that outcome occurring. [20]

9.1.2. Cost/Benefit Analysis

Cost/Benefit Analyses are a number of analysis techniques where the expected costs and benefits are calculated.[15] In general these analyses determine the relevant costs and benefits and calculate the Net Present Value (NPV) which represents the combination of the discounted costs and benefits over the projected lifespan of the system or project. The NPV is calculated in the equation below. The lifespan of the project is divided into periods which are represented by t in the equation, where T is the final period, and the letter δ_t represents the discount rate for the period t. [155] In essence the metric is used to compare the various alternatives with each other to determine future potential costs and benefits. [15]

$$NPV = \sum_{t=0}^{T} \frac{1}{(1+\delta_t)^t} \left(Benefits_t - Costs_t \right)$$ **Equation 9-2**

$$E[NPV] = E\left[\sum_{t=0}^{T} \frac{1}{(1+\delta_t)^t} \left(Benefits_t - Costs_t \right) \right]$$ **Equation 9-3**

Often cost benefit analyses are deterministic in nature, but Reference 155 discusses potential ways that risk and uncertainty can be incorporated into the analysis. The expectation of the NPV can be calculated to incorporate uncertainty and the associated risk into the analysis. Other techniques include using a risk-adjusted discount rate, certainty equivalents, and the use of safety margins. For additional information about these techniques see Reference 155.

9.1.3. <u>Regret Theory</u>

Regret Theory considers the consequences of making one decision over another. It compares the level of satisfaction of considering one alternative over another. References 151 and 120 show how the expected value of satisfaction of choosing one alternative over another can be determined. For example consider the equations below. The first equation is the satisfaction of selecting Alternative 1 over Alternative 2 (assuming the objective is to maximize the satisfaction) for one possible scenario. The second equation calculates the expected value of satisfaction of the choice between these two alternatives for N possible scenarios. Each scenario occurs with a probability of π_n.[151,120]

$$s*_i = s_{1i} - s_{2i}$$ **Equation 9-4**

$$E[s*] \equiv \sum_{n=1}^{N} \pi_n s*_i$$ **Equation 9-5**

Such a technique is useful in comparing between multiple alternatives, and a variation of this can be used in determining the potential risk associated with selecting a particular alternative over another. However, this theory does not specifically address the need for modeling the value of reducing the uncertainty.

9.1.3.1. Expected Value of Information

The value of information and the cost of obtaining information are of great importance in the health sciences field. An example of a technique for valuing information in this industry can be seen in Reference 201. In this reference instead of selecting between different alternatives as with Regret Theory, the authors are developing a method to determine if clinical trials with a new treatment should be used. In this example the authors are valuing the potential benefit of the new treatment with the additional cost. The similarity is that the new treatment can be considered analogous to gaining new knowledge.

In this technique the Expected Opportunity Loss (EOL) is determined for the scenario where the best decision is to continue with the existing treatment (or keep the existing knowledge) and the EOL is also determined where the best decision is to adopt the new treatment (gain new knowledge).[201] In this reference it is determined that the expected opportunity loss is minimized when continuing with the current treatment if the monetized effectiveness is less then the cost of changing treatments (gaining new knowledge). However the EOL is minimized if the new treatment is pursued if the monetized effectiveness is greater than the difference in cost.[201]

This reference also identifies the maximum it would be potentially worth to gain additional information through the new trial as the Expected Value of Perfect Information (EVPI). [201]

9.1.4. <u>Expected Value of Perfect Information</u>

This Expected Value of Perfect Information (EVPI) describes the value added to a decision making process by knowing the exact value of a particular uncertain variable. This technique uses Bayesian Decision Theory to include uncertainty into a decision making process. [134] The EVPI determines the difference between the expected loss resulting from the decision made with the available information and the expected loss of resulting from the decision that is made when all uncertainty is removed from the problem. The EVPI measures the loss due to the uncertainty in the problem. [134]

For many design problems it would be very useful to know the EVPI and the general concept is similar to the idea of determining the value of reducing uncertainty. However, for the type of problems considered in this research there is no effective technique for determining the expected loss resulting from the decision that is made with perfect information.

9.1.5. Expected Value of Including Uncertainty

A similar technique calculates Expected Value of Including Uncertainty (EVIU), which is the expectation of the difference in loss resulting from using an optimized alternative considering no uncertainty and an alternative that was found while considering the uncertainty. [134] This can be shown in the following equation where $E[L_{NOUnc}]$ represents the expectation of the loss where uncertainty is not considered and $E[L_{UncIncluded}]$ is the uncertainty where it is considered.

$$E[L(d,x)] = \int_X L(d,x) f(x) dx$$ **Equation 9-6**

$$EVIU \equiv E[L(d_{NOUnc},x)] - E[L(d_{UncIncluded},x)]$$ **Equation 9-7**

$d \in D$	d is the decision selected from space D
$x \in X$	x is an uncertain empirical variable from space X
$L(d,x)$	L is the loss which is a function of d and x
d_{NOUnc}	This is the decision selected when no uncertainty is considered
$d_{UncIncluded}$	This is the decision selected when uncertainty is considered

$f(x)$ This is the probability function for x

This technique is not appropriate because the original information for a selected alternative already includes uncertainty. Instead a designer is interested in evaluating the estimated loss that might result from making a decision with the current level of knowledge as opposed to a more informed decision. So, instead of determining the EVIU the designer needs to determine a new value called the Expected Value of Reducing Uncertainty (EVRU).

9.2. Requirements for Determining the Value of Reducing Uncertainty

While each of the previously discussed theories/techniques has relevant aspects, no existing theory/technique can completely address the problem of determining the value of reducing uncertainty in a design problem. A set of requirements for a process to determine this value was developed by considering the differences between the different existing techniques and the identifying where each of the techniques was lacking. These requirements are listed below. The technique must be:

- **Capable of evaluating and ranking the effect of uncertainty in a design problem**
- **Capable of identifying how much the uncertainty in the problem can be reduced**
- **Capable of estimating how much it will cost to reduce the uncertainty**
- **Capable of evaluating the benefit associated with reducing the uncertainty**
- **Capable of evaluating the cost of reducing the uncertainty with the benefit gained from reducing the uncertainty**

Figure 9-1 illustrates the effectiveness of each technique in meeting these requirements. While each technique is capable of addressing part of the requirements, no technique satisfies all of the required capabilities.

	Expected Utility Theory	Cost/Benefit Analysis	Regret Theory	EVI	EVPI	EVIU
Capable of evaluating and ranking the effect of uncertainty	Fair	Poor	Fair	Poor	Fair	Fair
Capable of identifying how much the uncertainty in the problem can be reduced	Poor	Poor	Poor	Poor	Fair	Poor
Capable of estimating how much it will cost to reduce the uncertainty	Poor	Fair	Poor	Fair	Poor	Poor
Capable of evaluating the benefit associated with reducing the uncertainty	Fair	Fair	Fair	Good	Fair	Fair
Capable of evaluating the cost of reducing the uncertainty with the benefit gained from reducing the uncertainty	Poor	Good	Poor	Good	Poor	Poor

Poor | Fair | Good

Figure 9-1: Evaluation of Related Techniques and Theories

- Good indicates that technique can handle the requirement directly or with only a few modifications
- Fair indicates that the technique could be used to handle the requirement with some modification or additional information
- Poor indicates that it either cannot handle the requirement or that it would require significant modification to handle the requirement

Based upon the requirements it is possible to determine a set of techniques that are required.

Technique for evaluating and ranking the effect of uncertainty in a design problem.

In general this can be accomplished through a sensitivity analysis. There are a variety of types of sensitivity analyses but in general this analysis measures the local effect of a specific input parameter on a specific output. Reference 164 discusses the general steps that need to be taken to complete a sensitivity analysis and discusses a number of potential methods such as: Local method, Regression method, Morris, Variance based method, and Monte Carlo filtering technique. Saltelli et al. in Reference 164 discuss the characteristics of these methods and provide suggestions for selecting the appropriate analysis for a given problem.

Process for identifying how (or if) the uncertainty can be reduced and how much of it can be reduced.

The first aspect of this process is to determine how the uncertainty could be reduced. Additional information could be from experts, intelligence reports, additional experimental tests, construction of a prototype, etc. This part of the process is highly dependent on the type of design problem and the types/sources of uncertainty being considered. For this process it is suggested that the designer look to the relevant literature or experts to identify which of the uncertainties can be reduced, how they can be reduced, and by how much. Reference 63 provides guidance for information gathering.

Process for identifying the cost of reducing the identified uncertainty.

To determine the cost, the designer would need to consider all of the costs associated with gaining additional information including costs associated with materials, labor, taking resources from other projects, additional time required for gaining information and iterating through the design process. Reference 87 provides a general discussion over cost estimation. This process is highly dependent upon the design problem being analyzed. It should also be noted that cost in this case may not refer to a monetary value at this stage of the process. The cost term may be related to time, specific resources utilized, or some other metric representing an expense of some sort.

Technique for evaluating the benefit of reducing identified uncertainties.

The existing techniques discussed earlier in this chapter primarily focus upon determining the benefit between alternatives while considering uncertainty or the benefit to considering uncertainty in the decision making process. Despite the variety of available techniques, there is not a process or technique for evaluating the benefit that would be gained by reducing the uncertainty. A technique for accomplishing this task is presented in later in this chapter and is demonstrated with a persistent strike fleet design example. This technique utilizes many of the aspects of the existing techniques and determines the benefit based upon the potential expected loss of selecting one alternative over another.

Process for converting cost and benefit information into common units.

While it is possible to compare a variety of metrics, the easiest metric to use is often a monetary value. To compare the cost and benefit associated with reducing the uncertainty in a design problem, the designer would need to convert all of the cost information to a monetary value or a common unit. Reference 201 uses a multiplier to transform metrics into a monetized value.

In the literature, references such as Reference 15 discuss the difficulty in transforming non-economic consequences to monetary values. For instance, how should the loss of life be valued? Or, as another example, what is the value of designing an aircraft that can takeoff from a 2000ft runway versus the value of developing an aircraft that needs a 5000ft runway? How much more value is gained by having an aircraft that can loiter for one more hour? The metrics relating to these questions can be difficult to compare subjectively and even more difficult to value.

In most cases, transformations for these metrics are likely to be highly subjective and controversial (depending on the metric being converted). However such a transformation can still provide useful comparative information to the designer in determining if reducing the uncertainty is of value. In cases where the transformation may skew the results or would result in significant controversy, it is also possible to use a non-monetary value as the common metric throughout the analysis with this technique. For example, the cost would be measured in lives lost instead of dollars lost.

A technique for transforming different metrics in a design process based upon the relative weights provided by the designer or stakeholder is discussed later in this chapter.

Technique for comparing the cost of reducing the identified uncertainty with the value of reducing that uncertainty.

Reference 201 discusses a technique in the health sciences industry where the potential benefit (b) of gaining additional knowledge[28] is calculated by comparing the monetized benefit of gaining this knowledge with the cost of gaining the knowledge. This reference illustrates how a cost/benefit analysis can be used to identify the benefit associated with reducing uncertainty. However the difficulty to this part of the process is not necessarily in comparing the cost with the benefit but rather in determining what the actual benefits and costs would be for the design process where the uncertainty is reduced before a decision is made.

9.3. Expected Value of Reducing Uncertainty (EVRU)

This research focuses on developing the following techniques/processes:

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[28] For Reference 201, the new knowledge would come from running new clinical trials testing new medical treatments

- **Technique for evaluating the benefit of reducing identified uncertainties.**
- **Process for converting cost and benefit information into common units.**
- **Technique for comparing the cost of reducing the identified uncertainty with the value of reducing that uncertainty.**

From the literature, the EVIU is similar in concept to what is needed for determining the benefit, but this technique needs to be modified to calculate the Expected Value of Reducing the uncertainty rather than including it in the design process. The modifications to the EVIU equations are presented below.

$$EVRU = E[L(d_{org}, x_{org})] - E[L(d_{new}, x_{new})]$$ **Equation 9-8**

$$E[L(d,x)] = \int_X L(d,x) f(x) dx$$ **Equation 9-9**

Or, for the discrete case:

$$E[L(d,x)] = \sum_X L(d,x) f(x)$$ **Equation 9-10**

These equations compare the loss from selecting one alternative over another with the loss that would be obtained if a different decision was made with additional information. The subscript "org" indicates the original decision and "new" represents the new decision. One of the main characteristics of this technique is that it emphasizes that fact that multiple decisions could be made.

However, these equations do not fully address the issue of determining the potential benefit to reducing the uncertainty. The potential loss is based upon the designer making a different decision with the additional information. In some cases, even after the uncertainty is reduced, the designer would select the same alternative. For this scenario the Actual Value of Reducing Uncertainty (AVRU) would be 0. In some cases it is possible for the AVRU to be negative because of the additional cost associated with reducing the uncertainty. But, there is some value with increasing the certainty in a decision. This concept is not addressed in this research but is certainly an area of interest for future research. The calculation for the AVRU is shown below. The case when the original decision is the same as the new decision (decision made with additional information) is also presented.

$AVRU = L(d_{org}, x_{act}) - L(d_{new}, x_{act})$ where $x_{act} \in X_{act}$ **Equation 9-11**

if $d_{new} = d_{org}$ **Equation 9-12**

$L(d_{org}, x_{act}) - L(d_{new}, x_{act}) = 0$ **Equation 9-13**

AVRU = 0 **Equation 9-14**

If the value of the AVRU is equal to zero then the value of the EVRU is also zero. Therefore, if the original decision is the same as the decision made with additional information, the EVRU is equal to 0. With this in mind, the following equation presents the new equation for the EVRU.

$EVRU = E[L(d_{org}, x_{org})] - E[L(d_{new}, x_{new})] \cdot g(d_{org}, d_{new})$ **Equation 9-15**

where

$g(d_{org}, d_{new}) = 0$ when $d_{new} = d_{org}$

$g(d_{org}, d_{new}) = 1$ when $d_{new} \neq d_{org}$

The challenge to using the EVRU as shown in the equation above in the design process is that the designer does not actually know how the uncertainty will be reduced once the additional information is obtained. In other words, $E[L(d_{new}, x_{new})]$ is unknown and it becomes necessary to approximate this value without actually reducing the uncertainty.

Research Question: How do we approximate $E[L(d_{new}, x_{new})]$?

To approximate this term it is necessary to consider the different possible ways that the uncertainty can be reduced. For instance a number of different uncertainty reduction scenarios can be created where each scenario selects a different point in the uncertainty range about which to reduce the uncertainty. If only one scenario is selected then the uncertainty would be reduced about the mean of the uncertainty range. If two scenarios are selected then the range would be divided into two equal intervals and the uncertainty would be reduced about the mean for each interval.

For this technique the EVRU can be calculated as shown in Equation 9-16. In this equation, "NS" represents the number of scenarios.

Equation 9-16

$$EVRU = \frac{\sum_{k=1}^{NS} \left(E[L(d_{org}, x_{org})] - E[L(d_{new,k}, x_{new,k})] \right) \cdot g(d_{org}, d_{new,k})}{NS}$$

where

$g(d_{org}, d_{new}) = 0$ when $d_{new} = d_{org}$

$g(d_{org}, d_{new}) = 1$ when $d_{new} \neq d_{org}$

Research Question: How many scenarios should be used to approximate $E[L(d_{new}, x_{new})]$?

It is expected that as the number of uncertainty reduction scenarios increases, the accuracy of the approximate value for the expected loss associated with the decision made with reduced uncertainty ($E[L(d_{new}, x_{new})]$) will also increase. However, it is also anticipated that after a number of scenarios the accuracy of the value will remain constant or only increase minutely. It is expected that after a certain point the effects from additional uncertainty reduction scenarios will average out.

To determine the number of scenarios necessary for evaluating a reduction in uncertainty, four different functions, which model the uncertainty characteristic of an uncertain variable (x), are evaluated for an increasing number of uncertainty reduction tests. The number of uncertainty reduction tests varies from 1 to 20 reductions. The different functions were used to determine if the number of scenarios is function dependent. The four different functions are a uniform function, an approximated normal function, and two asymmetric beta functions.[29] The functions are shown in Figure 9-2.

To simplify the problem it is assumed that there is a 1/1 relationship between an uncertain variable (x) being reduced and the resulting effectiveness (the design metric). In other words, the value of the uncertain variable x is equal to the effectiveness. This direct relationship was selected so that the resulting effects are more apparent.

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[29] All of the functions are actually beta functions modeled in MATLAB with the betapdf function.

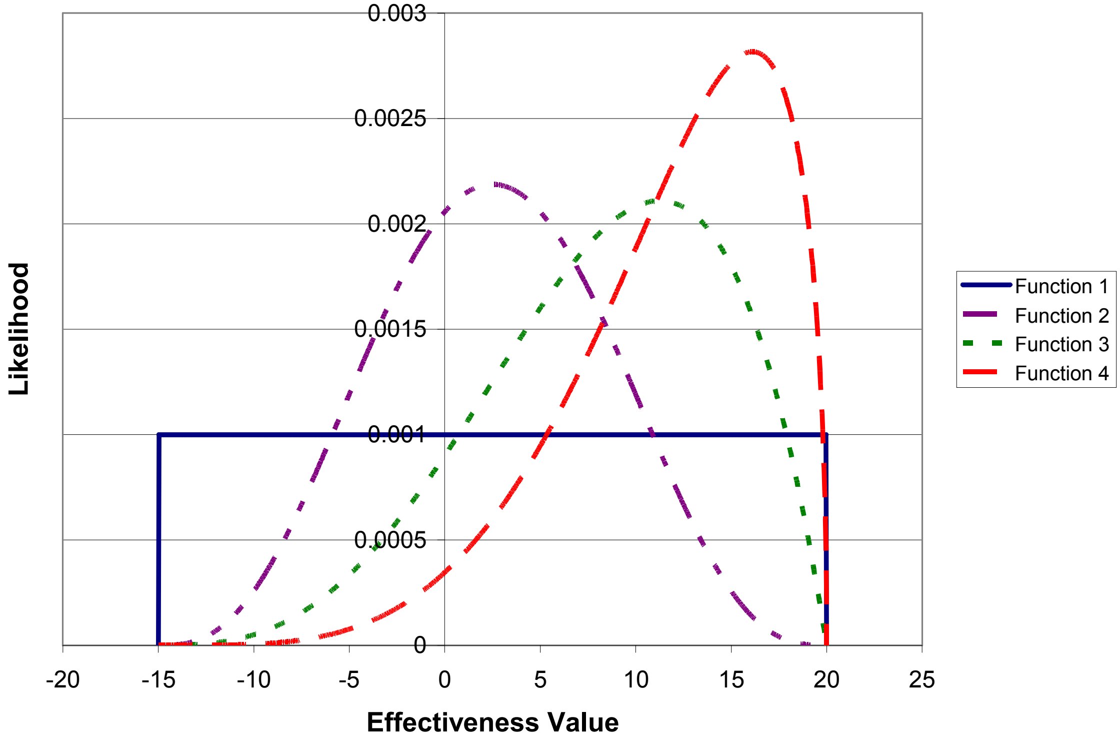

Figure 9-2: Distributions used in Number of Scenario/ Uncertainty Reduction Test

For each function the value of $E[L(d_{new}, x_{new})]$ was calculated for uncertainty reduction scenarios from 1-20. The results from these analyses are shown in Figures 9-3 through 9-6. Each figure is associated with a different function.

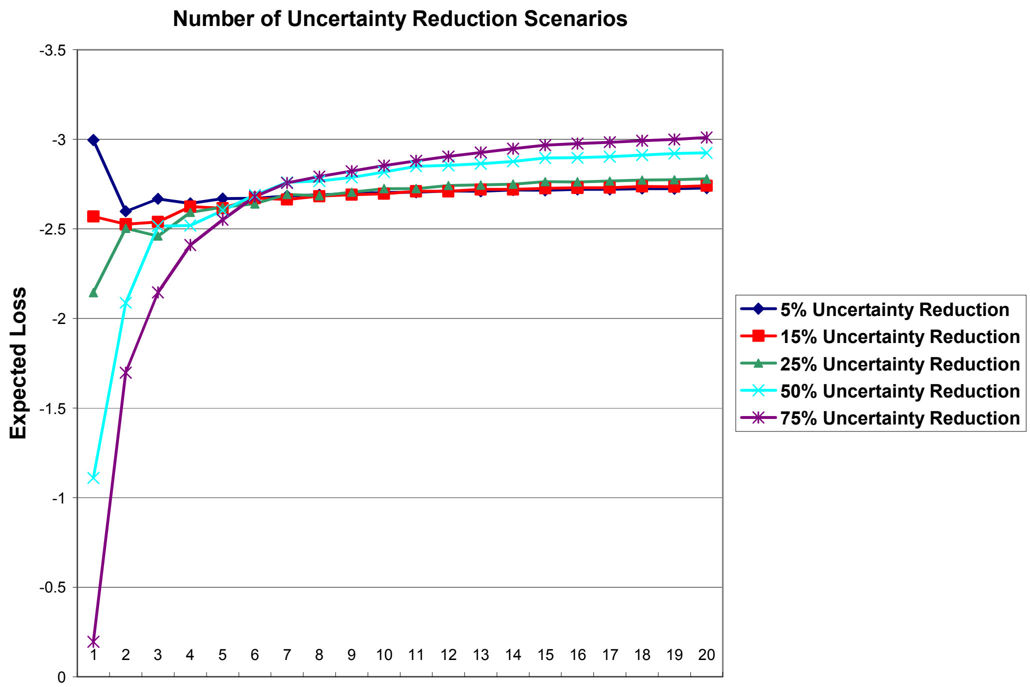

Figure 9-3: Estimated Expected Loss Calculated for every Scenario Reduction Group (Function 1 – Uniform Distribution)

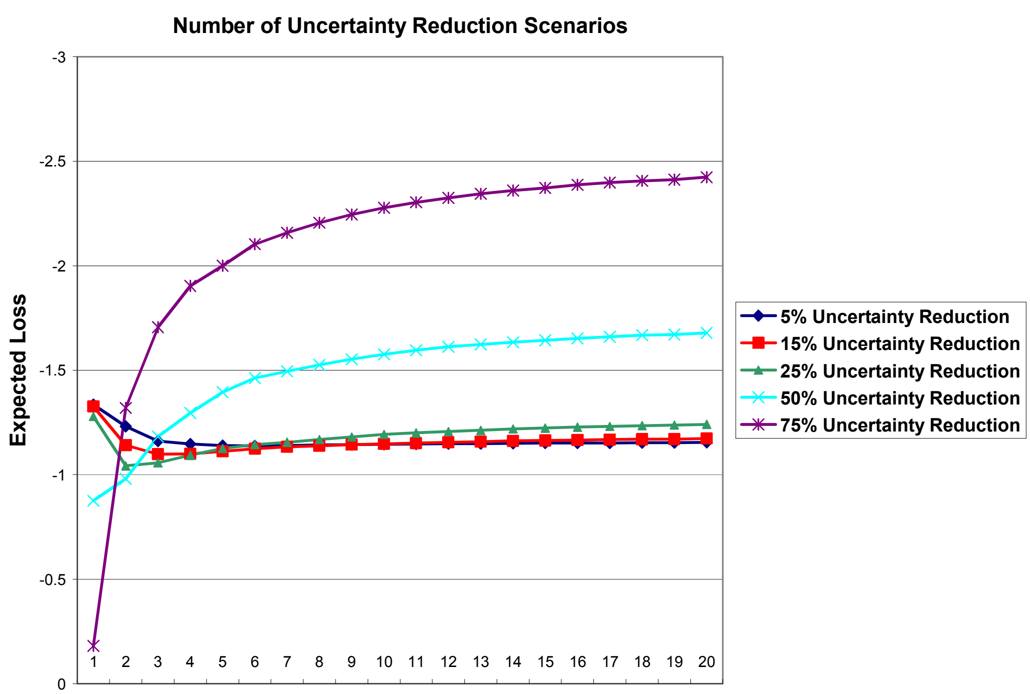

Figure 9-4: Estimated Expected Loss Calculated for every Scenario Reduction Group (Function 2 – Approximated Normal Distribution)

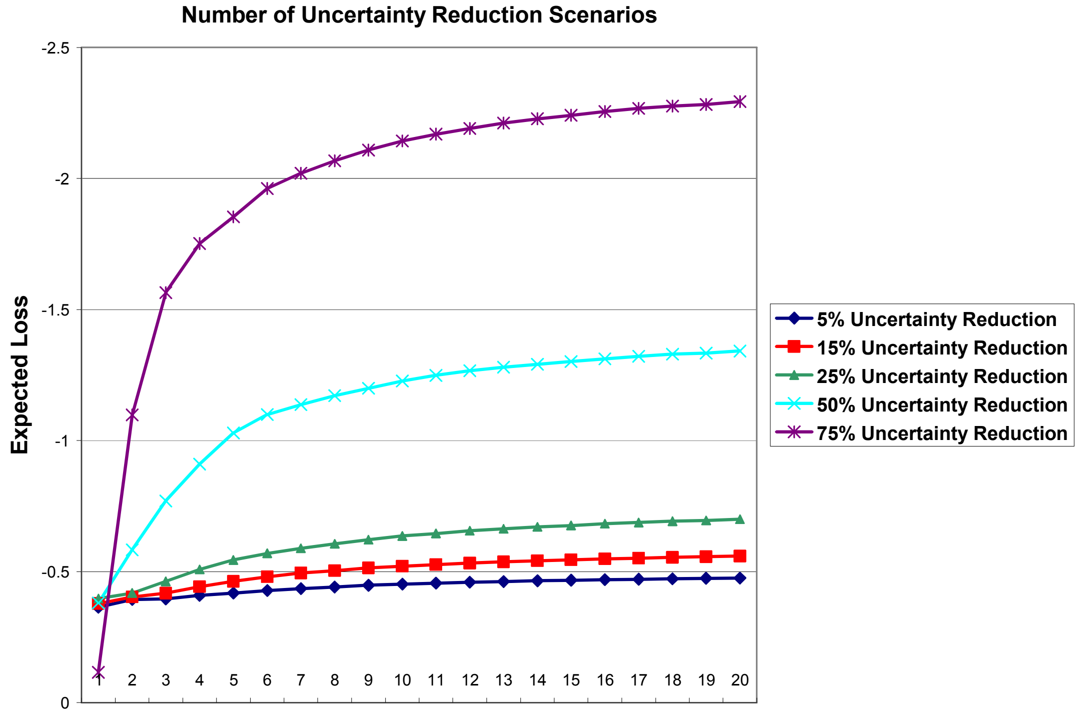

Figure 9-5: Estimated Expected Loss Calculated for every Scenario Reduction Group (Function 2 – Asymmetric Beta Distribution, P1 = 4, P2 = 2)

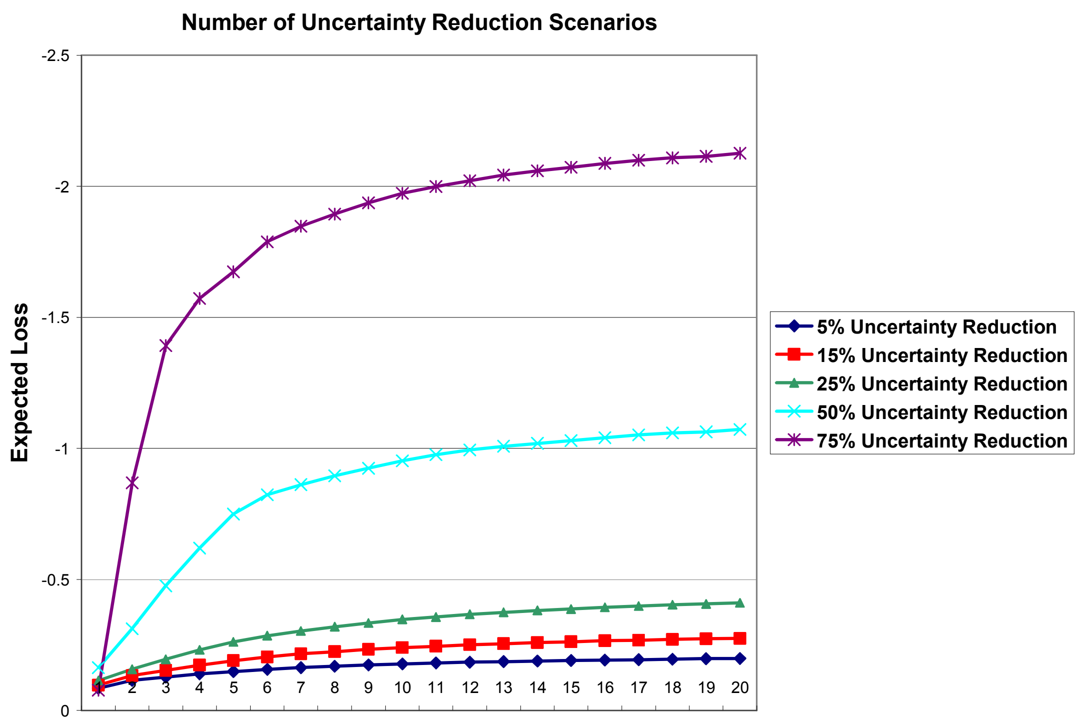

Figure 9-6: Estimated Expected Loss Calculated for every Scenario Reduction Group (Function 2 – Asymmetric Beta Distribution, P1 = 5, P2 = 1.5)

9.4. Value of Reducing Uncertainty Method (VRUM)

The Expected Value of Reducing Uncertainty (EVRU) can be used in a design process to aid the designer in determining if it is necessary to gain more information, or reduce the uncertainty, before a final design decision should be made. A process called the Value of Reducing Uncertainty Method (VRUM) has been developed around the EVRU technique and is shown in Figure 9-7. This technique/process is only applicable for reducible uncertainty, which means that it is only applicable for epistemic uncertainty and not aleatory uncertainty. A discussion over the differences between these types of uncertainty is presented in Chapter 3.

As presented earlier in this chapter there are a number of requirements that need to be satisfied within this process. The techniques need to be capable of:

- Evaluating and ranking the effect of uncertainty in a design problem
- Identifying how much the uncertainty in the problem can be reduced
- Estimating how much it will cost to reduce the uncertainty
- Evaluating the benefit associated with reducing the uncertainty
- Evaluating the cost of reducing the uncertainty with the benefit gained from reducing the uncertainty

The process satisfies the first requirement by incorporating a sensitivity analysis to evaluate the effects of the uncertainty and to determine where reducing the uncertainty would be of the most value. The second and third requirements are included in Task 8 of the process. This step is highly problem dependent and for the most part the specifics are determined by the designer. The meat of VRUM is associated with requirement four. The technique for determining the EVRU is used to calculate the benefit associated with reducing the uncertainty. And finally the last requirement is addressed in Task 12. This task is where the EVRU is compared with the Expected Cost to Reduce Uncertainty

(ECRU) and it is determined if additional information should be obtained before a decision is made.

VRUM was developed to be used in conjunction with Probability Theory, Evidence Theory, and Info-Gap Theory. The types of uncertainty addressed by Fuzzy Set Theory are not specifically addressed in this process. However, it may be possible with some modification to the process to consider various uncertainty reductions that affectively change the membership sets. This possibility is left open for future research.

This process is demonstrated with the fleet design example (Example F) from Chapter 8.

Persistent Strike Fleet Design VRUM Example

Problem Definition:

The purpose of the original design problem was to design a conceptual UAV for a general persistent strike scenario based upon aircraft specific constraints and fleet related performance and economic requirements. The specifics of the problem are detailed in Chapter 8.

Given Information:

The given information is information from previous steps in the design method.

Metrics of Interest:

For this example problem the metrics of interest on listed in Table 9-1.

Figure 9-7: Value of Reducing Uncertainty Method (VRUM)

Table 9-1: Constraint and Desirement Definition

Traditional Design Metric	Design Constraint	Design Desirement	Relative Importance of Metric
Fleet Cost	< \$5000 Million	< \$500 Million	0.5
$T/W_{Available}$-$T/W_{Required}$ for Takeoff Requirements	> 0	< 0.05	0.25
$T/W_{Available}$-$T/W_{Required}$ for Maximum Mach Requirements	> 0	< 0.05	0.25

Ranked Set of Design Alternatives:

One hundred potential design alternatives were selected from the design space bounded in Table 9-2.

Table 9-2: Definition of Design Space

Design Variable	Minimum Value	Maximum Value
Wing Area (ft^2)	200	600
L/Dmax	15	35
Thrust at sea level (lbs)	2,000	50,000
Cruise velocity (kts)	200	500

The top design alternative from the RandO Design approach is presented in Table 9-3.

Table 9-3: Original "Top Ranking" Alternative

Design Variable	Top Ranking Alternative
Wing Area (ft^2)	543.01
L/Dmax	31.06
Thrust at sea level (lbs)	13850
Cruise velocity (kts)	346.64

The uncertainty variables and their parameters are provided in Table 9-4.

Table 9-4: Uncertainty Variable Characteristics

Uncertainty Variable	Uncertainty Type	Distribution	Minimum Value	Maximum Value	Uncertainty Modeling Technique
AMPR Weight Factor (%)	Ambiguity	normal	60	70	Probability Theory
CD0	Ambiguity	unknown	0.01	0.019	Evidence Theory
TOP	Ambiguity	unknown	100	250	Evidence Theory
Payload (lbs)	Ambiguity	unknown	2,000	5,000	Evidence Theory
C_{Lmax}	Ambiguity	unknown	1.2	2	Evidence Theory
M_{MAX}	Ambiguity	unknown	0.6	0.9	Evidence Theory
Cost per pound ($/lb)	Ambiguity	unknown	4500	5000	Evidence Theory
Mission Radius (nm)	Ambiguity	unknown	unknown	unknown	Info-Gap Theory
Type of Engine	Vagueness	NA	NA	NA	Fuzzy Set Theory

This process does not currently address uncertainty modeled by Fuzzy Set Theory, so the type of engine will not be considered for uncertainty reduction.

<u>Data from original design analysis including uncertainty</u>

The data from the original design analysis for the RandO approach will be used in this analysis. Depending on the uncertainty modeling techniques used, the data produced from the original design analysis may not be compatible with this technique and may require modification.

Consider a design problem where the uncertainty is modeled with Probability, Evidence, and Info-Gap theory. If all three techniques are used, Info-Gap Theory models the uncertainty first, followed by Evidence Theory and Probability Theory. With Info-Gap theory each alternative was ranked based upon the α and β values associated with a given nominal value for each uncertainty variable. The alternative selected is the alternative with the largest α and the smallest β value, thereby selecting an alternative that is both robust and opportunistic relative to the aspects of uncertainty. But, the α and β values associated with each metric are based upon value of the uncertain variable and do not represent the resulting values of the traditional design metrics. This technique is based upon calculating the difference in potential loss between different design decisions, and requires values based on the traditional metrics. To provide suitable information, it is possible to calculate the value for the metric of interest using the nominal uncertainty values in conjunction with values for the other uncertainty variables not modeled by Info-Gap Theory.

It is important for the designer to realize that this is a large assumption included in the EVRU process. The nominal value was the best guess for modeling the expected value of the uncertain variable when very little was known about it. This assumption is necessary in order to compare relative losses between different potential decisions. Additionally, this assumption will not actually affect the design decision. This assumption is only a factor in determining if additional information is necessary before making the design decision.

Using the nominal values from Info-Gap Theory, for every Evidence Theory run value, and every Probability Theory run value, the traditional design metrics are calculated. This information is then sorted based upon the Evidence Theory run values to determine the plausible and believable values for each alternative. It is necessary to determine the plausible and believable values because there is not enough information available to determine a single value. This effectively bounds the problem space and will result in a plausible and believable value for the final result of the EVRU. If Evidence Theory is not used in the design process, there will be only one value for the EVRU.

At the end of this step there are two lists of data for each potential design alternative. The first list is associated with Plausible metric values and the second list is associated with Believable metric values. Each value is associated with a specific combination of values for the uncertainty variables modeled by Info-Gap, Evidence, and Probability Theory. Additionally based on the information from both Probability Theory and Evidence Theory, each value is associated with a certain likelihood of occurrence.

Task 1 - Monetize Metrics of Interest

This task is where a multiplier, such as λ from Reference 201, would be used to transform the metric of interest into a monetized value. The best source of multiplier would be literature or expert opinion.

However, when converting metrics to monetary values, it is important to stay true to the original weights used in this process. These weights are important and not following this scale with the metric conversion will distort the actual estimated loss (or gain) of the selection of one alternative over another.

One option is to use the original weights in converting metric values to a monetary value, especially if one or more of the metrics is already in monetary form. This option is used for this example. For instance, in the example problem there is one monetary metric (fleet cost) with weight 0.5, and two nonmonetary metrics (T/WTakeoff) weight 0.25 and (T/Wmaxspeed) also with weight 0.25.

The first sub-task is to determine the maximum and minimum values for all of the metrics. This information is available from the original design data. Before the process can continue, all of the data from the original design process needs to be scaled such that the objective for the each metric is either to be maximized or minimized. Metrics where the constraint and desirement have a competing relationship need to be converted into a monotonic function through the use of a penalty function and a penalty factor if desired.

After the metrics have been converted, if necessary, it is possible to transform the nonmontonic metrics based upon the supplied weight fractions and the maximum and minimum values of the monetary metric.[30]

The relationship shown in the equation based upon Reference 86 is used to determine the maximum monetary value for each metric. This equation states that the ratio of two measures of performance (c_1 and c_2) is equal to the ratio of weights associated with each measure of performance. This equation is used in the following equations to determine the maximum and minimum monetary value for metric "i" designated by $\overline{M}_{MAX,i}$ and $\overline{M}_{MIN,i}$ respectively. The bar over the M, indicates that this value is now in monetary terms. $\overline{M}_{MAX,ORGMonetaryMetric}$ represents the maximum value of the original monetary metric for the problem. $\overline{M}_{MIN,ORGMonetaryMetric}$ represents the minimum value for the original monetary metric.

$$\frac{c_1}{c_2} = \frac{w_1}{w_2}$$ **Equation 9-17**

$$\overline{M}_{MAX,i} = \frac{\overline{M}_{MAX,ORGMonetaryMetric} \cdot w_i}{w_{MonetaryMetric}}$$ **Equation 9-18**

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[30] If a design problem consists of multiple monetary metrics, only one can be selected as the monetary metric to convert all of the other values. If the problem has been weighted appropriately, the transformation will not affect the other monetary metric values.

$$\overline{M}_{MIN,i} = \frac{\overline{M}_{MIN,ORGMonetaryMetric} \cdot w_i}{w_{MonetaryMetric}}$$ **Equation 9-19**

Now that the maximum and minimum monetary values ($\overline{M}_{MAX,i}$ and $\overline{M}_{MIN,i}$) have been determined it is possible to transform the metric values to a monetary value. The final transformation equation is shown below in Equation 9-22.

$$Slope_i = \frac{\overline{M}_{MAX,i} - \overline{M}_{MIN,i}}{M_{MAX,i} - M_{MIN,i}}$$ **Equation 9-20**

$$b_i = \overline{M}_{MAX,i} - Slope_i \cdot M_{MAX,i}$$ **Equation 9-21**

$$\overline{M}_{i,j} = Slope_i \cdot M_{i,j} + b_i$$ **Equation 9-22**

In the equations above, j indicates the specific metric value from the data analysis being transformed. This equation is then used to convert all of the original nonmonetary metrics to monetary metrics.

Task 2 – Determine Δcost between "best" alternative / other alternatives and likelihood of occurrence

The objective of this task is to determine the Δcost between the "best" alternative / other alternatives and the likelihood of occurrence. As discussed earlier, the data is included in two data groups, one for the plausible values and one for the believable values. This data is also organized by each of the potential design alternatives. For each alternative there are a set of "n" values, where "n" is equal to the number of runs from the Probability Theory DOE discussed in Chapter 7. For the example problem there was only one variable modeled with Probability Theory and it was modeled by seven different intervals. As a result for each alternative in each of the data groups, there are seven values relating to the seven runs in the Probability Theory DOE.

Because these values are in monetary terms it is possible to estimate the monetary loss for each metric associated with selecting one alternative over another. The calculation for this is shown in the equation below. In this equation, “i" designates the metric of interest, “j” designates the interval value being considered, and “k” represents the alternative number.

$$\Delta\overline{M}_{i,j}(k,TopAlternative) = \overline{M}_{i,j,k} - \overline{M}_{i,j,TopAlternative}$$ **Equation 9-23**

Task 3 - Determine potential loss associated with selecting “best” alternative over another with given information

The total Expected Utility for selecting the top ranking alternative over alternative j is calculated in the equation below. It is expected that this value will be positive, indicating that there is more utility in selecting the top alternative over the other alternative. Otherwise, the top alternative would not have been selected originally. However, in some cases due to the transformation to a monetary scale this will not be true. This shows a disparity between the original weights assigned to each metric and weights now inherent in the monetary scale. This is most likely to occur when a monetary conversion factor is used to transform the values to the common scale. The designer should be aware of this disparity and consider that this will skew the final results. In this equation “NI” designates the total number of intervals.

$$E[U_i(k,TopAlternative)] = \sum_{j=1}^{NI} \Delta\overline{M}_{i,j}(k,TopAlternative) \cdot \Pi_j))$$ **Equation 9-24**

The potential loss can be calculated by only considering the values from the above equation that are negative. This indicates that for some of the interval values, the other alternative has a better utility than the top ranking alternative.

$$E[L_i(k, TopAlternative)] = \sum_{j=1}^{NI} \Delta\overline{M}_{i,j}(k, TopAlternative) \cdot \Pi_j \cdot f(\Delta\overline{M}_{i,j}(k, TopAlternative))$$ **Equation 9-25**

where

$f(\Delta\overline{M}_{i,j}(k, TopAlternative)) = 0$ when $\Delta\overline{M}_{i,j}(k, TopAlternative) > 0$

$f(\Delta\overline{M}_{i,j}(k, TopAlternative)) = 1$ when $\Delta\overline{M}_{i,j}(k, TopAlternative) < 0$

This process is repeated for both the Plausible and Believable data groups.

Task 4 - Calculate Expected Loss Ratio (ELR) for "best" alternative compared with other alternatives

The Expected Loss Ratio (ELR) is based upon the expected utility of selecting the top alternative over another and the potential loss associated with this decision. The calculation for the ELR is shown below in Equation 9-26.

$$ELR_i(k, TopAlternative) = \frac{E[L_i(k, TopAlternative)]}{E[U_i(k, TopAlternative)]}$$ **Equation 9-26**

This value is not actually used in the calculation for the Expected Value of Reducing Uncertainty (EVRU), but it is useful in indicating the likelihood that there will be a loss associated with a given decision. If the ELR is equal to zero for specific alternative then there is no need to consider that alternative in subsequent tasks. Even if the uncertainty is reduced, that alternative would not be selected over the top ranking alternative.

Task 5 - Identify maximum potential loss if "best" alternative selected

Recall that the potential loss was determined for the "best" alternative versus all of the other potential alternatives in Task 3. In this task, the maximum value of these potential losses is determined. This represents the maximum loss that could potentially occur if the "best" alternative is selected. This task can be done by using Equation 9-27.

$$E[L_i(TopAlternative)]_{MAX} = \max(E[L_i(k, TopAlternative)] \qquad \textbf{Equation 9-27}$$

Task 6 - Rank sensitivity of metrics to uncertainty variables

Considering a number of uncertainty reduction scenarios for every possible uncertainty variable for most design problems would be a cumbersome process. However, based upon the Pareto Principle, it is common for a small subset of the variables to cause a majority of the variance in the traditional design metrics.[107] In many cases, it is possible to reduce the number of uncertain variables that should be considered through a screening test or other sensitivity analysis.

A screening test was performed for this problem following the technique presented in Reference 107. The Pareto Plots from this analysis are presented in Appendix C.

Task 7 - Identify uncertainty to be reduced

From the screening test results it was possible to determine that the following variables are the primary cause of variance in the design metrics: mission radius (or cruise distance), L/Dmax, cruise velocity, maximum Mach number, the thrust at sea level, and the wing area. Four of these variables are design variables, which leaves two uncertain variables that should be considered: mission radius and the maximum Mach number.

Task 8 – Determine Characteristics associated with Uncertainty Reduction

In this task the designer answers the following questions:

- How uncertainty can be reduced?
- How much uncertainty can be reduced?
- How much will cost to reduce uncertainty?

For the example problem two uncertainty variables are considered for reduction: mission radius and the maximum Mach number.

The mission radius is considered uncertain because the aircraft and the fleet scenario are for a general persistent strike scenario. The aircraft fleet could be used for a huge range of mission radii depending upon target and base locations. To reduce this variable information would need to be gathered about likely future scenarios. Considering the enormous expense related to the purchase of a fleet of unmanned aircraft, it is likely the information would be obtained from the military intelligence community on this issue.

It certainly could be possible to reduce this uncertainty considerably, by identifying likely future missions for the aircraft. However reducing the uncertainty related to the range will limit the flexibility of the future aircraft. But, it is still necessary to determine how much could be gained by reducing the uncertainty for the full range of possibilities. For this reason the uncertainty was reduced for this variable by the following percentages: 10,20,30,40,50,60, and 70%.

The cost associated with reducing this variable, is based upon the cost required for the intelligence community to provide better estimates for the mission radii for this type of aircraft. For this example problem a notional value of $10,000 per every 10 percent reduction in uncertainty was selected for comparative purposes. For actual applications of this process, an estimate for the cost of this task would need to be provided by the intelligence community. The calculation for the Expected Cost to Reducing Uncertainty (ECRU) for the mission radius is shown in the following equation.

$$ECRU(MissionRadius) = \$1{,}000 \cdot (\%\text{ Uncertainty Reduced}) \qquad \textbf{Equation 9-28}$$

The uncertainty relating to the maximum Mach number is also based on future scenarios. Factors such as the type of targets, the mobility of the targets, the terrain and the environment affect the most appropriate maximum Mach number. The intelligence community would again need to be questioned in order to determine expected values for types of targets, the mobility of the targets, the terrain and the environment for future scenarios. However this information alone will not provide a solution to the appropriate range of likely Mach numbers. This number would most likely need to be calculated based upon simulation results that consider all of the relevant factors. The information from the intelligence community would be used as inputs for the simulation. For this example problem the uncertainty relating the value for the maximum Mach number will be reduced by 30, 40, and 50%.

The cost associated with reducing the uncertainty for the maximum Mach number is composed of both variable and fixed costs.[87] A notional value of $10,000 per every 10 percent was again selected for gaining information from the intelligence community. An additional $150,000 was estimated for the development of a modeling and simulation environment to determine appropriate maximum Mach numbers. And finally $50,000 was estimated for the final analysis determining the Mach numbers with the simulation environment. Neither the development of the modeling and simulation environment nor the analysis is expected to be dependent on the amount of uncertainty to be reduced. Therefore these are fixed costs associated with the process. The calculation for the

Expected Cost to Reducing Uncertainty (ECRU) for the maximum Mach number is shown below.

$$ECRU(M_{MAX}) = \$1{,}000 \cdot (\text{\% Uncertainty Reduced}) + \$200{,}000 \qquad \textbf{Equation 9-29}$$

Task 9 - Identify scenarios for potential ways that uncertainty can be reduced

Based upon the analysis presented earlier in this chapter it is apparent that there are diminishing returns as the number of scenarios increases. For small to moderate reductions in uncertainty it is reasonable to run between 3 and 6 scenarios. For higher uncertainty reductions it is more appropriate to run around 7 to 10 scenarios.

For this example problem, to demonstrate the concept, 3 uncertainty reduction scenarios were considered for each possible reduction in uncertainty. For the mission radius, 21 total uncertainty reduction scenarios were considered, and 9 scenarios were considered for the maximum Mach number.

Task 10 – Repeat process for each Uncertainty Reduction Scenario

In this task, the following steps are repeated for each uncertainty reduction scenario:

- Rerun analysis with new uncertainty variable parameters
- Rank alternatives
- Identify “best” alternative
- Determine if there is a change in “best” alternative
- Repeat Tasks 1-5 for each scenario

The five subtask steps were completed for the example problem. The results from this analysis are presented in the next section.

Task 11 - Calculate EVRU

In this step, the following equation (9-16) is used to calculate the EVRU. The equation is presented below for completeness.

$$EVRU = \frac{\sum_{k=1}^{NS}\left(E[L(d_{org}, x_{org})] - E[L(d_{new,k}, x_{new,k})]\right)\cdot g(d_{org}, d_{new,k})}{NS}$$

Where

$g(d_{org}, d_{new}) = 0$ when $d_{new} = d_{org}$

$g(d_{org}, d_{new}) = 1$ when $d_{new} \neq d_{org}$

Task 12 - Determine if additional information is required before decision is made

In this task it is determined if the EVRU is greater than the ECRU identified in Task 8. If the EVRU is greater than the ECRU then it is likely that additional information should be acquired, otherwise the decision should be made with the available information. The results from the example problem are presented in the Tables below. All of the gain values are in $Million.

Table 9-5: Uncertainty Reduction Results – Cruise Range[31]

	Metric 1		Metric 2		Metric 3	
Uncertainty Reduction	**Minimum Gain**	**Maximum Gain**	**Minimum Gain**	**Maximum Gain**	**Minimum Gain**	**Maximum Gain**
10	0	223.71	0	4990	0	134.85
20	0	518.66	0	3917	0	137.31
30	0	703.18	0	3750	0	130.9
40	0	370.92	0	3734.5	0	88.662
50	0	26.867	0	3002.2	0	144.09
60	0	271.66	0	1735.9	0	133.94
70	0	501.35	0	0	0	195.3

[31] Gain values are in $Million

Table 9-6: Combined Uncertainty Reduction Results – Cruise Range[32]

Uncertainty Reduction	Minimum Gain	Maximum Gain
10	0	5348.6
20	0	4573
30	0	4584.1
40	0	4194
50	0	3173.1
60	0	2141.5
70	0	696.64

Table 9-7: Combined Uncertainty Reduction Results – Maximum Mach Number[33]

	Metric 1		Metric 2		Metric 3	
Uncertainty Reduction	Minimum Gain	Maximum Gain	Minimum Gain	Maximum Gain	Minimum Gain	Maximum Gain
30	1.03E-13	67.954	0	0	7.4767	58.84
40	1.01E-13	30.967	0	0	5.8817	58.225
50	9.93E-14	30.533	0	0	3.775	57.245

Table 9-8: Combined Uncertainty Reduction Results – Maximum Mach Number[34]

Uncertainty Reduction	Minimum Gain	Maximum Gain
30	7.4767	126.79
40	5.8817	89.192
50	3.775	87.778

[32] Gain values are in $Million
[33] Gain values are in $Million
[34] Gain values are in $Million

Table 9-9: Fleet Design Example Problem Results

	Uncertainty Reduction (%)	ECRU
Mission Radius	10	$10,000
	20	$20,000
	30	$30,000
	40	$40,000
	50	$50,000
	60	$60,000
	70	$70,000
M_{MAX}	30	$230,000
	40	$240,000
	50	$250,000

Table 9-10: Final VRUM Results

	Uncertainty Reduction (%)	ECRU ($Million)	EVRU Minimum ($Million)	EVRU-ECRU Minimum ($Million)	EVRU Maximum ($Million)	EVRU-ECRU Maximum ($Million)
Mission Radius	10	0.01	0	-0.01	5348.6	5348.6
	20	0.02	0	-0.02	4573	4573
	30	0.03	0	-0.03	4584.1	4584.1
	40	0.04	0	-0.04	4194	4194
	50	0.05	0	-0.05	3173.1	3173.1
	60	0.06	0	-0.06	2141.5	2141.4
	70	0.07	0	-0.07	696.64	696.57
M_{MAX}	30	0.23	7.4767	7.2467	126.79	126.56
	40	0.24	5.8817	5.6417	89.192	88.95
	50	0.25	3.775	3.525	87.778	87.53

CHAPTER 10: CONCEPTUAL DESIGN OF OPPORTUNISTIC AND ROBUST SYSTEM-OF-SYSTEMS

The primary focus for this research is to address the gaps discussed in Chapter 6. The techniques developed to fill these gaps can be used in conjunction with a general design method framework based off of existing methods. The result is a method for the CONceptual Design of Opportunistic and Robust System-of-Systems (CONDOR-SS).

The primary capability gaps to be addressed by this method are that:

- Existing SoS Design Methods are incapable of modeling all of the different types of relevant uncertainty
- Existing SoS Design Methods do not specifically address the fact that there can be propitious effects from uncertainty as well as pernicious.
- Existing SoS Design Methods focus on identifying the most effective design alternative with respect to the relevant uncertainty. However, none of these methods focus on determining if the uncertainty should be reduced before making the final design decision.

10.1. CONDOR-SS

As discussed in Chapter 2, a SoS problem can be broken into different design levels. The first level is the OES level of the SoS. This level is composed of different systems that must either be designed or selected. For the persistent strike battlespace scenario, this level is the entire battlefield system which may consist of satellites, ground based sensor systems, aircraft to both search for targets and eliminate targets, the base of operations, and the targets for a given coverage area and coverage period.

As illustrated in Figure 10-1, the design process begins with this top level and continues down to the next level (Intermediate Level A). For the battlespace problem, the next level would be the aircraft involved in the persistent strike mission. Once the system has been developed at this level, this information flows back up into the OES Level to determine if the objectives are being met and if constraints are being appropriately handled. For instance, now that the aircraft has been designed from a top level perspective, the designer has a better idea of the actual performance of this aircraft. This will reduce some portion of the uncertainty in the OES Level, and this information must be reevaluated to make certain that the appropriate aircraft has been selected and that it is still possible to meet the objectives of the OES Level.

If the system design of Intermediate Level A is acceptable, the design process continues on into Intermediate Level B, where the design of the subsystems of Level A is addressed. For instance, if the top level characteristics of the aircraft are acceptable, then the designer would consider the performance and characteristics of potential engines and weapon systems for the aircraft. This process is repeated through all of the subsequent levels until the Base Level is developed and approved for the entire SoS.

For the design of each SoS Level there are a set of specific steps that must be considered. The steps include: defining the problem; defining systems, system architecture, and scenarios; identifying system and scenario uncertainties; creating or selecting a Modeling and Simulation (M&S) environment; identifying critical uncertainty and design variables; creating surrogate models; identifying new alternatives; evaluating alternatives for SoS level; quantifying value of reducing uncertainty; gaining more information; and selecting a concept or concepts for SoS design level. These steps are discussed in more detail in the following sections of this chapter. A flow chart of these steps is presented in Figure 10-2.

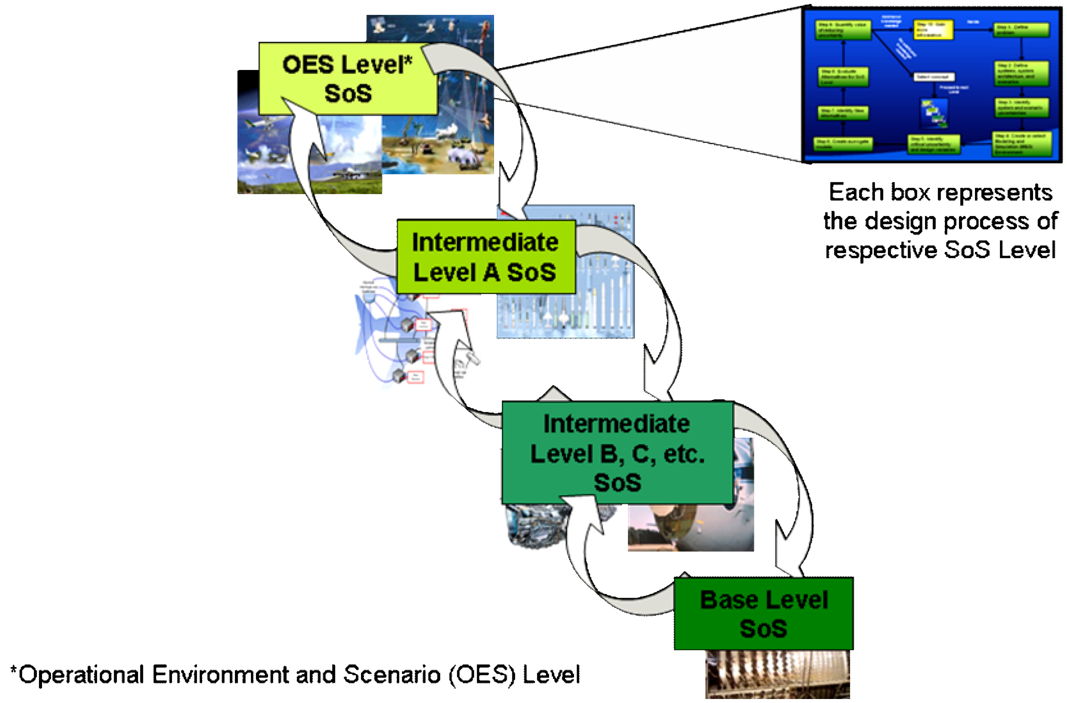

Figure 10-1: SoS Design Process

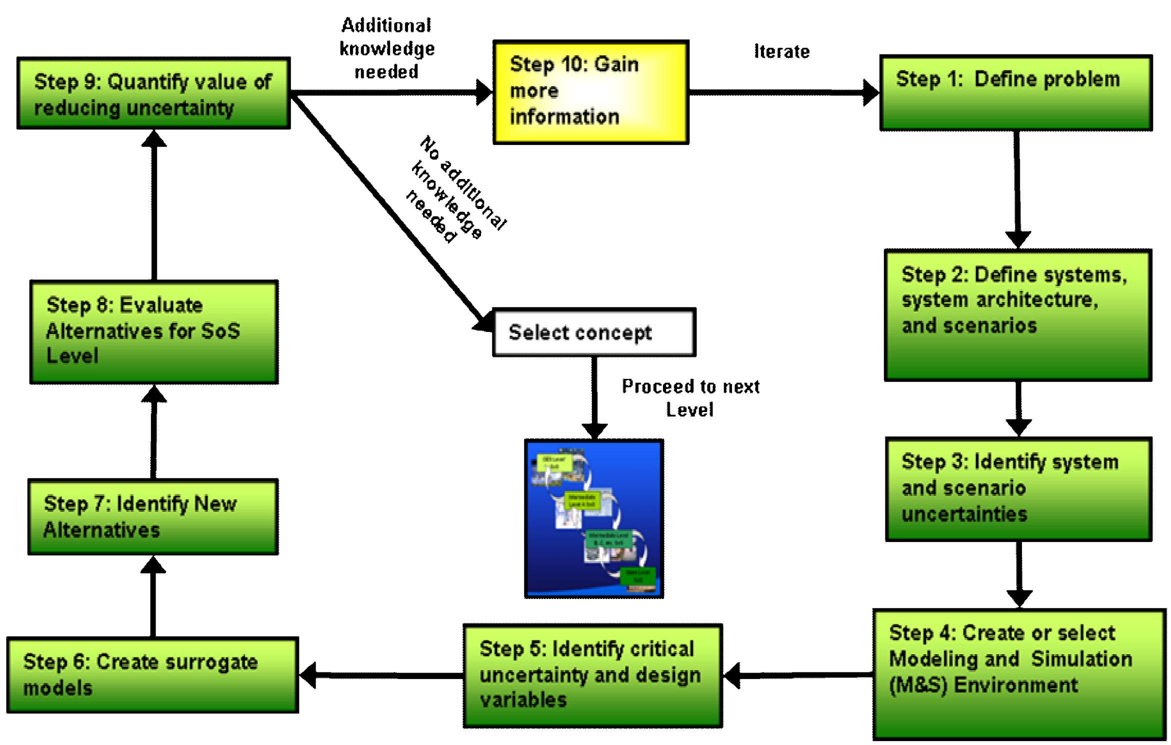

Figure 10-2: CONDOR-SS for each SoS Design Level

Step 1: Define the problem

This step consists of determining or identifying: the objectives for the SoS, the requirements, the constraints, and stakeholder defined scenarios.

Step 2: Define systems, system architectures, and scenarios

The next step of the process is to define the different levels of the SoS, the potential systems for each level of interest, the interactions/relationships between the systems, and the possible operational scenarios of the SoS. During this step the designer should create a matrix of alternatives identifying the different possible options (capabilities / characteristics) for the SoS and its subsystems. Different technologies can be considered as different options in the matrix. The designer should also identify the control factors (design variables) and noise factors (uncontrollable factors) as well as the output values that are going to be evaluated.

Once the systems have been defined, it is possible to develop potential operational and situational scenarios. Operational scenarios pertain to how the SoS is operated, for instance, this pertains to how each aircraft is used in the battlespace and how they interact. An example is that a MQ-9 Reaper could be used as a sensor aircraft, a strike aircraft, or both. Situational scenarios relates to bad weather scenarios, scenarios with high numbers of targets, etc.

Because it is likely that a SoS will need to be robust and opportunistic against a wide variety of scenarios, it is also important for the designer to create a matrix of scenario alternatives. This matrix can be used to identify the primary operational scenarios and the secondary scenarios, which are those that should be considered, but should not be weighted as heavily as the primary scenarios.

Step 3: Identify system and scenario uncertainties

The fourth step in the process is to identify system and scenario uncertainties so that these uncertainties can be modeled and considered in the design process. This step is critical because in the example problem, as with most design problems, there is significant uncertainty. Systems and system characteristics cannot be selected until the uncertainty has been quantified and propagated through the system so that the final effects can be determined.

Often in the design process, uncertainty is described as coming from either parameter uncertainty or model structure uncertainty. Parameter uncertainty describes the uncertainty relating to the either the variability or lack of information pertaining to either input or model parameters. [66,143] For the battlespace problem, examples could be either variability or lack of knowledge relating to the location of a target or the defensive/offensive capabilities of a target. Many parameters in the design analysis, such as aircraft acquisition cost, are likely to be estimated. There is a significant amount of uncertainty relating to what the actual acquisition cost of the aircraft would be and how it compares with the estimated parameter. Model structure uncertainty pertains to the fact that any model, despite its fidelity, will be a simplification of the actual system. This is where the uncertainty relating to the validity of assumptions built into the model is considered. [66, 68] There is uncertainty pertaining to how well a model can approximate the results of the true battlespace.

In addition to these two types of uncertainty, in order to quantify the effects on the design and design process for a SoS, it is necessary to further categorize the sources of uncertainty. The designer must consider sources of system uncertainty relating to system interaction, emergent behavior, situation, operations, independent agents, and new systems. As discussed in Chapter 3, system interaction uncertainty deals with uncertainty pertaining to how the different systems interact. Emergent behavior uncertainty pertains to uncertainty in unmodeled effects from systems interacting. Situational uncertainty is uncertainty relating to the environment of SoS. Operational uncertainty is the uncertainty dealing with how systems will be operated. Independent agent uncertainty deals with fact that systems can be autonomous. And, new system uncertainty is related to how new systems may be integrated into the system in the future.

Step 4: Create or select a Modeling & Simulation Environment

This step uses the information from the previous steps to either create a modeling and simulation (M&S) environment or to select an existing environment. At this stage it is important to use an M&S environment that models the main interactions and effects without going into more detail than necessary and without expending more resources than necessary. If it is found that more detail is needed in future steps, then this part of the process should be repeated with a higher fidelity analysis.

The M&S environment goes beyond simply modeling the SoS, it must be capable of allowing the uncertainty to be quantified, propagated, and tracked throughout the design problem. There are a wide variety of theories that can be used for modeling uncertainty including Classical Sets, Probability, Statistics, Fuzzy Sets, Possibility, Evidence, Info-Gap, etc. As discussed in Chapter 4, all of these theories have their advantages and can be used with this method. No one theory is appropriate for modeling all types of SoS uncertainty. During this step the designer must select the appropriate methods for modeling the uncertainty to be incorporated into the M&S environment.

In this research, four different theories will be used to cover the range of sources of SoS uncertainty. These theories are: Probability Theory, Evidence Theory, Fuzzy Set Theory, and Info-Gap theory. As discussed in Chapter 4, these theories are applicable to all of the different types of uncertainty problems in the development of a SoS and are used throughout the engineering industry.

Step 5: Identify critical uncertainties and design variables

While modeling the uncertainty is a critical aspect of most design projects, it will require additional resources. Depending upon the number of uncertainty and design variables that must be considered and the resources required for the M&S environment, it may be necessary to identify the critical uncertainties and variables through a screening test. This test will allow the designer to determine which of the uncertainties and variables do not significantly contribute to the variance of the design metrics for each scenario and therefore can be eliminated from future analyses. [107] If there are only a few design variables and uncertainties in the original problem or if the M&S environment is not resource intensive, then this step can be skipped.

Step 6: Create a surrogate model incorporating uncertainties

If the M&S environment is computationally expensive or it requires a significant amount of resources, the designer should create a surrogate model (or metamodel) of the M&S and uncertainty modeling environment. Examples of surrogate models includes Response Surface Equations (RSEs), Neural Networks, or Kriging.[54,31] These models are different types of equations which model the output of the M&S and uncertainty modeling environment for a given input. The purpose of using a surrogate model is so that it is possible to obtain the results from a large number of test cases without having to run the M&S environment for each case. This technique will make it possible to run a variety of detailed analyses without requiring a great deal of computational time or additional resources. [107]

Step 7: Identify New Alternatives

Often legacy systems can be used effectively in a SoS, and in many cases it is expected that existing systems will compose the SoS. This system is a special kind of SoS called a Family of Systems (FoS). A FoS is a collection of legacy systems which have each been designed for a specific purpose or mission in isolation of the other systems. [129] For this type of system there is an emphasis on the capabilities of the operation of the independent system and on the interoperation of all of the systems. Many existing SoS are FoS. A FoS designer will typically not have control over the design of specific system design parameters. Instead their purpose will be to develop the capabilities of the system based upon the parameters that they can control or select. [188] In the case of a FoS, the Base Level will be the lowest level of detail that the designer can control.

However, in cases where designers are interested in comparing existing systems against potential new systems, these new systems must first be conceptualized and modeled.

Even with surrogate models, it is impossible to analyze every possible alternative for a SoS design, especially when considering the complexity of a design problem with uncertainty included. Before the design process can continue it is necessary to identify a group of potential alternatives using either the M&S environment or the surrogates of the environment. Each of these potential alternatives would then be evaluated in Step 8.

In general, design concepts should be selected in a manner where the entire design space is evenly covered. While the entire design space should be considered, in circumstances where the designer is particularly interested in a certain region of the design space, it is acceptable to concentrate a number of solutions in this region. An example of how this could be accomplished is by using the Monte Carlo based design space exploration technique that was proposed by Ender in Reference 2. [71,188]

Step 8: Evaluate system alternatives for SoS Level

At this point it is necessary to evaluate the system alternatives to determine which concepts are the most robust and opportunistic and should be continued on with in the design process. Techniques from Info-Gap Theory allow a designer to make this kind of decision with the identified uncertainty. Info-Gap Theory allows designers to evaluate concepts on two levels. First the designer is trying to maximize the robustness of the alternative and they are also trying to minimize the uncertainty required for a concept to meet and exceed desired levels for the output metrics. This is done using the robustness and opportunistic functions that were discussed in Chapter 8.

The process for this step is illustrated in Figure 10-3. First, a set of alternatives is selected from the design space using either a DOE or Monte Carlo sampling technique as described in Reference 71. The HUMM is used to analyze the uncertainty for each alternative and to determine the Robustness and Opportunity Functions. A weight determination process, as described in Chapter 8, is used to determine the appropriate weights for the design metrics and then a MADM technique such as TOPSIS is used to evaluate the various concepts.[204]

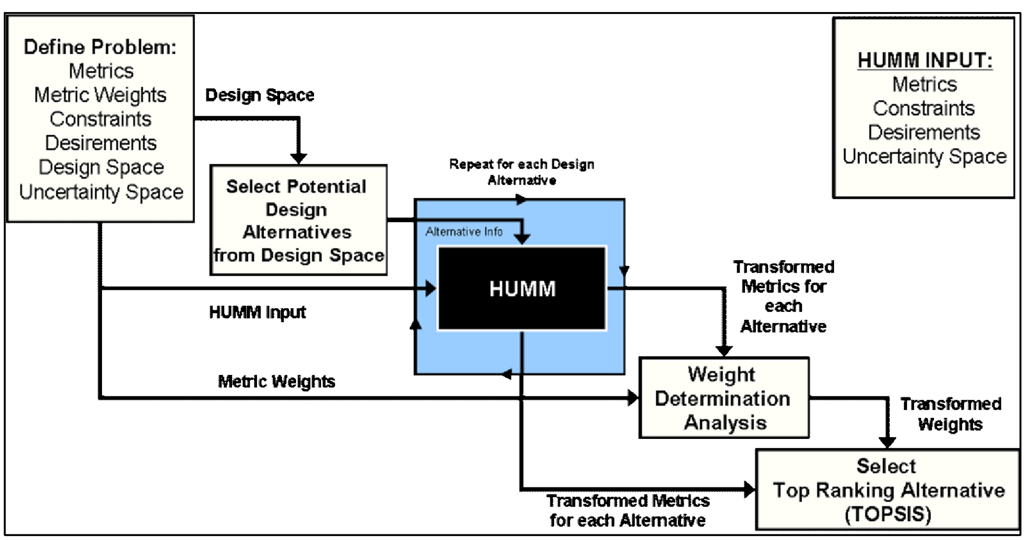

Figure 10-3: General Design Process for Robust, Opportunistic, and RandO Design Approaches

Step 9: Quantify the value of Reducing Uncertainty

In this step the designer determines if additional information should be gained about the problem to reduce the uncertainty or if a decision should be made with the available information. In some cases, it is not worth the additional resources required for further analyses, but in other cases it is necessary. This relates directly to both the resources required for further analyses and the risk associated with selecting the "wrong" alternative.

This type of analysis is done in a process called the Value of Reducing Uncertainty Method (VRUM) and is based on a modified form of the Expected Value of Including Uncertainty (EVIU) called the Expected Value of Reducing Uncertainty (EVRU). The EVIU is the expectation of the difference in loss resulting from using an optimized alternative considering no uncertainty and an alternative that was found while considering the uncertainty. [134] Because the original information for an alternative in this step already includes uncertainty, a designer instead is interested in evaluating the estimated loss that might result from making a decision with the current level of knowledge as opposed to a more informed decision. Instead of determining the EVIU the designer needs to determine the EVRU.

To determine the loss, the designer must identify the primary factors increasing the uncertainty in the problem and the sources of this uncertainty. With this information the designer can determine what additional knowledge is required and how this knowledge should be obtained. This information will be used to calculate the loss associated with the EVRU.

The cost of reducing the uncertainty is estimated as the Expected Cost of Reducing Uncertianty (ECRU). A cost benefit analysis between the EVRU and the ECRC is conducted to determine if additional knowledge should be obtained and the uncertainty reduced. If it is determined that additional knowledge should be gained, the designer should proceed on to Step 10. If not, a design concept (or concepts) should be selected with the available information and carried forward in the design process.

Step 10: Gain additional knowledge

The designer should use the information gained from Step 9 to determine what additional knowledge is needed. This information may come from additional research, experimentation, high fidelity analyses, or a higher fidelity M&S environment. In this step the designer may either obtain the additional knowledge required and then iterate the process by beginning again with Step 1, or the designer will immediately proceed to Step 1 but will utilize a higher fidelity M&S environment in Step 4.

This entire process should continue until a concept is selected to be passed on to the next SoS Design Level.

CHAPTER 11: EXAMPLE PROBLEM

The main motivation for this research resulted from the realization that there is significant uncertainty in the conceptual design process of a SoS and that the design process could be improved by accounting for this uncertainty. In Chapter 1 it was discussed how the design of a persistent strike SOS is an excellent application for demonstrating the techniques and processes developed in this research. Not only are persistent strike SoS currently of interest to the aerospace community but all types of uncertainty can be potentially included in the design problem.

A number of example problems focusing on different aspects of the persistent strike SoS design problem have been explored in previous chapters. The purpose of this chapter is to demonstrate the entire conceptual design process for a persistent strike SoS. This will demonstrate the utility of the techniques and processes presented in this research and serve as an example for how the processes could be used for other conceptual design problems.

In order to fully demonstrate the utility of these techniques and processes it is necessary to compare results of the new analyses with the performance of an actual SoS. No data exists for future persistent strike SoS, so it is necessary to turn towards historical events for an appropriate design problem.

An excellent example of a historical persistent strike SoS was the fleet of aircraft used to search and destroy mobile scud missile launchers in Desert Strom. The SoS operated under considerable uncertainty. For instance, as a few examples, there was uncertainty that pertained to the targets, threats, operation of the systems, and the environment itself. [176,177]

Not only does this historical scenario contain significant uncertainty making it an appropriate problem for the Robust and Opportunistic (RandO) Design Method, but there is considerable documentation over the systems, tactics, events, and outcomes of the scenario. [47,48,49,50,51,52,176,177] This information can be used to accurately develop a modeling and simulation environment for the example problem and can be used to verify the results from the analysis. For these reasons, the Desert Storm Scud Hunt was selected as the example problem to demonstrate the complete design method.

11.1. Background

In the 1980s Iraq purchased a large number of Scuds from the Soviets. In the Iran-Iraq War, Iraq demonstrated its willingness to use the weapons against a civilian population, by launching missiles at Iranian cities. [49] The scud attacks significantly affected the outcome of the war, and with this in mind, it was recognized long before Operation Desert Storm that Saddam Hussein would be likely to launch the missiles at Israel in an attempt to draw Israel into the conflict. [49] It was foreseen that this would have led to war between Jordan and Israel, thereby creating another Arab-Israeli conflict. If this were to happen, the unity of the Coalition would be severely tested. [49]

To prevent this scenario from unfolding, fixed scud launch sites were attacked in the beginning of Desert Storm. [49] Intelligence had assumed early in the planning stages that the scuds would be launched from fixed sites rather than the mobile missile launchers. Therefore, by eliminating the launch sites in the beginning of the offensive, the threat from the scuds would be dramatically reduced if not eliminated entirely. [47] Additionally intelligence assumed that if the mobile launchers were used, the launch preparation procedures would produce signatures that could be easily detected allowing coalition forces time to attack the launcher before the missile was fired. [47]

Based on these assumptions and the limited capabilities of the scuds themselves, the military leaders viewed the missiles as “militarily irrelevant”. [48] Other than the strikes on fixed launch sites, there was no preparation of plans for scud hunting. [48]

However, the assumptions made early in the planning process were quickly found to be incorrect. In the month before Operation Desert Storm intelligence determined that mobile missile launchers had been dispersed to unknown, prepared hiding sites within range of Israel and Saudi Arabia.[49] And, on the second day of Operation Desert Storm, Iraq used its mobile missile launchers to fire Scuds at Israel. [48] While there were only minor injuries from the launch there were huge political impacts. It was greatly feared that Iraq would use chemical weapons against Israel.[49]

Because it was greatly feared that Israel would enter the conflict, and indeed they were preparing for a counterattack, the President directed that "unprecedented steps be taken to persuade Israel not to exercise its unquestioned right to respond to Iraqi attacks." [49] It was determined within the first couple of days that the current efforts to suppress the Scud attacks were not effective and that it was necessary for some other solution.[49] As a response it was necessary to organize aircraft to hunt continuously for the mobile missile launchers and eliminate the targets. In other words, it was necessary to create a SoS to *persistently* cover the region where the launchers were located and then *strike* any located targets.

11.2. Robust and Opportunistic (RandO) Design Method

The SoS solution for the Scud Hunt begins with the OES Level. For this particular problem, by the time it was realized that a new SoS was needed, there is no time for the design, development, construction, and procurement of new systems. For this reason the OES Level is also the base level of the SoS to be designed. Only existing systems are to be utilized. The design of the SoS is based on selecting the appropriate systems and specifying the operational characteristics of the system.

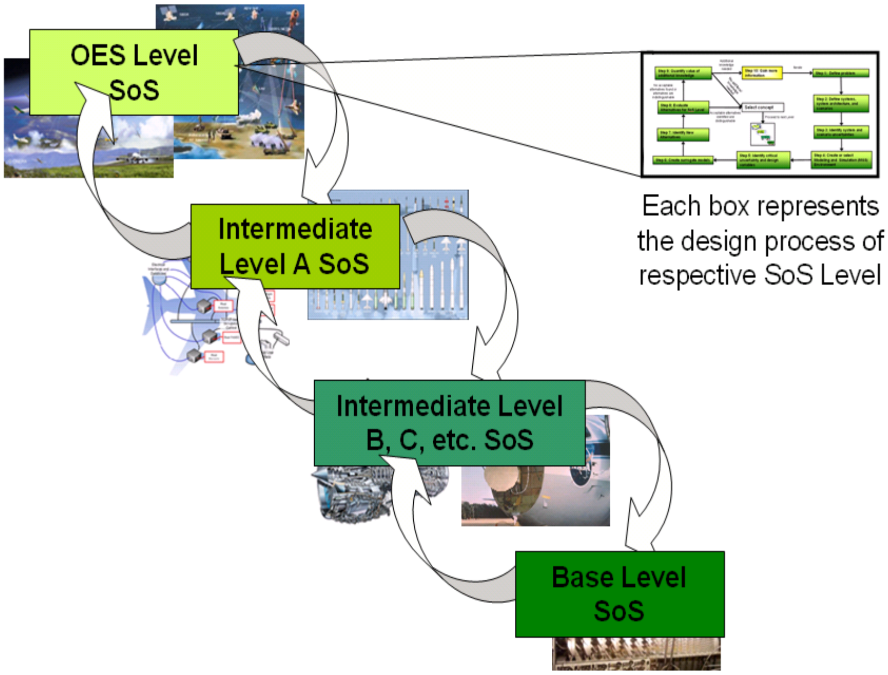

Figure 11-1: RandO SoS Design Process Overview

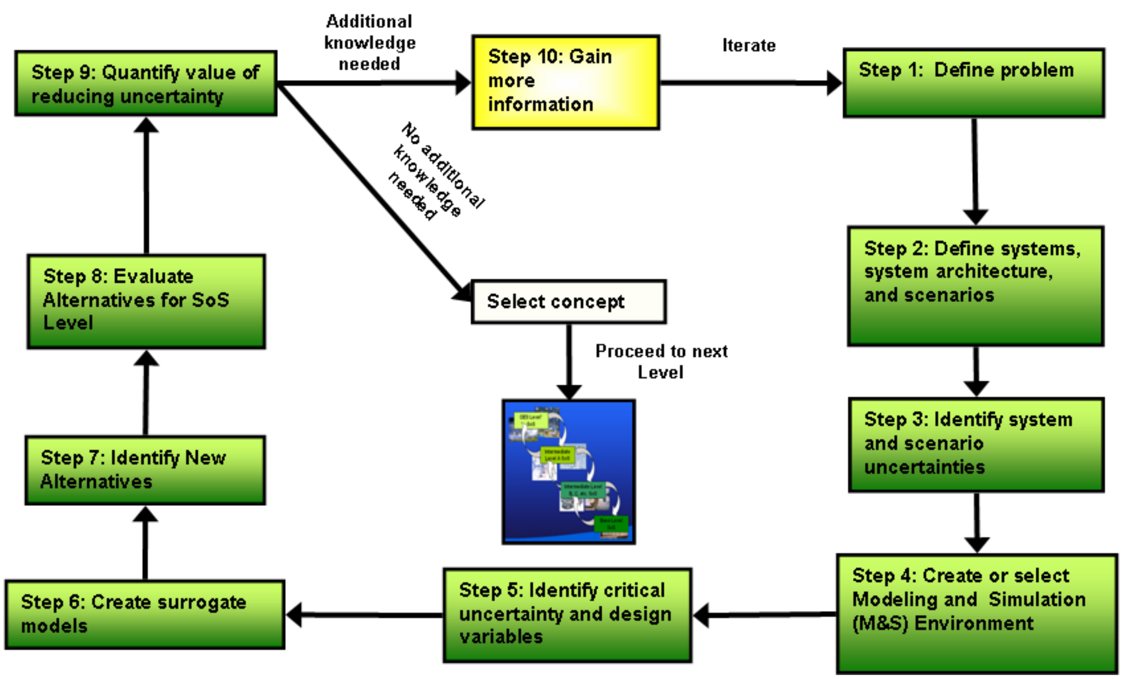

Figure 11-2: Design Process for each SoS Level

Step 1: Define the Problem

The overall objective for the SoS is to prevent the launch of Scud missiles. For this reason one of the design metrics is the number of Scuds launched (NumScudLaunched). The physical damage from the scud being launched (unless chemical weapons were employed) is not nearly as significant as the launch of the missile itself. The constraint and desirement are shown in the equations below.

Constraint 1 : $NumScudLaunched = 0$ **Equation 11-1**

Desirement 1: $NumScudLaunched = 0$ **Equation 11-2**

Ideally the SoS would be able to locate and destroy the launchers so that the threat is eliminated and so the systems within the SoS can be used for other purposes. However previous studies had demonstrated the difficulty associated with trying to find individual launchers in their operational area. [49] Even so the most effective SoS for this problem would locate the missile launchers and eliminate them.

The actual number of scud launchers was unknown, but planners used a working estimate of 600 Scud missiles (and variants) and 36 mobile launchers. [49] While the number of missile launchers destroyed would be an appropriate metric, it does not fully capture the complexity of the problem.

Initially planners assumed that decoys or other sources of “background” noise would not impact the process of identifying scud launchers. [47] This was also quickly recognized to be a false assumption and it was realized that in the beginning stages of the Scud Hunt that the Iraqis were camouflaging their missile launchers as part of the hiding techniques.

For instance, pilots were told to look at Bedoiun tents to determine if they were actually hiding missile launchers. [176]

In addition to hiding the missile launchers the Iraqis used decoys, in some cases high fidelity decoys.[51] However their use of high fidelity decoys was not known. In fact many, if not all, of the missile launchers that were reported destroyed through the efforts of the Scud Hunt were most likely decoys. [49]

Because of this complexity, the combined number of missile launchers and decoys killed (NumTargetDestroyed) is the most appropriate metric for measuring the effectiveness of the SoS. The desirement and the constraint values are set based on the estimated number of scud launchers.

Constraint 2: $NumTargetDestroyed > 20$ **Equation 11-3**

Desirement 2: $NumTargetDestroyed > 35$ **Equation 11-4**

The problem is also constrained by the number of systems used. Aircraft are needed for other missions and there are only a limited number of each kind of aircraft available. For this scenario it is very likely that two types of aircraft will be used. One type of aircraft to search/destroy in the day light, and another type of aircraft to search/destroy at night. The sensor systems required for night hunting limits the types of aircraft available and the number of aircraft available. For this reason two additional design metrics are number of aircraft (hunter/killers) used in the day (NumHK1) and the number of aircraft (hunter/killers) used at night (NumHK2).

The constraint value for these metrics was set by the number of aircraft available. [51,52] The desirement values were set by the number of aircraft in a squadron for deployment and organization purposes.

Constraint 3: $NumHK1_{Available} > NumHK1_{Required}$ **Equation 11-5**

Desirement 3: $NumHK1_{Required} < 24$ **Equation 11-6**

Constraint 4: $NumHK2_{Available} > NumHK2_{Required}$ **Equation 11-7**

Desirement 4: $NumHK2_{Required} < 24$ **Equation 11-8**

Step 2: Define systems, scenarios, and system architectures

Define Systems

There are a number of different types of systems to be considered in the development of the SoS including various types of aircraft and various types of weapons. There are also a number of enemy systems that must be considered as well including the different variants of the Scud missiles available, decoy systems, and potential threats.

For the SoS design problem, information available from the planning stages of Desert Storm and the beginning of Desert Storm was used in the system selection process. Information learned in the beginning of Desert Storm is appropriate to use because the official scud hunt did not begin until a few days after the first offensive strike.

Coalition Systems

Aircraft

There were a variety of aircraft that could have been involved in the scud hunt including: [51]

- Air-to-Ground Aircraft (Potential Hunter/Killer Aircraft)

- Air-to-Air Aircraft
- Forward Air Control Aircraft
- Electronic/Reconnaissance Aircraft
- Tanker Aircraft

There were a number of air-to-ground aircraft used in desert storm, but only a subset of these systems was appropriate for a persistent strike mission. For day operations it was possible to use: A-10, F-111F, F-16, and F-15Es. [51] For night operations, a number of both F-16s and F-15Es were equipped with the LANTIRN targeting pods. [51] These pods allowed the aircraft night and all weather weapons delivery capability. [51]

There was some question as to the threat that would be posed by the Iraqi air force before Desert Storm. It could have been possible for Iraqi fighters to interfere with the scud hunt and to require the need for protection provided from air-to-air aircraft. However, since the Iraqi tactics focus on deterrence over offensive combat, it was determined in the first few days of Desert Storm that this would not be a significant factor. [51] For this reason, no air-to-air aircraft were considered in the SoS.

In Desert Storm, especially in the beginning before the SAM sites stopped using RADAR, aircraft such as the F-4G were extremely useful in destroying surface to air missile (SAM) sites. [51, 177] However, most of the Iraqi threats were not focused in the region where the scuds were located. For this reason, these assets were not considered to be a significant factor in the scud hunt scenario for this example problem. For a similar reason jammer aircraft were not considered as a significant factor.

Reconnaissance aircraft were not considered to play a major role in the scud hunt. Not only were the mobile launchers difficult to locate but any information provided by a reconnaissance aircraft would quickly become obsolete by the time that a strike aircraft would arrive at the location. [177]

Air refueling, on the other hand, was assumed to play a significant role in the scenario. Based upon the distance from the aircraft bases to the killboxes (designated search area), it is assumed that aircraft will refuel on the way to the killbox and mid-mission to reduce the number of aircraft necessary for continuous coverage. Numerous air refueling tracks were setup near the border and several tankers were stacked within a single orbit at 500 ft intervals so that multiple aircraft could be refueled simultaneously. [51] Additionally as one aircraft was being refueling another aircraft waited "on wing" to quickly move into position once the tanker was available. [51]

Weapons

There were two types of weapon systems that offered the most potential for the scud hunt: general purpose bombs and laser-guided bombs. The general purpose bombs were the most commonly used. The MK-80 series were developed in the 1950s, are free falling non-guided bombs, and had both nose and tail fuzes to "ensure reliability and produce effects of blast, cratering, or fragmentation". [51] Bombs from this series weigh from 500-2000 lbs. [51] All of the air-to-ground aircraft were capable of carrying the MK-84, a 2,000lb bomb.

Laser guided bombs are general purpose bombs with an additional guidance kit. These weapons are maneuverable, free fall weapons with a guidance system that detects laser energy. [51]

Enemy Systems

It is important to consider the enemy systems when developing a military SoS. The systems considered in this example are the scud missiles, the missile launchers, the decoys, and the potential threats.

Targets

There were several Scud types including the Scud B, Al-Husayn, Al-Abbas, and the Al-Hijarah. These missiles had ranges varying from 160 nm to 430 nm.[49] These missiles could be fired from standard Scud transporter-erector launchers or from the Iraqi mobile erector launcher. [49] The mobile launcher is as large as a medium-sized truck and a missile in its traveling position looked like a oil tanker from 10,000ft. [49,177] The maximum speed of the mobile launcher was 30 nm/hr.[182]

Intelligence predicted that if the mobile launchers were used that they would require several hours to setup based on launch procedures used by Soviet Scud units in Europe. [49]

Decoys

Intelligence initially predicted that decoys would not be a significant factor in the analysis. However, it was determined in the early stages of Desert Storm that the Iraqis were using various deception tactics to hide scud launchers and confuse the searching aircraft. [176]

Threats

There were a number of potential threats that existed in Desert Storm from SAM to AAA. The primary SAMs are as follows: SA-6, SA-2, SA-3, SA-8, Roland, and the SA-9. These missiles had various ranges from 6 nm to 27 nm. Some of the SAMs, such as the SA-6, were used against very-low to medium altitude threats while the SA-2 was to be utilized against high-altitude targets. [51] The Iraqis also had Man Portable Air Defense SAMS with a range of 2.5 miles.

One of the more dangerous threats was the anti-aircraft artillery (AAA), which posed a threat below 15,000 ft. The primary systems included the ZSU-23/2 (23mm cannon systems, and 14.5mm) or the ZSU-23/4. [51] However, the threat from AAA was not as significant in the region where the scuds were located. [176]

Define Scenarios / System Architectures

Based upon the literature there were a wide variety of potential operational and situational scenarios that needed to be considered. [47] The purpose of identifying these scenarios is to help determine the underlying sources of uncertainty and to determine what factors need to be considered in the modeling and simulation environment.

Situational scenarios

The situation scenarios can be broken down into three different categories: targets, threats, and the weather/environment.

<u>Targets</u>

- Scenarios: Only fixed launchers used / only mobile launchers / fixed and mobile launchers both used

By the third day of Operation Desert Storm it was evident that only the mobile launchers were being used. [49] Additionally, by this time in the operation the known fixed launch sites had already been attacked thereby eliminating the likelihood of future use.

- Scenarios: Rapid set up time for launch / Extended set up time for launch

It was estimated in the planning stages that the launch process would require and extended period of time. [47] By the time the scud hunt was initiated it was apparent that the initial planning assumptions were incorrect and the variables that were originally known may need to be modeled with some variability.

- Scenarios: Short strike window of opportunity / Long strike window of opportunity

Due to the mobility of the targets it is possible that the window of opportunity for striking a target after it is identified will close before the aircraft can strike. This possibility suggests that the mobility of the target should be considered in the analysis.

- Scenarios: Easy to locate launchers / Difficult to locate launchers
- Scenarios: No decoys or deception used / Decoys and deception used

Planners had assumed that it would be easy to locate the mobile launchers based previously identified signatures for these systems. It was also generally assumed in the planning stages that no decoys or other background noise would hinder the location process. Again their assumptions were found to be incorrect and it was recognized that these values may be considerably different than first supposed.

- Scenarios: Daytime launch / Nighttime launch

Night launches were highly likely because of the additional stealth offered by the cover of night. In the early days of the engagement this assumption was found to be fairly accurate and reduced the need to consider the multiple options of this scenario in the analysis.

Threats

- Scenarios: High threat environment / Low threat environment

Initially it was uncertain if the systems would be operating in a high or low threat environment for the scud hunt. For this reason both scenarios need to be considered.

Weather / Environment

- Scenarios: Poor visibility / High visibility
- Scenarios: Heavy winds / Low winds

In the months leading up to Operation Desert Storm there had been no indication that weather would be an issue. [50] But, historical data for the region suggested that weather may be more of an issue in the winter months. According to Reference 130 approximately 90% of Iraq's rain falls between November and April, and heavy cloud cover is common in January and February. [100] Additionally, heavy winds are also normal for this time of year and can lead to large sandstorms. [100] Reference 100 discusses how low ground fog may also be an issue in this period.

While weather was not considered initially to be a significant factor in the planning stages of the operation, based upon the available historical information, it would be useful to model its potential effects.

- Scenarios: Terrain conducive to hiding / Terrain hinders hiding

It was well known that the western region of Iraq is rugged and ideal for concealing mobile missile launchers. Figures 11-3 and 11-4 illustrate the various terrains of Iraq. There are a number of ravines, culverts, and highway underpasses that offer excellent hiding locations. [49] Based upon this information it is unnecessary to consider the scenario where it is difficult to conceal the mobile launcher.

- Scenarios: Terrain conducive to target mobility / Terrain hinders target mobility

Because of the rugged nature of the terrain, the mobility of the targets is hindered. However it is expected that the roads in the region will be used for the targets to change locations.[101] The mobility of the mobile launcher has been documented, and this information combined with the likely effects on its mobility from the terrain can be considered in the analysis.

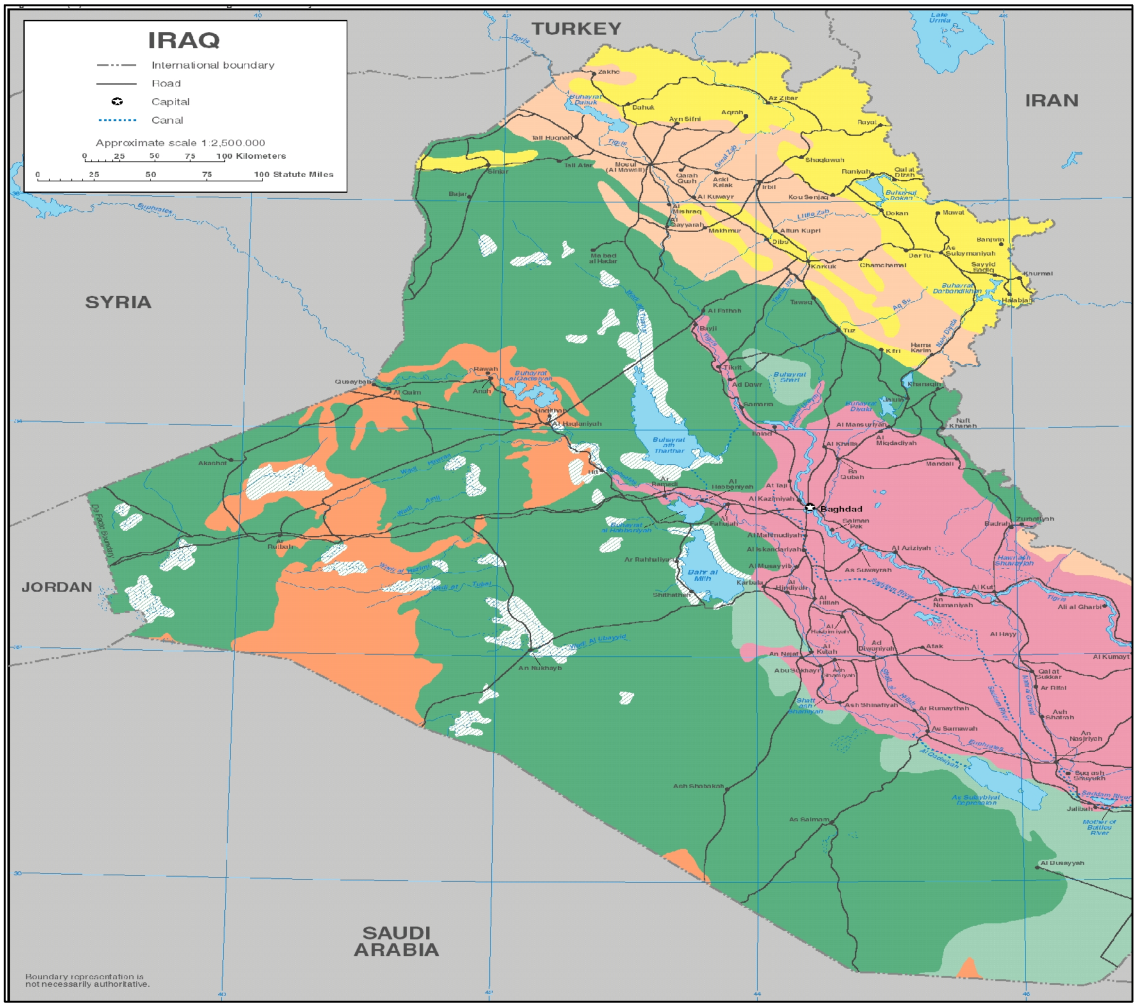

Figure 11-3: Terrain of Iraq [101]

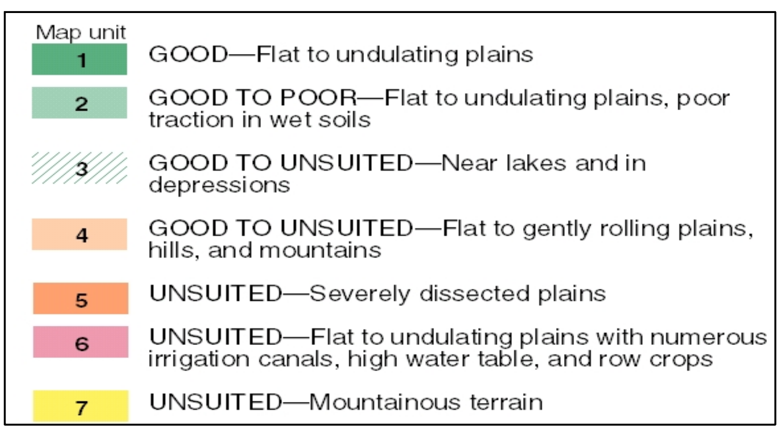

Figure 11-4: Legend for Map showing Terrain of Iraq [101]

Operational Scenarios

While the situational scenarios are based upon material and physical aspects of the environment, the operational scenarios are based upon the functions or actions to be performed by the SoS. The main categories for these scenarios are:

Refueling

- Scenarios: aerial refueling / No aerial refueling
- Scenarios: One tanker in refueling orbit / Multiple tankers in refueling orbit
- Scenarios: Tankers available / Tankers unavailable

In Operation Desert Shield it was apparent that aerial refueling would be an important factor in Desert Storm. The available air space was filled with tanker refueling tracks. [47] Multiple tankers were used in the same orbit so that multiple aircraft could refuel, but it is not certain how many tankers should be used in the refueling tracks associated with the scud hunt. [47]

It is possible that there may be situations where for some reason there are no tankers available for refueling.. [47,177] This possibility must be considered throughout the mission.

Search

- Scenarios: One aircraft searches killbox (search area) / multiple aircraft search killbox

The number of aircraft searching a designated area is uncertain. It is assumed that having more aircraft in the region is likely to increase the likelihood of locating and destroying a scud launcher, but this will also significantly increase the number of aircraft required.

- Scenarios: Continuous coverage of killbox / Intermittent coverage of killbox

Due to the heavy political implications associated with Israel entering the conflict, the area of interest should be continuously covered. [47,177] Any lapses in coverage would provide opportunities for a launch or additional movement.

- Scenarios: Aircraft searches specified killbox within entire region of interest/ Aircraft searches entire region of interest

For any given airspace in the region, there is likely to be quite a bit of traffic. At night when the aircraft are flying without lights there is an increased chance of collision if aircraft are searching unspecified regions. Additionally if an aircraft repeatedly covers the same area, the pilot is more likely to be able to identify changes that would indicate movement. [176]

- Scenarios: Aircraft search region of interest in set pattern systematically / Aircraft search region of interest randomly

By searching a killbox systematically it is less likely for any particular region to be missed in the search. However, the set search pattern may also provide the opportunities for the targets to predict the search pattern of the aircraft and move accordingly. It is expected that there will be some randomness in the search pattern based upon the behavior of a human pilot.

- Scenarios: Aircraft fly to location of missile launch / Aircraft only search designated search area

It is logical for the nearest aircraft to fly to the estimated launch location of any scud that is fired. The aircraft is trying to identify the location of scud launchers and for this scenario the pilots would know that there is a scud launcher somewhere in the nearby region.

- Scenarios: Targets/decoys are located / No targets/decoys are located

Based upon the terrain, sensor capabilities, and deception tactics of the Iraqis it is possible that no scud launchers (targets) or decoys will be identified. The objective is to determine the SoS that will minimize this likelihood of this scenario occurring, but it is a possibility.

- Scenarios: Aircraft fly at low altitudes / Aircraft fly at high altitudes

The uncertainty of threats in the area, resulted in the minimum altitude being set for the missions in the early stages of the war. In general, aircraft were ordered to fly above 10,000ft to avoid the threat posed by AAA. [177] This information should be included in the analysis.

Design Variable Selection

Based upon the information gathered in this step it is possible to identify potential design variables to be considered. For this example problem the aircraft characteristics are known but the aircraft utilized for either day or night operations is uncertain. Additionally there are a number of operational variables that should be considered including killbox size, number of refueling patterns, the amount of time searching in a particular region after a scud fires, the number of tankers in a individual refueling pattern, and the number of aircraft flying together in the same killbox.

Table 11-1: Potential Design Variables

Potential Design Variables
Type of H/K Aircraft for Day
Type of H/K Aircraft for Night
Size of Killbox (nm)
Number of Refueling Patterns
Search Time after Launch (min)
Number of Tankers per Pattern
Number of H/K Aircraft in same Killbox

Step 3: Identify system and scenario uncertainties

Chapter 3 discusses the various sources of uncertainty for SoS design problems. These sources are related to: component interactions, system interactions, emergent behavior, situational uncertainty, operational uncertainty, independent agent, and new systems.

System and Component Interactions

Within the problem there are a variety of system interactions to consider. For instance, if tankers are not available to provide fuel, the range and endurance of the aircraft will be dramatically reduced for that sortie. This situation can be built into the model itself and is affected by the number of tankers deployed to each tanker orbit.

Another example of how systems interact in the SoS, is with malfunctions. If one component of the aircraft malfunctions the entire aircraft may become unusable or uncontrollable. Based upon data from the Vietnam War, it is possible to estimate reasonable values for the uncertainty related to malfunctions. [6] It was estimated that the number of aircraft that malfunction is between 0.6 aircraft per 1000 sorties and 0.9 aircraft per 1000 sorties.

A malfunction with the aircraft primarily affects the scenario by increasing the number of aircraft required to meet the persistent coverage requirement of the persistent strike mission. Another uncertain element that needs to be considered is the repair time required before an aircraft will be available for service. In planning for Operation Desert Storm it was estimated from previous research that 50% of aircraft return to service within 4 hours. [50] Additionally according to the fix rate for a F-15, 75% of aircraft will be fixed within an eight hour period. [73,74] Based upon this information the average repair time was conservatively estimated to be between 7.6 and 16.4 hours.

Another source of uncertainty pertaining to system/component interactions is weapon malfunctions. If a weapon system malfunctions after it has been released the target will not be hit. If the launch system for a weapon malfunctions, not only will the target not be affected, but now the aircraft is carrying dead weight. The effects associated with this failure scenario can be modeled within the M&S environment to determine the full impact. To better estimate the weapon malfunction uncertainty, it is likely that the manufacturer will have statistical test data to describe the rate of failure. For this example problem, no data was available from the manufacturers, instead estimates based on the performance of the weapon systems from Desert Storm were used. The total probability of a weapon malfunction was set to a range of 30-50%. [39]

Emergent Behavior

Emergent behavior within the system is defined in this research as unmodeled or unplanned behavior. As such, the uncertainty associated with this behavior cannot be modeled, but potential emergent behaviors can be identified and the potential for them to be tracked should be built into the M&S environment.

An example of potential emergent behaviors could be launchers remaining hidden and missiles not being launched due to aircraft patrolling nearby. While the purpose of the aircraft searching is to locate and eliminate the launchers, it is possible that the act of searching will encourage the target never to emerge.

Another example could be the development of a pattern where the missiles are launched in between "shift changes" of aircraft or the development of a pattern where missiles are launched because of a regular search pattern.

For this problem, from the perspective of the metrics, constraints, and desirements identified in Step 1 of the design process, only the behaviors associated with the missiles actually being launched is of interest. It is not necessarily of interest at this time to predict when they will be launched, considering the main goal of the system is to prevent them from ever being fired.

Situational Uncertainty

To an extent some of the uncertainties associated with the situation were addressed in the previous step. These uncertainties are due to environmental or external factors. There is no control over the uncertainty related to the situation. For the scud hunt the primary variables associated with the situational uncertainty relate to the weather (cloud cover/visibility); the number and behaviors of the targets; the number and behaviors of the decoys; and the threats.

Weather

The statistical weather data for the region indicated that typically there were 20-30% ceilings at 10,000 ft or below throughout the day. [50] Since this is just an average value, and the weather could significantly affect the ability of a hunter/killer aircraft in finding targets, it is necessary to model a range of potential values for the weather. For this example the weather was modeled as having between 10-40% cloud cover throughout the engagement period.

Number of Targets

While the actual number of scud launchers was unknown, intelligence estimated that the number of targets ranged from the high twenties to the mid-thirties. [49] For the uncertainty analysis it was estimated that there were between 25 and 35 potential scud launchers.

Behavior of Targets

It was originally estimated that it would take a few hours for the pre-launch procedures based on the operations of Soviet Scud units in Europe. [49] But, after the first few days of the engagement, it was apparent that the assumptions made by the intelligence community were inaccurate and the possibility of a drastically reduced pre-launch setup time should be considered in the analysis. In the uncertainty analysis values from 30 min to 3 hours were considered.

Other behavior characteristics of the targets were also unknown. For instance, how long, on average, would the Iraqis wait after firing a missile to either move the missile launcher or launch again. This characteristic affects the overall scenario and should be considered even though little information is available about the behavior.

Because of the number of hiding locations in the area of interest and the mobility of the targets, the window of opportunity to strike a target could certainly be a factor. As such, this should be a variable considered in the analysis.

Number of Decoy

It was originally thought that decoys would not affect the Scud Hunt. However, since it was determined that the Iraqis were using deception to protect the mobile launchers, a number of decoys should be considered in the analysis as well. There is no intelligence suggesting appropriate values.

Threats

The threat from enemy systems can be modeled in the scenario through attrition. Based on data from the Vietnam War, it is possible to develop estimates for attrition. Based on this data, there was a loss rate of 0.4288 aircraft per 1000 sorties. [6]

In this engagement aircraft were lost due to AAA, SAM, and enemy aircraft. After a few days of engagement, and based upon Iraqi tactics, it was doubtful that enemy aircraft would be a large factor, but the other threats were relevant. A range of possible attrition values was assumed for the hunter/killer aircraft based on the Vietnam Data.

Operational Uncertainty

These uncertainties are typically identified in the previous step when the operational scenarios are identified.

Refueling

It is possible that for a given sortie that there will not a tanker available for refueling. This would dramatically limit the endurance of the aircraft and cause the original flight plan to change. The aircraft would not be able to cover the search region for the originally planned period of time and would have to return to the base. This particular aspect of uncertainty would not necessarily be modeled by a specific uncertain variable, but rather it would be an effect from the scenario itself. This possibility should be modeled in the M&S environment.

Search

There is a certain amount of uncertainty regarding the actions and decisions of the pilots in this scenario. This uncertainty is discussed in the next section.

For this example problem there is the possibility that targets/decoys may or may not be located. Once a target or decoy is located the aircraft would stop searching and instead strike the located target. This is likely to affect the endurance of the aircraft to an extent, and if all of the aircrafts weapons are expended the aircraft would return to base. None of these elements is specifically related to a particular variable, but they are ambiguous elements that should be modeled in the scenario.

Independent Agent Uncertainty

Within the actual operation of the SoS, there are independent free thinking systems (pilots). The pilots would be continuously making decisions to try and maximize the effectiveness of the hunter/killer aircraft. These decisions are difficult to predict because they are based on all of the available information about the scenario, from the pilot's perspective, at that instant in the scenario. For instance, a pilot may receive some information or stimulus that encourages the pilot to fly to another region of the area of interest. Or, another option is that the pilot may simply get restless and want to modify the original search pattern.

While these actions can be extremely difficult to predict, or even model, it is possible to include a certain element of randomness into the actions of the pilot. The amount of randomness can be modeled in the scenario and can be used to determine if there if this would be a factor in the effectiveness of the SoS.

Another way that the decision making of the pilot would affect the example problem is the case when multiple targets are identified. There may be a limited window of opportunity and the pilot may have to decide which target to strike. Considering it is possible that decoys may be used, it is possible that the pilot will strike the decoy and not the target. This would be difficult to capture with a specific uncertainty model but the possibility of this occurrence, and the uncertainty of the pilot's final decision, should be modeled in the analysis.

New System Uncertainty

Often this uncertainty is related to the characteristics of future systems. However, this uncertainty may also relate to factors such as system upgrades. Or, this uncertainty may relate to existing systems where the performance is either variable due to learning curves or untested and therefore unknown.

In Operation Desert Storm, most of the systems utilized were well tested legacy systems. However, the F-15E was a relatively new system. It had only been introduced into service approximately three years before Desert Storm and it was not yet capable of its full potential. [177]. For instance, the aircraft was specifically designed to be used with smart bombs, however when the engagement first started it was only certified to use dumb bombs. However, by the start of Desert Storm a subset of the aircraft were equipped with targeting pods and were able to drop smart bombs. [177]

It essence the pilots were forced into "on the spot" job training with the new weapon systems. The learning curve associated with the pilot's becoming familiar with the new systems was expected to play a factor in the accuracy of striking the targets. While this factor is difficult to specifically model with a particular variable this can be modeled, to an extent, in the modeling and simulation analysis.

Uncertainty Variables

Not all of the uncertainty in the design problem is addressed by uncertain variable values. Often the uncertainty in the system will be linked to specific interactions within the system itself. By identifying the uncertainty in this step it is possible to make certain that the important interactions are capable of being modeled in the modeling and simulation environment.

On the other hand, there are some factors that are possible to model through the use of uncertainty variables. Based on the previous analysis the following forty-two variables were identified as potential uncertain variables for the analysis:

- Number of targets
- Number of decoys
- Minimum target setup time
- Minimum window of opportunity to strike target after missile has been launched
- Average target hiding time
- Average groundtime of hunter/killer aircraft
- Attrition of hunter/killer aircraft -daytime operations
- Malfunction of hunter/killer aircraft - daytime operations
- Average repair time for hunter/killer aircraft - daytime operations

- Attrition of hunter/killer aircraft -nighttime operations
- Malfunction of hunter/killer aircraft - nighttime operations
- Average repair time for hunter/killer aircraft - nighttime operations
- Probability of malfunction with weapon launch system
- Probability of weapon missing target (due to malfunction, pilot error, etc.)
- Probability of malfunction with weapon detonation system
- Factor modeling randomness of pilot actions
- Length of engagement
- Minimum likely cloud coverage
- Maximum likely cloud coverage
- Length of killbox
- Number of hunter/killer aircraft flying together in same killbox
- Hunter/Killer aircraft TOGW - daytime operations
- Hunter/Killer aircraft loiter airspeed – daytime operations
- Hunter/Killer aircraft cruise airspeed – daytime operations
- Hunter/Killer aircraft dash/strike airspeed – daytime operations
- Hunter/Killer aircraft propulsion system type – daytime operations
- Hunter/Killer aircraft sensor radius – daytime operations
- Hunter/Killer aircraft range – daytime operations
- Hunter/Killer aircraft maximum mission time – daytime operations
- Hunter/Killer aircraft TOGW - nighttime operations
- Hunter/Killer aircraft loiter airspeed – nighttime operations
- Hunter/Killer aircraft cruise airspeed – nighttime operations
- Hunter/Killer aircraft dash/strike airspeed – nighttime operations
- Hunter/Killer aircraft propulsion system type – nighttime operations
- Hunter/Killer aircraft sensor radius – nighttime operations
- Hunter/Killer aircraft range – nighttime operations

- Hunter/Killer aircraft maximum mission time – nighttime operations
- Range of weapon
- Weight of weapon
- Weapon type, smart bomb or dumb bomb
- Number of tankers per refueling orbit
- Maximum time to spend searching for scuds after missile launch

Step 4: Create or select a Modeling & Simulation Environment

There are several tools and existing models that could be used to create a modeling and simulation environment for this problem. The available options are ATMAS Version 2, FLAMES, or SEAS. There is also the option of significantly modifying ATMAS Version 2 to overcome any limitations of the original version.

To determine the appropriate modeling and simulation (M&S) environment a set of five requirements was considered. The requirements are as follows. The M&S environment must be:

- Capable of modeling systems and relevant uncertainty with necessary fidelity
- Capable of being run on multiple computers
- Capable of being easily modified
- Available for development and use

Additionally there needed to be no restrictions on distribution or use.

ATMAS Version2

Aircraft Time critical target Mission Analysis Simulation (ATMAS) was developed by the Aerospace Systems Design Laboratory at the Georgia Institute of Technology. The original purpose behind the development of ATMAS Version 1, which is described in

Reference 189, was created to simulate a Time Critical Target (TCT) search and destroy mission scenario in order to evaluate morphing aircraft versus conventional fleet of strike and sensor aircraft. ATMAS Version 2 extended the capability of ATMAS Version 1 by modeling a larger variety of aircraft fleets.

The code was developed to easily model various aircraft fleets for a hunter/killer mission using simple performance equations and vehicle characteristics. While performance and physical characteristics for these existing aircraft can be used in the simulation, all aircraft characteristics can be easily modified within the code in order to incorporate future generations of aircraft.

Any computer that has a MATLAB license will be able to run ATMAS. There would be no restrictions in either its development, use, or future distribution. However, the primary limitation to this tool is that it does not model many of the necessary systems and uncertainty characteristics identified in Steps 2 and 3. It was developed to compare multiple types of fleets (of the same type of aircraft) against a simplified TCT search and destroy mission. The fidelity of the ATMAS Version 2 is limited. In its current state, it is not appropriate for this example problem.

FLAMES

FLexible Analysis, Modeling, and Exercise System (FLAMES) is a object-oriented architecture that can model a variety of scenarios with significant level of fidelity, if desired. It includes a number of base classes for a number of models including vehicles, sensors, weapons, communication devices, and jammers. Additionally it is capable of simulation models of cognitive processes like decision making. [78]

FLAMES has been used to model SoS scenarios as discussed in Reference 78 and is a highly capable modeling and simulation environment. The main downside to this tool is that there is a fairly high learning curve associated with using the product, significant development time, and there are restrictions to its use (limited licenses).

SEAS

System Effectiveness Analysis Simulation (SEAS) is an agent-based analysis tool specifically for military operation analyses. Small or large scale military operations can be modeled with this tool and it can be used in analyzing effects-based operations, network centric warfare, and various warfighting concepts. [168] This tool includes an agent library and incorporates probabilistics to account for uncertainty in a given scenario. It also was set up to model communications between different systems. [168]

This tool is relatively easy to learn and use due to its framework. Considering it was specifically developed to model military operations it would be a good choice for the modeling and simulation environment for this problem. However, there are only a limited number of licenses available for this tool and there are restrictions to its use.

ATMAS Version 3

Another option is to take the original framework of ATMAS Version 2 and modify it to meet the specific needs of this problem. Because of the flexibility of MATLAB, it is possible to model all of the necessary systems and uncertainties involved in this problem. Additionally any computer that has a MATLAB license will be able to run either version of ATMAS. There would be no restrictions in either its development, use, or future distribution.

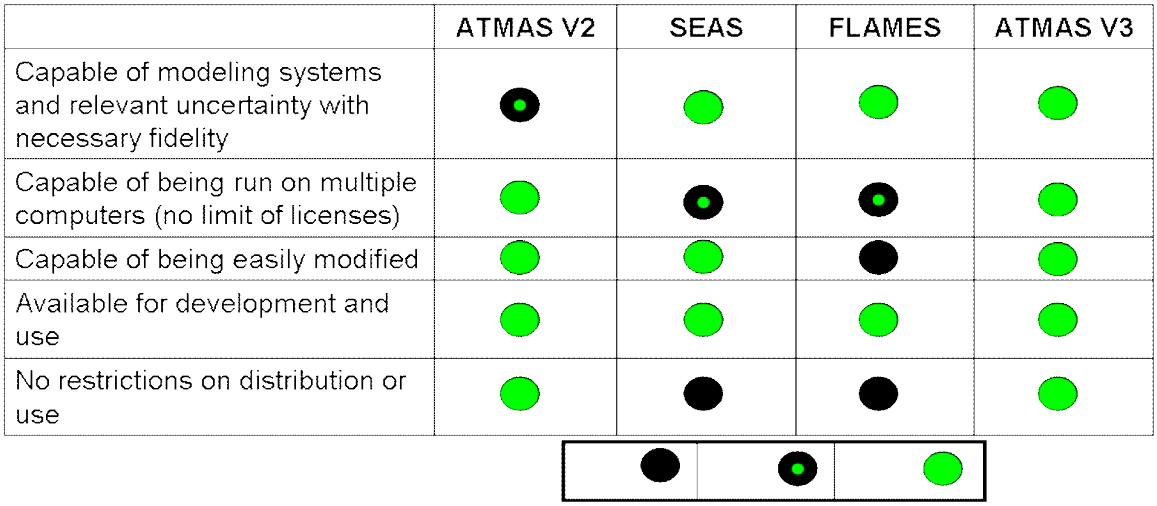

Figure 11-5: Evaluation of Possible M&S Tools

After evaluating all of the tools against the requirements for a modeling and simulation environment it was determined that the best solution was to modify the original version of ATMAS and create ATMAS Version 3.

A detailed overview of ATMAS Version 3 is provided in Appendix D.

Comparison of Modeled Effects to Historical Trends

For this example problem, it is necessary to consider the ability of the modeling environment to realistically capture the trends of interest. In this particular case, for a particular SoS, we are interested in being able to model the number of scuds launched and the targets destroyed. For this example problem, it is possible to compare the M&S environment with the historical results of the event.

The most important trend that was found was that the number of scuds launched per day dramatically decreased once the scud hunt was initiated. The increase in scuds launched towards the end of the war has been linked to the desperation of the Iraqi army as the end was approaching. This particular aspect of the engagement was not modeled in the M&S environment, but it is possible to look for the trend of decreased launches with the beginning of the scud hunt.

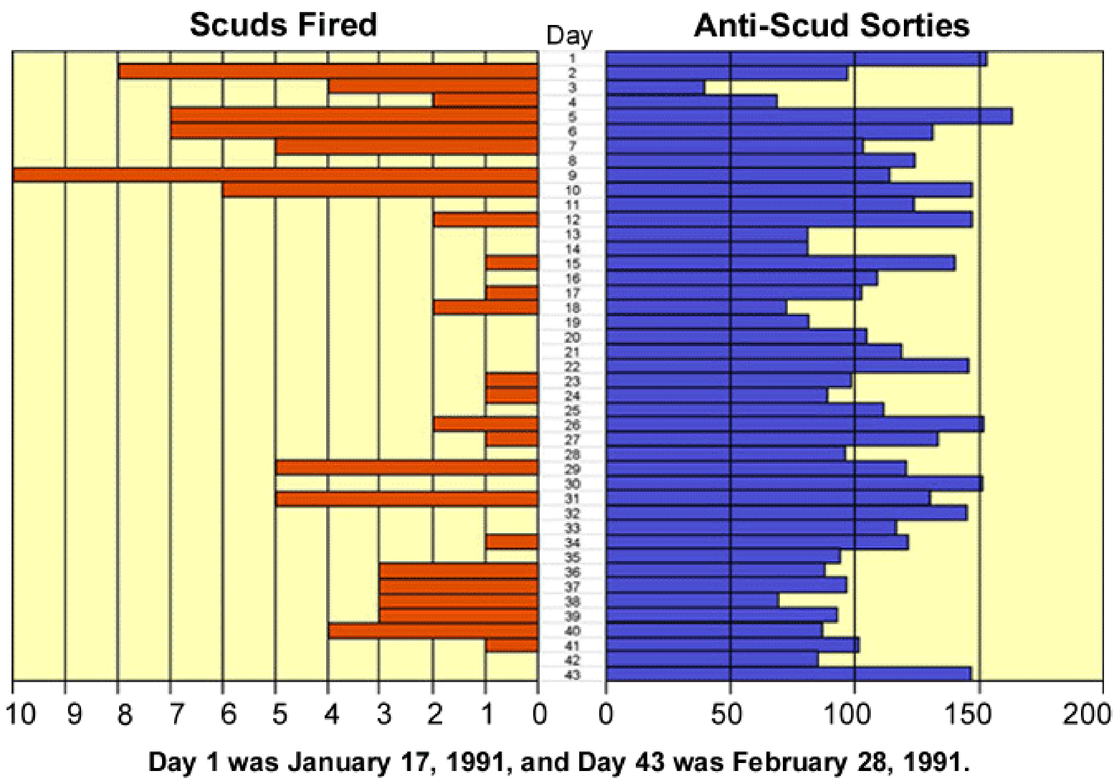

Figure 11-6: Comparison of Scuds Fired to Anti-Scud Sorties for Desert Storm [77]

In the M&S environment when no aircraft were used to cover the region for the period of 40 days, 311 scuds were launched in the simulation. In the simulation where the historical SoS was utilized, 31 missiles were launched. This demonstrates a dramatic decrease in launches which is consistent with the historical trends. Based upon the actual scud launch rate for the modeled period of the Scud Hunt, there would have been 35 missiles launched for the historical SoS. This difference of only four missiles indicates that the simulation is a fairly accurate representation of the events of interest.

Additionally, after the war it was determined that most, if not all, of the 80 targets that were reported as destroyed by the pilots of the hunting aircraft, were decoys. In the simulation of the historical SoS, no actual launchers were destroyed and all of the decoys

(100 were estimated) were destroyed by the hunter/killer aircraft. Again these results match up well with the actual events.

Step 5: Identify critical uncertainties and critical design variables

To determine the critical uncertainties and design variables associated with this problem, a screening test was utilized. The process for conducting this screening test is described in Reference 107.

A Pareto analysis was conducted in JMP for each of the traditional design metrics.[105] The respective Pareto plots can be seen in Appendix E. Based upon the results from this analysis the critical design variables and uncertainty variables were selected. Table 11-2 lists the design variables and Table 11-3 provides information about the identified uncertainty variables.

Any variable not listed in these tables was set to an average value based upon the available literature within the M&S environment.

Table 11-2: Design Variables and Design Space

Design Variables	Alternatives						
Type of H/K Aircraft for Day	A-10	F-111F	F-16	F-15E			
Type of H/K Aircraft for Night	F-16	F-15E					
Size of Killbox (nm)	10	20	30	40	50	60	
Number of Tankers per Pattern	1	2	3	4	5	6	
Number of H/K Aircraft in same Killbox	2	3	4	5	6	7	8

Table 11-3: Characteristics of Uncertainty Variables

Uncertainty Variable	Distribution	Minimum Value	Maximum Value	Uncertainty Modeling Technique
Number of Targets	Unknown	25	35	Evidence Theory
Minimum target setup time	Unknown	0.5 hrs	3 hrs	Evidence Theory
Average Repair time for Hunter/Killer Aircraft	Unknown	7 hrs	17 hrs	Evidence Theory
Probability of weapon missing target	Unknown	20%	50%	Evidence Theory
Average cloud coverage	Unknown	10%	40%	Evidence Theory
Malfunction of Hunter/Killer Aircraft	Unknown	Unknown	Unknown	Info-Gap Theory
Number of Decoys	Unknown	Unknown	Unknown	Info-Gap Theory
Average Target Hiding Time	Unknown	Unknown	Unknown	Info-Gap Theory
Factor modeling Random Pilot Actions	Unknown	Unknown	Unknown	Info-Gap Theory

Step 6: Create a surrogate models

Before the design and uncertainty analysis can be conducted, due to the computational expense of an ATMAS simulation, it is necessary to create a surrogate model of the M&S environment. In this step surrogate models were created for each of the four design metrics. As discussed previously, there are three main types of surrogate models that are commonly used: Response Surface Equations (RSEs), Neural Networks (NNs), and Kriging. [54,22,31]

Neural Networks (NN) were selected to model the M&S environment in this problem because of the nonlinear nature of the metric data and because of the significant amount of data to be used in the development of the surrogate model. The NN were created using a MATLAB code called BRAINN (BRAINN) that was developed by the Aerospace Systems Design Laboratory at the Georgia Institute of Technology.[36]

Step 7: Identify new alternatives

Because of the nature of the problem no new SoS alternatives were considered in this example problem. However Reference 188 illustrates how the basic elements of design method can be used to design a modern persistent strike aircraft where the design of new systems is considered. The main difference with the technique presented in Reference 188 and this research is that a Monte Carlo Simulation is used to model the uncertainty and that the problem is addressed from the perspective of Robust Design instead of RandO Design.

Step 8: Evaluate and Select Alternatives for SoS Level

This step is where the process presented in Chapter 7, as shown in Figure 11-7, primarily maps to this design method.

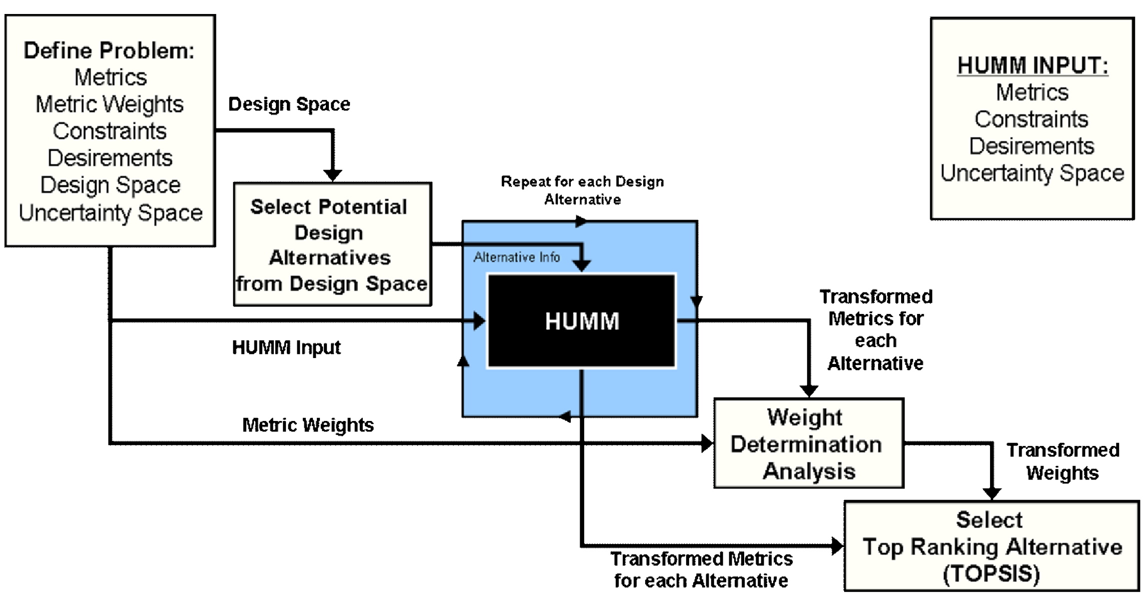

Figure 11-7: General RandO Design Process

Information about the metrics, constraints, desirements, and the relevant uncertainty are from the first seven steps of the CONDOR-SS method and are the inputs for the process. Design alternatives from the design space are selected and a HUMM analysis is completed for each alternative.

HUMM Process Task 1 – Define the Design Metrics, Constraints, and Desirements

This step was completed in Step 1 of CONDOR-SS.

HUMM Process Task 2 – Define the uncertainty characteristics

This step was completed in Step 2 of CONDOR-SS.

Because no actual values are known for the Info-Gap Variables, the following characteristics were estimated based on maximum possible values from the literature.

Table 11-4: Parameters for Info-Gap Analysis

Uncertainty Variable	Nominal Value	Minimum Possible Value	Maximum Possible Value
Malfunction of Hunter/Killer Aircraft	0.8 (aircraft / 1000 sorties)	0.3 (aircraft / 1000 sorties)	2 (aircraft / 1000 sorties)
Number of Decoys	20	0	100
Average Target Hiding Time	8 hours	2 hours	48 hours
Factor modeling Random Pilot Actions	30 %	10%	90%

HUMM Process Task 3 – Determine the number of analysis runs for each Uncertainty Variable

Table 11-5: Uncertainty Analysis Runs

Uncertainty Variable	Uncertainty Modeling Technique	Number of interval values for each variable
Number of Targets	Evidence Theory	2
Minimum target setup time	Evidence Theory	2
Average Repair time for Hunter/Killer Aircraft	Evidence Theory	2
Probability of weapon missing target	Evidence Theory	2
Average cloud coverage	Evidence Theory	2
Malfunction of Hunter/Killer Aircraft	Info-Gap Theory	9
Number of Decoys	Info-Gap Theory	11
Average Target Hiding Time	Info-Gap Theory	9
Factor modeling Random Pilot Actions	Info-Gap Theory	7

As presented in Table 11-5, based on the uncertainty characteristics of the uncertain variables, it is appropriate to only include Evidence Theory and Info-Gap Theory in the HUMM analysis for this example problem.

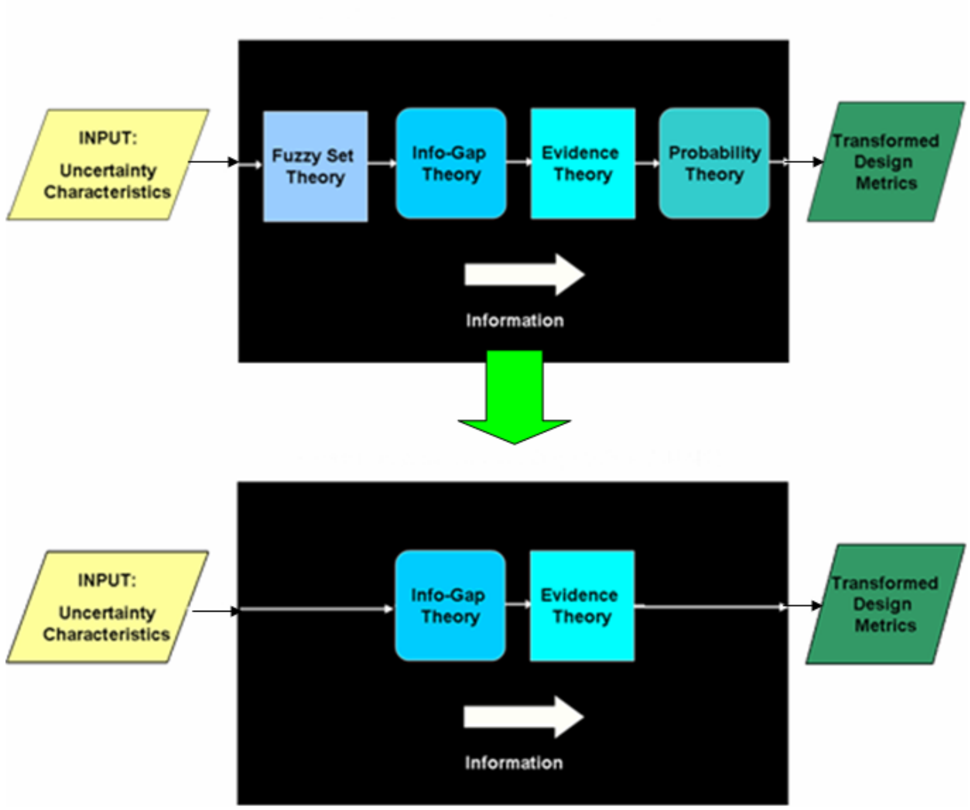

Figure 11-8: HUMM with only Info-Gap Theory and Evidence Theory

HUMM Theory Process Task 4 – Setup a Full-Factorial DOE

In this analysis, due to the characteristics of the uncertainty, there are only two types of uncertainty modeling techniques that were required: Evidence Theory and Info-Gap Theory. For each of these theories a separate full factorial DOE is created.

For Info-Gap Theory there are 6238 cases to be run (6237 DOE cases plus an additional case modeling the nominal values). Evidence Theory requires only 32 DOE cases since it only models the bounds of the uncertain variable space.

As a result, for the full analysis there are 199,616 separate analysis runs to be conducted (6238 IG DOE runs x 32 ET DOE runs).

HUMM Theory Process Task 5 – Run Model and Simulation Environment for all of the DOE Runs and Calculate Final Design Metrics

Using the process described in Chapter 7, the 199,616 analysis runs are conducted by using the surrogate models created earlier in the process. This process is repeated for each of the design alternatives, selected by the latin hypercube sampling technique, and the final Plausible α, Believable α, Plausible β, and Believable β for each of the traditional design metrics (Number of Scuds Launched, Number of Targets Located, Number of Daytime Hunter/Killer Aircraft, and Number of Nighttime Hunter/Killer Aircraft) are determined for each alternative.

Weight Determination

After HUMM has been completed for all of the identified alternatives, Step 8 of CONDOR-SS continues by using the metric information from HUMM in a weight determination exercise. In this exercise 1500 possible weight combinations are considered. The different weight combinations were selected by again using the latin hypercube sampling technique.

For every weight combination a separate TOPSIS analysis was used in order to determine the "best" alternative for this combination. Each of these alternatives was then compared for three different uncertainty scenarios. Because the actual uncertainty is unknown, for each scenario a range and distribution for the uncertainty variables is assumed for the problem. These assumed uncertainty characteristics are used only in estimating the most "effective weight combination". These values were not used in the uncertainty modeling analysis.

The first scenario is based upon average uncertainty values, the second scenario is based upon "ideal" uncertainty values (or less challenging uncertainty values as the case may be), and the third uncertainty scenario is based upon challenging uncertainty values. For each of the alternatives selected based upon the potential weight combination and for each scenario, a Monte Carlo Analysis consisting of 1000 runs is conducted using the provided range and distributions for the uncertainty variables. For this analysis 4.5 million analysis runs are conducted using the surrogate models.

The distance between the resulting design metric calculated by the MC analysis, which is the traditional metric and not one based on α or β, and the desirement was calculated. If a constraint was violated the distance was increased through the use of the same penalty function applied in the original uncertainty modeling analysis. The average resulting distance for each alternative was then compared and evaluated with TOPSIS to rank the effectiveness of each of the weight combinations for each of the uncertainty scenarios. The top ranking weight combinations were determined and averaged to calculate the estimated best weight combination for each set of metrics.

The weights identified are presented in Table 11-6.

Table 11-6: Calculated Weights from Weight Determination Analysis

Traditional Metric	HUMM Metric	Weight
Metric 1: Number of Scuds Launched	**α Plausible**	0.22
	α Believable	0.22
	β Plausible	0.32
	β Believable	0.25
Metric 2: Number of Targets Destroyed	**α Plausible**	0.25
	α Believable	0.25
	β Plausible	0.25
	β Believable	0.25
Metric 3: Number of H/K Aircraft (Daytime)	**α Plausible**	0.24
	α Believable	0.26
	β Plausible	0.26
	β Believable	0.24
Metric 4: Number of H/K Aircraft (Nighttime)	**α Plausible**	0.25
	α Believable	0.26
	β Plausible	0.26
	β Believable	0.23

Identification of “most robust and opportunistic” design alternative

Using the data provided from HUMM and the weights determined in the previous section, it is possible to identify the most robust and opportunistic design by conducting a Multi-Attribute Decision Making (MADM) technique. For this example problem TOPSIS, out of the various available MADM techniques, was selected because its foundation is similar to the fundamental concept of the RandO Method where the distance between the alternative and a negative outcome is maximized and the distance between the alterative and a positive outcome is minimized.[204]

The characteristics of the alternative selected by utilizing TOPSIS are shown in Table 11-7.

Table 11-7: Characteristics of most Robust and Opportunistic SoS Alternative

Design Variables	
Type of H/K Aircraft for Day	F-16
Type of H/K Aircraft for Night	F-16
Size of Killbox (nm)	59
Number of Tankers per Pattern	4
Number of H/K Aircraft in same Killbox	5

Step 9: Quantify the Value of Additional Information (Reducing Uncertainty)

For this example problem, all available information up to the point when the system needed to be deployed was considered. This is a fair representation of the actual historical event, because it was originally assumed from analyses conducted in the planning stages that the threat from the mobile scud launchers would not be a major factor in the engagement.

When this was determined in the first few days of the war, a new solution was needed immediately and had to be made with the available information. While a temporary solution based on past experience could be used while a design analysis such as the one conducted in the previous sections is conducted (assuming there is already a M&S environment capable of modeling the scenario), it would not be possible to wait for the additional information that would be required to reduce the uncertainty since all available information would have been used in the planning process.

For this reason, and because there is no historical data that could be used to verify this section of the analysis, it was determined that the alternative selected in Step 8 should be selected. However for completeness, the tasks required for the VRUM are briefly reviewed.

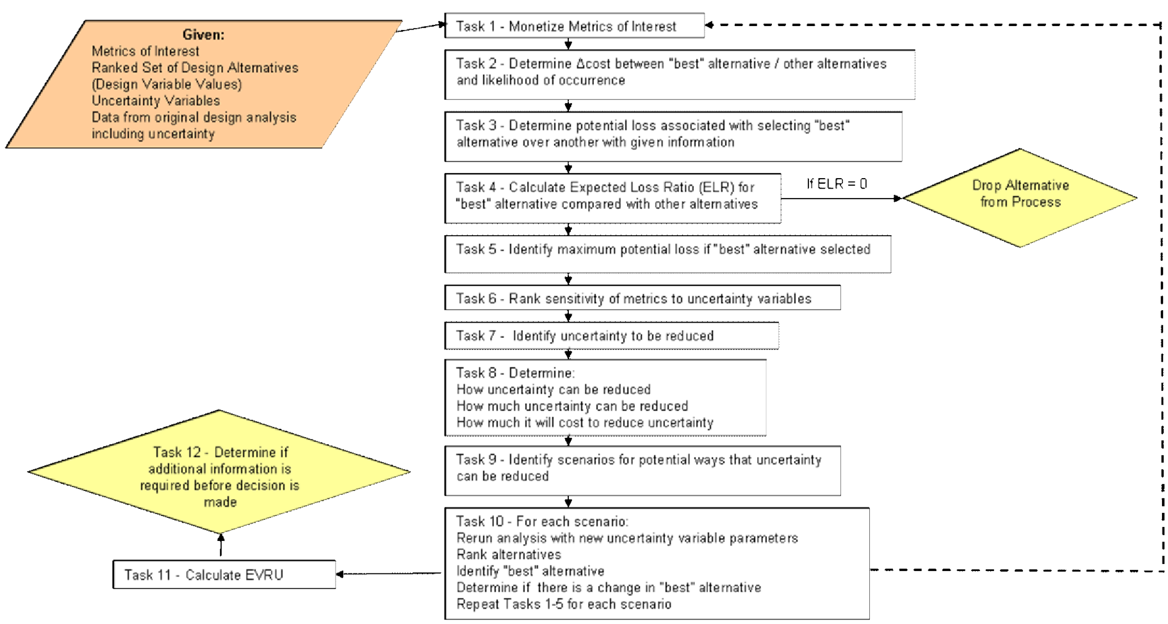

Figure 11-9: VRUM Process

VRUM Task 1 - Monetize Metrics of Interest

To determine the value of reducing uncertainty, it is necessary to convert the design metrics and the cost associated with reducing uncertainty into a common scale. One of the most applicable scales, is to transform all values for comparison into a monetary scale. Two options for accomplishing this task are discussed in detail in Chapter 9.

VRUM Task 2 – Determine Δcost between "best" alternative / other alternatives and likelihood of occurrence

The purpose of this task is to determine the difference in the cost (or loss) between "best" alternative / other alternatives and likelihood of occurrence for each of the runs from the HUMM. Since the metrics have already been converted into a monetary scale, or another common scale, it is possible to estimate the total loss for each metric associated with selecting one alternative over another.

VRUM Task 3 - Determine potential loss associated with selecting "best" alternative over another with given information

The information from Step 9, Task 2 can be used to determine the total potential loss associated with selecting the originally identified "best" alternative over the other alternatives. This loss is equal to the expectation of the difference in cost, when there is a loss, based upon the information from the previous task. Note that all of the data is used in this calculation. Only the values where there is a loss associated with selecting the original "best" alternative over another are considered.

The total utility associated with selecting the original "best" alternative over another is calculated by determining the expectation of all of the data (for this alternative pair) from Task 2.

VRUM Task 4 - Calculate Expected Loss Ratio (ELR) for "best" alternative compared with other alternatives

The Expected Loss Ratio (for each alternative pair) can be determined from the ratio of the loss and the total utility calculated in Task 3. If the ELR is zero, there is no possibility of the other alternative being a better option than the original "best" alternative. In this case the other alternative can be dropped from future analyses.

VRUM Task 5 - Identify maximum potential loss if "best" alternative selected

The maximum potential loss is the maximum value of the potential loss for each alternative pair identified in Task 3. This information in conjunction with the ELR data from Task 4, can allow a designer to evaluate if there is any possibility of an improved decision by reducing the uncertainty. If the maximum potential loss is inconsequential or all of the other design alternatives are eliminated, then there is no need to continue with this process. The "best" alternative has already been selected.

VRUM Task 6 - Rank sensitivity of metrics to uncertainty variables

Not all of the uncertainty variables significantly affect the metrics. A sensitivity test or screening test should be utilized in order to reduce the number of variables to consider. This step reduces the computational expense of this process.

VRUM Task 7 - Identify uncertainty to be reduced

Based on the results of the sensitivity test from Task 6, the important uncertainties should be selected.

VRUM Task 8 – Determine Characteristics Associated with Reducing Uncertainty

In this task there are four main questions to be answered:

- Can the uncertainty be reduced?
- How can the uncertainty be reduced?
- How much uncertainty can be reduced?
- How much will it cost to reduce uncertainty?

The purpose of this task is to determine the characteristics associated with actually reducing the uncertainty. All of this information is obtained from either the literature or subject matter experiments and is highly problem dependent. In this task information from subject matter experts / literature is used to determine the Expected Cost of Reducing Uncertainty (ECRU).

VRUM Task 9 - Identify scenarios for potential ways that uncertainty can be reduced

As an example, while it might be possible to estimate that with additional testing the uncertainty can be reduced by 10%, it is not known without actually doing the test, how the uncertainty would be reduced. In order to estimate this value, it is necessary to run a number of different uncertainty reduction scenarios. In each scenario the uncertainty is reduced about a different location within the range of values for the variable. In this task the number of uncertainty reduction scenarios is determined. Chapter 9 provides guidance over the number of scenarios to be used depending on the amount of uncertainty to be reduced.

VRUM Task 10 – Uncertainty Reduction Analysis

The main steps involved with this task are to:

- Rerun analysis with new uncertainty variable parameters
- Rank alternatives
- Identify "best" alternative
- Determine if there is a change in "best" alternative
- Repeat Tasks 1-5 for each scenario

For each of the previously identified scenarios the HUMM process is repeated and a new alternative is selected using a MADM technique. This new "best" alternative is compared with the original best alternative to determine if a new decision might be made if the uncertainty is reduced.

VRUM Task 11 - Calculate EVRU

In this task the Expected Value of Reducing Uncertainty (EVRU) is determined from the results from Task 10. The EVRU is essentially the average value of the expected loss from selecting one alternative over another with the expected loss that would be obtained if a different decision was made with additional information.

VRUM Task 12 - Determine if additional information is required before decision is made

The final task of the process is to compare the EVRU with the ECRU. If the cost is greater than the value associated with reducing the uncertainty, then there is no value in reducing the uncertainty before making the final decision. Otherwise the uncertainty should be reduced before a final selection is made.

Step 10: Gain additional knowledge or Select Concept

As discussed in the previous section, it was determined that the decision would be made with no new additional information. This means that the decision would be made without reducing the uncertainty; therefore, the final design decision was to select the SoS alternative with the characteristics listed in Table 11-7.

11.3. Evaluation of Design Method for Desert Storm Scud Hunt

In the previous sections of this chapter the RandO Design Method was used to determine the most robust and opportunistic SoS for the Scud Hunt scenario. This analysis used the information that was available to the analysts and military leaders in the actual scenario, so that an "apples to apples" evaluation of the identified SoS and the historical SoS can be conducted. The following sections review the actual values for the uncertainty and compares the effectiveness of the SoS original solution with the effectiveness of the RandO solution.

11.3.1. Desert Storm Scud Hunt Characteristics

There was considerable uncertainty involved in the scud hunt scenario. The types of uncertainty that were identified as the most critical were: the number of targets, minimum target setup time, average repair time for hunter/killer aircraft, the probability of weapon missing target, the average cloud cover, the number of malfunctions of hunter/killer aircraft, the number of decoys, the average target hiding time, and the randomness (unpredictability) of a pilots actions.

The publicly available literature can be used to determine/approximate the actual values for most of these uncertainty variables. These values are provided in Table 11-8.

Table 11-8: Actual Values to Uncertain Variables

Uncertainty Variable	Actual Value
Number of Targets[35]	19
Minimum target setup time[36]	30 min
Average Repair time for Hunter/Killer Aircraft[37]	7 hours
Probability of weapon missing target[38]	60%
Average cloud coverage[39]	~ 50%
Malfunction of Hunter/Killer Aircraft[40]	0.775 aircraft /1000 sorties
Number of Decoys[41]	100
Average Target Hiding Time[42]	8
Factor modeling Random Pilot Actions[43]	0.3

11.3.2. Historical Scud Hunt SOS

The following aircraft were involved in the scud hunt: F-15E, A-10, F-16, and F-111F. For the scud hunt in the western region of Iraq, which is the scenario that the M&S was based on, the F-15E were primarily used for night scud hunting and the A-10s were used for Day scud hunting. [50,176,177] Twenty-five percent of the available F-15Es were used for this mission, and 7% of the A-10s. [50] For this example problem, the M&S environment determined that ½ of a squadron of A-10 and F-15E were used for the Scud Hunt. F-16s were one of the main aircraft used in other regions for the scud hunt.

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35 [49]
36 Estimated based on information from Reference 49.
37 Estimate based from data in Reference 50.
38 This is a very rough estimate based on data provided in Reference 39. Note that this is per bomb dropped. This value is also based on data for Laser guided bombs.
39 [50,81]
40 This value is actually for the number of aircraft damaged. For this example problem, malfunctions and battle damage where the aircraft was not lost were grouped together.[52]
41 No data exists for the number of decoys, however it was known than the launch areas were seeded with decoys. 80 launchers were reported destroyed so it is likely that there were a large number of decoys.[49]
42 No information is available that characterizes the behavior associated with the mobile launchers. However based on the firing data of the launches presented in the literature an estimate of 8 hours was used for this analysis.[52]
43 No data on this variable. This value was roughly estimated by synthesizing data from References 176 and 177.

All of the values actually used in the historical scenario are provided in Table 11-9.

Table 11-9: Historical SoS Characteristics

Design Variables	
Type of H/K Aircraft for Day	A-10
Type of H/K Aircraft for Night	F-15 with LANTIRN system
Size of Killbox (nm) (smallwood)	30
Number of Tankers per Pattern (weapons)[44]	3
Number of H/K Aircraft in same Killbox (smallwood)	2

A squadron (24 aircraft) of each type of aircraft was required for this SoS.

11.3.3. <u>RandO SoS</u>

Using CONDOR-SS, the RandO SoS solution was identified to have the characteristics shown in Table 11-10. Instead of A-10s and F-15s, F-16s are primarily used. Additionally the size of the killbox is increased as is the number of tankers per refueling orbit and the number of aircraft searching a single killbox. For this solution, five aircraft are continuously searching instead of two. This resulted in 61 F-16 for day operations and 62 aircraft for nighttime operations.

Table 11-10: RandO SoS Characteristics

Design Variables	
Type of H/K Aircraft for Day	F-16
Type of H/K Aircraft for Night	F-16 with LANTIRN system
Size of Killbox (nm)	60
Number of Tankers per Pattern	4
Number of H/K Aircraft in same Killbox	5

[44] Estimated from Reference 50.

11.3.4. Effectiveness of Original SoS and RandO SoS

The original SoS was not effective in destroying any actual scud launchers, but it was successful from the point of view that it dramatically reduced the number of scud launches. The historical SoS reduced the percentage of scud launches by 90%. The simulated launches are presented in Figure 11-10.

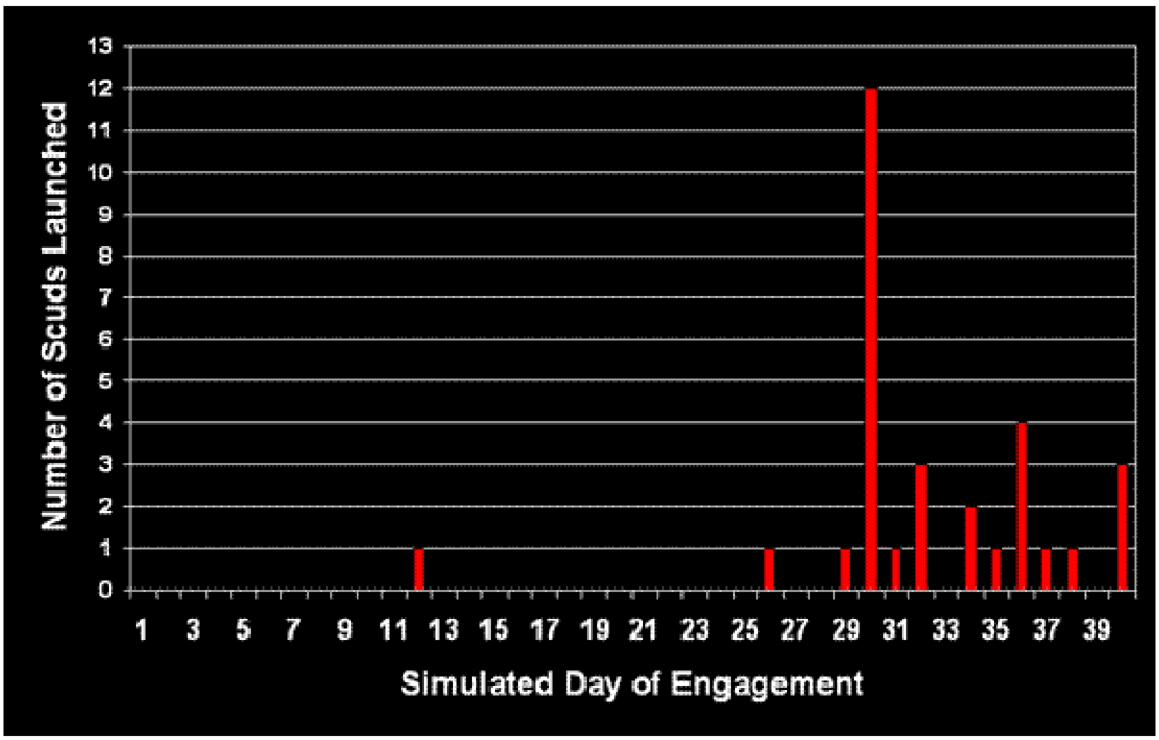

Figure 11-10: Number of Simulated Scuds Launched for Historical SoS

The RandO SoS was equally ineffective in destroying any actual scud launchers, though like the historical SoS, it was able to eliminate all of the decoys. However the RandO SoS was able to reduce the number of Scud launches by 97%. For this SoS there were only 8 scuds launched as illustrated in Figure 11-11. Figure 11-12 comparisons the results for the two different SoS solutions.

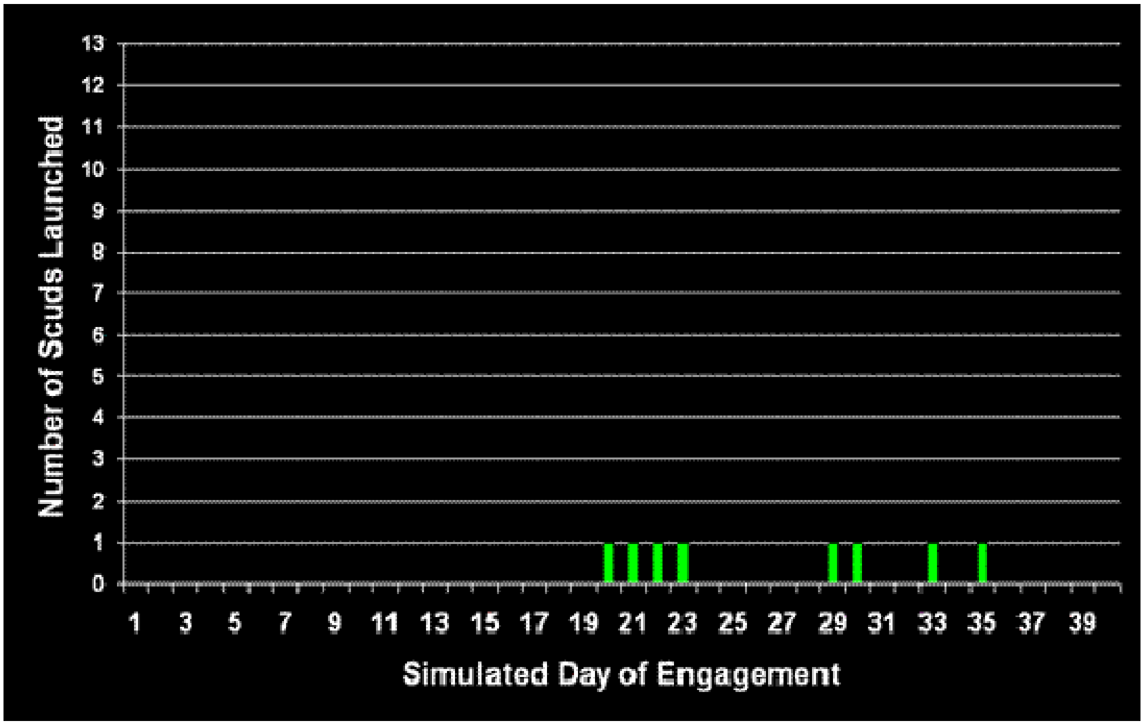

Figure 11-11: Number of Simulated Scuds Launched for RandO SoS

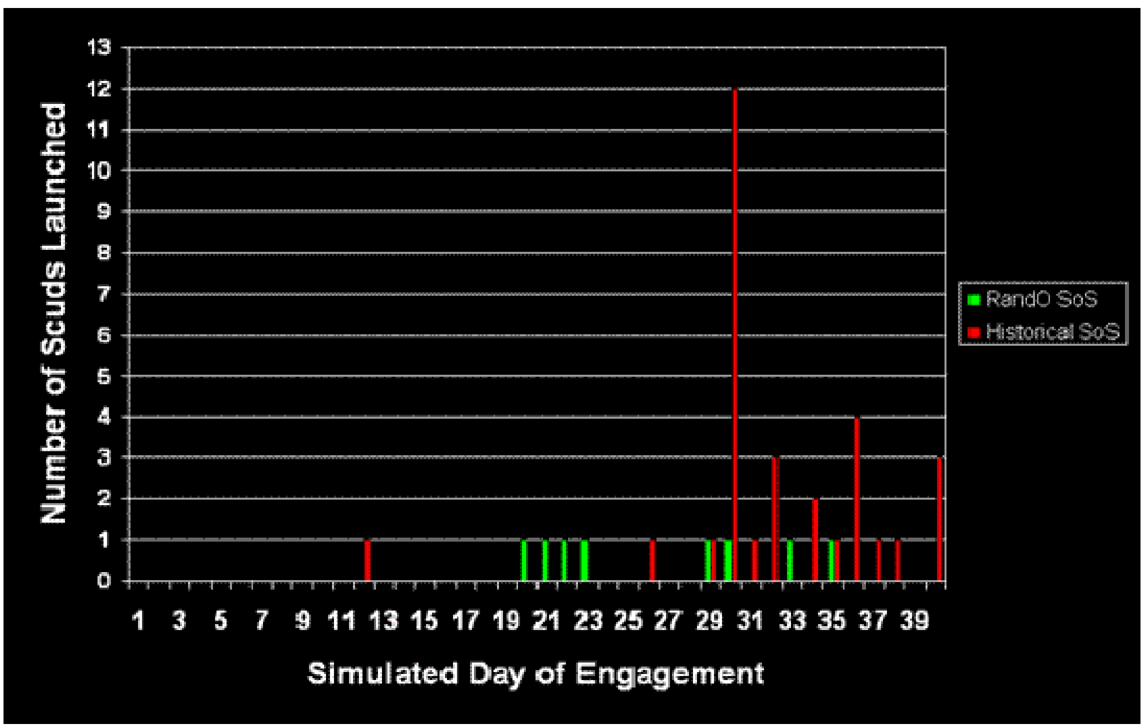

Figure 11-12: Comparison of Number of Simulated Scuds Launched between Historical and RandO SoS

Based on these results it is apparent that the RandO SoS is more effective in reducing the number of scud launches, which was the primary metric for this design problem. This clearly illustrates the value of this design method.

The historical SoS required less than half as many aircraft, but, it is important to recall that the design problem was only constrained by total number of aircraft available and rewarded if less than a squadron were used. If it is desired for fewer aircraft to be utilized, the design constraints should be modified and design method repeated.

CHAPTER 12: REVIEW OF PRIMARY RESEARCH QUESTIONS AND HYPOTHESES

The purpose of this chapter is to review the primary research questions and associated hypotheses identified in Chapter 6.

Primary Research Question 1 (A.4): Is it possible to create a hybrid uncertainty modeling technique that can combine Probability Theory, Evidence Theory, Info-Gap Theory, and Fuzzy Set Theory?

Hypothesis 1: A hybrid uncertainty modeling technique effectively combining Probability Theory, Evidence Theory, Info-Gap Theory, and Fuzzy Set Theory can be created by utilizing a full factorial DOE to model the possible uncertainty combinations and by transferring relevant information about the uncertainty between theories.

Chapter 7 presents a hybrid uncertainty modeling technique that utilizes the foundations of Probability Theory, Evidence Theory, Info-Gap Theory, and Fuzzy Set Theory along with a full factorial DOE to model the possible uncertainty combinations. This technique transforms the traditional design metrics into information that is compatible with each of the original uncertainty modeling theories.

This technique is fully documented for a simplified aircraft acquisition cost example which requires the use of all four uncertainty modeling techniques based upon the information available for the uncertain variables. This technique is also utilized for all of the example problems in Chapter 8 and in the Operation Desert Storm Scud Hunt Example in Chapter 11.

Primary Research Question 2 (B.1): Is there any benefit to considering both the pernicious and propitious qualities of uncertainty in a design process?

Hypothesis 2: For design problems characterized by competing constraint and desirement relationships, there is a benefit to considering both the pernicious and propitious qualities of uncertainty in a design process.

Six different example problems were used to demonstrate the differences between Robust Design, Opportunistic Design, and Robust and Opportunistic (RandO) Design in Chapter 8. For all of the example problems, the RandO Design approach (which considers both the positive and negative aspects of uncertainty) was found to identify the best design alternative, on average, for a variety of uncertainty scenarios.

The results also showed that for the design problems where there was a purely complementary relationship between the constraint and the desirement, the Robust Design approach and the Opportunistic Design approach also identified the best alternative. For this type of design problem, any of the three approaches could be used to identify the best solution and it is not necessary to consider both the pernicious and propitious qualities of uncertainty.

Primary Research Question 3 (B.2): How can both of the pernicious and propitious qualities of uncertainty be incorporated in a design process?

Hypothesis 3: The pernicious and propitious qualities of uncertainty can be incorporated in a design process by maximizing the Robustness Function (α), defined as the expected difference between the value of the design metric and the respective constraint, and minimizing the Opportunity Function (β), defined as the expected difference between the value of the design metric and the respective desirement, for a given design alternative.

This hypothesis is based loosely on the concept of the Robustness Function (α) and Opportunity function (β) from Info-Gap Theory. Instead of focusing only on the potential variance for the uncertain design variables as the design metrics, this hypothesis considers the variance of metrics in general. This expanded concept of the Robustness Function (α) and Opportunity function (β) is documented and demonstrated by the simplified aircraft acquisition cost example in Chapter 4.

The use of this technique for a design problem is demonstrated in all of the example problems in Chapter 8. These example problems also serve to illustrate the differences between only maximizing the Robustness Function (Robust Design), only minimizing the Opportunity Function (Opportunistic Design), and finding a compromise between these techniques (RandO Design).

Primary Research Question 4 (C.2): Is it possible, with the available information and analysis tools, to estimate the benefit associated with reducing the relevant uncertainty in the design process before a final design decision is made?

Hypothesis 4: The benefit associated with reducing the relevant uncertainty in the design process before a decision is finalized can be estimated by comparing the Expected Value of Reducing Uncertainty (EVRU) with the Expected Cost to Reduce Uncertainty (ECRU).

The Fleet Design example problem in Chapter 9 was used to demonstrate the process of determining the Expected Value of Reducing Uncertainty (EVRU) and the Expected Cost to Reduce Uncertainty (ECRU). These terms were compared in the example problem to estimate the benefit associated with reducing the uncertainty before a final SoS design concept is selected.

12.1. Validation of Methods

To fill the gaps identified in Chapter 6 several methods were developed: HUMM, RandO Design Method, and VRUM. These methods were then combined with elements from existing SoS design methods in CONDOR-SS to provide a complete cohesive design method for a conceptual SoS. These methods were all validated using the Validation Square technique, which guides the designer through the verification and validation process, as described in Reference 169.

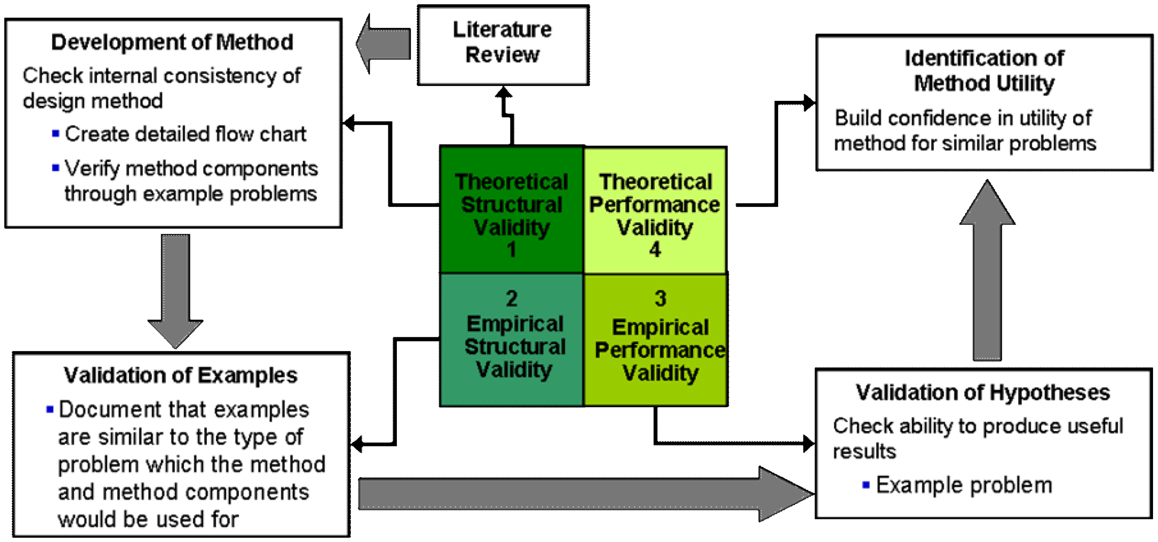

Figure 12-1: Validation Square Process [169]

Figure 12-1 illustrates that the validation square has four separate concepts: the Theoretical Structural Validity (TSV), Empirical Structural Validity (ESV), Empirical Performance Validity (EPV), and the Theoretical Performance Validity (TPV). [169]

12.1.1. Theoretical Structural Validity

The Theoretical Structural Validity (TSV) focuses on the internal consistency of the design method. To meet the requirements for this aspect of the square the tools and processes used in the design method must be considered valid within their specified ranges. Additionally, for specific applications, the internal structure of the design method must be considered consistent. [169]

To confirm the TSV of the method the first task was to create a detailed flow chart of the processes. These flow charts include information about the inputs and outputs relating to each process within the method. The flow charts for the four methods are presented in Chapters 7,8,9, and 10, and an overview is presented in Figure 12-2.

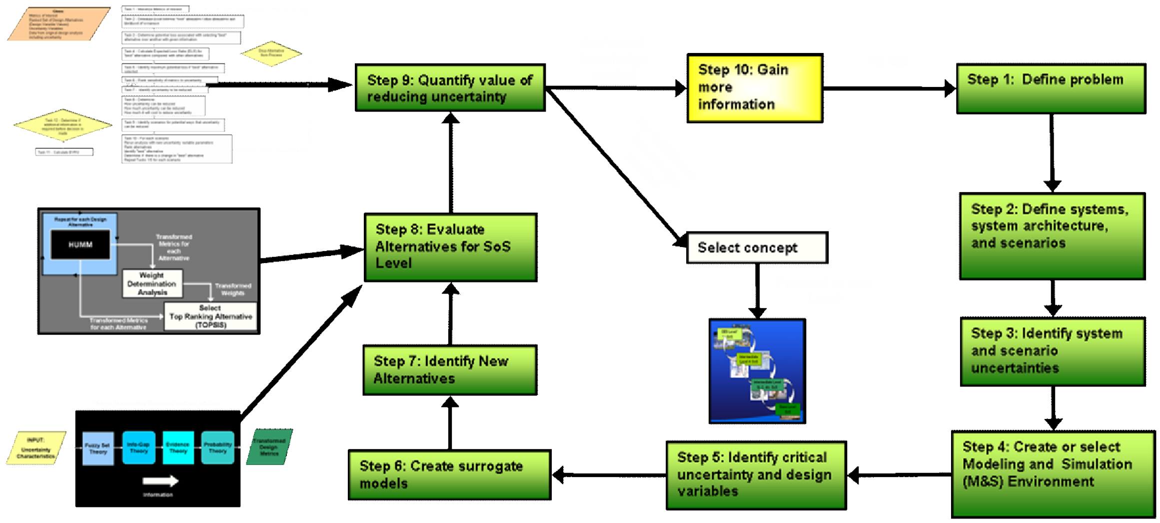

Figure 12-2: Overview of flow charts for 4 design methods/techniques

Once the flow chart has been reviewed to confirm that it is internally consistent, all of the various components of the method were validated through simple example problems. For example consider the HUMM, which is a core element of both the general RandO design approach and CONDOR-SS. All of the various uncertainty modeling components of HUMM were validated for the aircraft acquisition cost example in Chapter 4. Similarly basic example problems were used to test the other components of all the methods.

12.1.2. <u>Empirical Structural Validity</u>

This step is completed simultaneously with the tasks for confirming the Theoretical Structural Validity. The Empirical Structural Validity (ESV) checks the appropriateness of the chosen example problems intended to test the design method. The example problems are used to test elements of the concept that is being validated. In this part of the square the tools and processes used in the design method must be considered valid within their specified ranges. In order to meet the requirements of this construct, it must be shown that the example problems are similar in type to the problem that they will be applied to in practice. [169]

In order to determine the appropriateness of the chosen example problems from the TSV it is necessary to document the example problems and show that they are similar to the types of problems that will be solved when the method is used as a whole. This is simply making sure that this is an apples to apples comparison and that the final results will be consistent with what is expected.

In Chapter 1 the persistent strike SoS problem was identified as an appropriate example for the development of a conceptual design method of a SoS. It is both of interest to current engineering and defense communities and involves a significant amount of uncertainty. For this reason, the primary example problems used to validate this research are related to the persistent strike SoS design problem.

12.1.3. Empirical Performance Validity

The third step of the validation square is the where the Empirical Performance Validity is evaluated. In this step a comprehensive problem is solved, which means that all components of the concept are tested together. [169] The tasks associated with this process confirm the ability to produce useful results for the chosen example problems.

The empirical performance of HUMM was validated with a persistent strike UAV Aircraft Acquisition Cost example. A persistent strike UAV design problem and a persistent strike fleet design problem were used to validate the RandO design approach. The persistent strike fleet design problem was then used to validate the empirical performance of the VRUM. And finally, a SoS design problem based on the Scud Hunt from Operation Desert Storm was used to validate the CONDOR-SS method as a whole.

12.1.4. Theoretical Performance Validity

The objective of the Theoretical Performance Validity (TPV) part of the Validation Square is to confirm the ability of the method to produce useful results beyond the chosen example problem or problems. This is where the limitations and potential extensions of the method are articulated, and the main criterion for this construct is that the design method is deemed valid beyond the example problems. [169]

The particular task is primarily addressed in the final chapter of this document. This chapter not only reviews all of the methods and their capabilities, but also discusses their deficiencies and suggests areas appropriate for future research.

CHAPTER 13: CONCLUSIONS AND RECOMMENDATIONS

This chapter is organized is divided into three parts. The first part discusses the hybrid uncertainty modeling technique developed in Chapter 7 and also discusses potential areas for future work. The second part focuses on the conclusions from Chapter 8 that highlight the differences between the three design approaches (Robust Design, Opportunistic Design, and Robust and Opportunistic Design) for modeling uncertainty. The third and final section of this chapter reviews the research pertaining to valuing a reduction in uncertainty.

13.1. Uncertainty Modeling for a Conceptual System-of-Systems Design Method

Based upon the complex and highly interrelated nature of most system-of-systems (SoS), it is likely that there is to be a variety of types of uncertainty associated with the operation and development of such as system. As such it is expected that the conceptual design process of a SoS will be characterized by significant uncertainty. Because of the wide variety of systems and potential situational / operational scenarios that could be related to the general SoS all of the types of uncertainty (randomness and sampling, confusion and conflict, inaccuracy, ambiguity, vagueness, coarseness, and simplification) could affect the system and the design of the system.

Because all of the types of uncertainty could affect the design of the system, it is necessary for a conceptual design method for a SoS to be capable of modeling and analyzing all of these different types. Existing SoS design methods that consider uncertainty primarily use probabilistic techniques to consider uncertainty in the design process. However additional uncertainty modeling techniques are necessary to appropriately model all of the different types of uncertainty.

There are a large number of uncertainty modeling techniques that could be integrated into such a design process including: Probability Theory, Classical Set Theory, Fuzzy Set Theory, Possibility Theory, Evidence Theory, and Info-Gap Theory. It is possible to take various combinations of these theories in order model the different types of uncertainty. In fact if three of the more common theories: Probability Theory, Fuzzy Set Theory, and Evidence Theory were combined, they would be capable of modeling all of the different types of uncertainty.

Additionally, there are different levels of knowledge that also pertain to the types of uncertainty. Uncertainty can vary from well defined (where distributions can be created with statistical data describing the characteristics of the uncertainty) to very poorly defined (where information about the actual bounds or characteristics of the uncertainty can only be assumed). Certain uncertainty modeling techniques are more appropriate for modeling different levels of knowledge. Probability Theory is useful for modeling well defined uncertainty, Info-Gap Theory is useful for modeling uncertainty where there is a severe lack of information about its characteristics, and Evidence Theory can often be used for the case when the uncertainty is not well defined but there is enough information to predict bounds to the uncertainty.

Considering a conceptual SoS design problem could involve all of the types of uncertainty and is likely to incorporate uncertainties that have varying levels of knowledge, a hybrid uncertainty modeling technique was developed that combines the capabilities of all four uncertainty modeling techniques. The technique was developed to be modular so that only the relevant uncertainty modeling techniques would be included for each design problem.

The tasks for this process are as follows:

Task 1 – Define the design metrics, constraints, and desirements

Task 2 – Define the uncertainty characteristics

Task 3 – Determine the number of analysis runs for each uncertainty variable

Task 4 – Setup a full-factorial DOE

Task 5 – Run model and simulation environment for all of the DOE runs and calculate final design metrics

This technique utilizes a full factorial DOE to model the different possible combinations of all of the relevant uncertainty variables. In order to transfer knowledge between the different uncertainty modeling techniques in the process, the technique transforms the traditional metric values. For instance in the case of Info-Gap Theory the metric values are in terms of the Robustness and Opportunity Functions (α and β, respectively). When Evidence Theory is used, the plausible and believable values of the metric are determined. If both Info-Gap Theory and Evidence Theory are used the design metrics become the Plausible α, the Believable α, the Plausible β, and the Believable β. The different metrics are necessary because in the case of both Info-Gap Theory and Evidence Theory, neither theory has enough information about the uncertain variable to estimate a value for the traditional design metric without applying additional assumptions.

A simplified aircraft acquisition cost problem is used in Chapter 7 to demonstrate the utility of this technique. The technique is also used extensively in the design problems in Chapter 8 and 11. These example problems show that it is indeed possible to combine all of the techniques to appropriately model the different types of uncertainty for different levels of knowledge.

13.1.1. Future Work

While this technique has significant potential for the SoS design community, it is computationally expensive. Full factorial DOEs are used throughout the technique to model the different potential combinations of uncertainty, and if large number of uncertain variables are considered, this technique would be impractical to implement, even with the use of techniques like surrogate models to reduce the analysis time.

The purpose of this research was to demonstrate that it is possible to combine the four disparate techniques into one hybrid technique that can be used for a SoS design problem. However, future work is required to reduce the computational expense to make this technique feasible for large scale SoS problems.

Additionally there are potential avenues for research based upon specific aspects of the technique. For instance the normalization process used to convert the different Robustness and Opportunity Functions for each uncertainty variable to a common scale was found to be a significant factor. While numerous tests were performed in order to determine which normalization technique was the most appropriate, they were only performed on one class of problem. It is possible that another normalization technique will be more appropriate for the general design problem.

Because it is likely to be difficult to predict the best weights for metrics such as the Plausible α, the Believable α, the Plausible β, and the Believable β, a weight determination study was performed for all of the example problems in Chapter 8. From these example problems it was apparent that there was some benefit to performing a weight determination for each problem. However, additional weight determination studies may be able to provide general guidance for weights so that a full weight determination study is not required for every specific design problem.

In the Info-Gap Theory when the constraint or desirement is not found within the range of expected values for the uncertainty variable, the location to the constraint or desirement is linearly extrapolated. Chapter 8 discusses how penalty (or bonus) functions can be applied in these cases to emphasize the unknown location of the constraint/desirement. But, there are a number of other techniques that could be used. For instance, the data could be regressed to determine a more accurate model of the behavior of the uncertainty. This was not done in this research because the technique was already computationally expensive and preliminary studies illustrated for test cases that the linear extrapolation was a reasonable approximation considering the inherent uncertainty in the design problem. Another option is that the range of the uncertain variable can be iteratively extended until the constraint or desirement is reached. In all of these cases it is likely that the desirement or constraint is located well beyond the realistic bounds for the variable, considering the technique uses the expected maximum feasible range originally. However, these distances can still be used to determine α or β values for the design analysis.

Penalty and bonus functions are used throughout the hybrid technique to "reward" or "punish" any design alternative that crosses either the desirement or constraint. A general exterior penalty (or bonus) function was used based upon Reference 198. Additionally a "push off" factor could be included by the designer to penalize alternatives that just barely cross the threshold of the constraint. Additional studies, beyond the basic study done in Chapter 8, would be useful in determining the specific parameters for the penalty function in relation to various types of problems. Or, additional studies could compare the exterior penalty function and "push off" factor with other penalty functions for a variety of problems to determine which functions are most appropriate for different types of problems.

13.2. Robust and Opportunistic (RandO) Design

There are two sides to uncertainty in a design problem. Uncertain values and situations can have either propitious or pernicious effects on an associated system or the overall SoS. However, most design methods that consider uncertainty focus on developing designs that are robust, or insensitive, to the variation in uncertainty. These design techniques usually do not consider the positive potential of the uncertainty and as a result can lead to overly conservative designs.

To account for both sides of the uncertainty, it is possible to expand the general concept of the Robustness Function (α) and Opportunity Function (β) from Info-Gap Theory. Instead of focusing only on the potential variance for the uncertain design variables as the design metrics, this hypothesis considers the variance of metrics in general. The Robustness Function (α) is now defined as the expected difference between the value of the design metric and the respective constraint. And similarly the Opportunity Function (β) is defined as the expected difference between the value of the design metric and the respective desirement. From this point of view it is possible to use the Robustness Function (α) and Opportunity function (β) in a variety of uncertainty modeling techniques. Because it is expected that there will be a lack of knowledge pertaining to at least some of the uncertain variables in the conceptual design process for a SoS, this research primarily focuses upon techniques including Info-Gap Theory.

There are three potential approaches to considering uncertainty in a design process: Robust Design, Opportunistic Design, and Robust and Opportunistic Design. These three approaches are defined for this research as follows:

Definition of Robust Design:

Robust Design is a technique that identifies the design alternative that satisfies design constraints for a range of uncertainty values.

The focus of Robust Design is satisfying the design constraints and preventing negative effects from the uncertainty. Based upon the concept of α and β, Robust Design only uses the metrics based upon the Robustness Function (α). If Info-Gap Theory and Evidence Theory are used in the design process, as is the case for the example problems in this chapter, the design metrics for robust design are the Plausible α and the Believable α.

Definition of Opportunistic Design:

Opportunistic Design is a technique that identifies the design alternative that achieves the design desirements for a range of uncertainty values.

This approach specifically focuses on the positive aspects of uncertainty. The main metric for this technique is based upon the Opportunity Function (β) from Info-Gap Theory. If both Info-Gap theory and Evidence Theory are used in the design process, the design metrics for this approach are the Plausible β and the Believable β.

Definition of Robust and Opportunistic Design:

Robust and Opportunistic Design is a technique that identifies the design alternative that both satisfies the design constraints and achieves the design desirements for a range of uncertainty values.

RandO Design is a combination of Robust Design and Opportunistic Design and this approach focuses on both constraints and desirements. This approach to design is the compromise between Robust Design and Opportunistic Design. In a sense it is the approach that offers the “best of both worlds”. The main metrics for this design process are both α and β. If Evidence Theory and Info-Gap Theory are included in the design process, there will be four design metrics. These metrics are: the Plausible α, the Believable α, the Plausible β and the Believable β.

Six different example problems were used to demonstrate the differences between these three design approaches. For all of the example problems, the RandO Design approach was found to identify the best design alternative, on average, for a variety of uncertainty scenarios.

The results also showed that for the design problems where there was a purely complementary relationship between the constraint and the desirement, the Robust Design approach and the Opportunistic Design approach also identified the best alternative. For this type of design problem, any of the three approaches could be used to identify the best solution and it is not necessary to consider both the pernicious and propitious qualities of uncertainty.

13.2.1. Future Work

The experiments demonstrated that there could be considerable value to the RandO Design approach for many design problems. The general concept of RandO is not necessarily linked with the hybrid uncertainty technique developed in Chapter 8. It is anticipated that it could be utilized with a wide variety of uncertainty modeling techniques for future research.

The example problems from Chapter 8 indicated that in the case of purely complementary constraints and desirements that any of the design approaches could be used. And while it was shown that on average the RandO technique was found to identify the best alternative, there were a few uncertainty scenarios where the alternative identified by the Opportunistic Design approach was determined through the MADM technique to be the best. It is also possible to conceptualize situations where the most robust (and conservative) design alternative would be the best solution, such as situations where there is a very high cost to any failed constraint. It would be interesting to compare the various design approaches across a wide variety of problems to identify specific classes of problems where each design approach should be utilized.

13.3. Value of Reducing Uncertainty

In design problems with uncertainty, after a design alternative has been selected based upon the uncertainty analysis, there is the chance that another lower ranking alternative may actually be the best solution. In a nondeterministic decision process it is highly unlikely for there to be 100% confidence in any particular design alternative. If the designer had additional information (reduced uncertainty) they may determine that another alternative should be selected.

However, the original decision was made with all of the available information and only by gaining additional information/knowledge can the uncertainty be reduced. The fact that prevents designers from automatically acquiring this additional information is that there is some cost (be it monetary, related to time, etc) associated with the acquisition of additional information. The challenge becomes how to compare to unknown quantities: the value associated with reducing the uncertainty with the cost of reducing the uncertainty.

The value associated with reducing the uncertainty can be estimated by determining the value of the Expected Value of Reducing Uncertainty (EVRU). This can be found by first estimating the difference in expected loss associated with selecting the original top ranking design alternative with the expected loss associated with selecting the new top ranking alternative for a particular uncertainty reduction scenario. The EVRU is then the average value of the difference in expected loss for a set number of uncertainty reduction scenarios.

The cost of reducing the uncertainty is estimated in the term the Expected Cost to Reduce Uncertainty (ECRU). This particular term is highly problem dependent and needs to estimate all of the costs associated with gaining additional information. The important non-monetary costs such as resources expended, or time required to determine additional information, should be converted to a monetary value. Chapter 9 discusses options for the transformation of non-monetary values.

As discussed in Chapter 9, there are a couple of cases where there is no need to gain additional information. In the first case, even if the uncertainty is reduced, the design decision would be the same. In this case there is no need for additional information unless it would benefit the design process in the future. In the second case, the cost associated with gaining the new information/knowledge is greater than the likely potential savings by choosing the "best" alternative.

All of these techniques were utilized in a process called the Value of Reducing Uncertainty Method (VRUM). The tasks of this method include:

- Task 1 - Monetize Metrics of Interest
- Task 2 – Determine Δcost between "best" alternative / other alternatives and likelihood of occurrence
- Task 3 - Determine potential loss associated with selecting "best" alternative over another with given information
- Task 4 - Calculate Expected Loss Ratio (ELR) for "best" alternative compared with other alternatives
- Task 5 - Identify maximum potential loss if "best" alternative selected
- Task 6 - Rank sensitivity of metrics to uncertainty variables
- Task 7 - Identify uncertainty to be reduced
- Task 8 – Determine Characteristics Associated with Reducing Uncertainty
- Task 9 - Identify scenarios for potential ways that uncertainty can be reduced
- Task 10 – Uncertainty Reduction Analysis

- Task 11 - Calculate EVRU
- Task 12 - Determine if additional information is required before decision is made

The utility of the VRUM was demonstrated on a fleet design example problem in Chapter 9.

13.3.1. Future Work

It is possible to use existing informational and analysis tools to determine the value associated with reducing the uncertainty. But, there are several potential options for future work in this area.

There are two main techniques proposed for monetizing the non-monetary metrics in the problem. The first technique is to use a transformation factor and the second is to use the relative weights and a linear transformation to adapt the metrics. The first technique can be highly subjective and the second technique may overly distort the value of the metrics depending on the user supplied weights. Future research should consider different techniques for the monetization of non-monetary metrics and should determine which type of general problems would be appropriate for each of the techniques.

Another area of potential work lies in determining if there are other techniques for reducing the uncertainty for variables modeled with Info-Gap Theory. This theory is not conducive to uncertainty reduction scenarios and to effectively reduce the uncertainty it was assumed that the reduction would allow the variable to be modeled by Evidence Theory. From one perspective this is appropriate because Evidence Theory models the next highest level of information, but it is uncertain what range of variables to consider for the variable. Future research should be conducted to determine the best way to estimate the range for the uncertainty variable once it is converted from an Info-Gap variable to an Evidence Theory Variable.

And finally, additional research could be conducted to offer improvements in the uncertainty reduction scenario selection. Currently the range associated with the uncertainty variable is divided into evenly spaced intervals and the uncertainty reduction occurs about the mean value of the interval. However it may be possible to identify a better way to select the uncertainty reduction scenarios for different types of problems. For instance if a probability distribution is known for the variable, perhaps this distribution should be used in determining both the length and location of where the intervals should be located in the range of potential variables,

13.4. Concluding Remarks

While the primary motivation for this research was to address many of the gaps in existing SoS design methods, the techniques and concepts are general enough to apply to a broad range of design problems. After all SoS design problems are not the only problems that use a variety of different types of uncertainty. They are not the only problems that would benefit by considered both the propitious or pernicious characteristics, and they are not the only problems where it might be advantageous to reduce the uncertainty before a final decision is made.

Uncertainty is a fact of life and inherent in any design process. In light of this fact, the challenge for a designer often becomes, not necessarily how to eliminate the uncertainty, but instead how to manage it.

APPENDIX A

The charts in this section are for the Info-Gap Example Problem in Chapter 4. In Table A.1 the three uncertainty variables modeled by Info-Gap Theory are presented in the first three columns. The resulting cost for these values is presented in the fourth column and the difference between the constraint/desirement value and the calculated cost value is presented in the fifth and sixth columns, respectively.

Table A-1: Info-Gap Theory Example – Original Data

AMPR Weight Factor (%)	Cost($)/lb	TOGW (lbs)	Cost ($Million)	Constraint ($Million)	Success ($Million)
55	4000	10000	10.78	19.22	0.78
60	4000	10000	11.76	18.24	1.76
65	4000	10000	12.74	17.26	2.74
70	4000	10000	13.72	16.28	3.72
75	4000	10000	14.70	15.30	4.70
55	4375	10000	11.79	18.21	1.79
60	4375	10000	12.87	17.13	2.87
65	4375	10000	13.94	16.06	3.94
70	4375	10000	15.01	14.99	5.01
75	4375	10000	16.08	13.92	6.08
55	4750	10000	12.80	17.20	2.80
60	4750	10000	13.97	16.03	3.97
65	4750	10000	15.13	14.87	5.13
70	4750	10000	16.30	13.70	6.30
75	4750	10000	17.46	12.54	7.46
55	5125	10000	13.81	16.19	3.81
60	5125	10000	15.07	14.93	5.07
65	5125	10000	16.33	13.67	6.33
70	5125	10000	17.58	12.42	7.58
75	5125	10000	18.84	11.16	8.84
55	5500	10000	14.83	15.17	4.83
60	5500	10000	16.17	13.83	6.17
65	5500	10000	17.52	12.48	7.52
70	5500	10000	18.87	11.13	8.87
75	5500	10000	20.22	9.78	10.22
55	4000	15000	16.17	13.83	6.17
60	4000	15000	17.64	12.36	7.64
65	4000	15000	19.11	10.89	9.11
70	4000	15000	20.58	9.42	10.58

AMPR Weight Factor (%)	Cost($)/lb	TOGW (lbs)	Cost ($Million)	Constraint ($Million)	Success ($Million)
75	4000	15000	22.05	7.95	12.05
55	4375	15000	17.69	12.31	7.69
60	4375	15000	19.30	10.70	9.30
65	4375	15000	20.91	9.09	10.91
70	4375	15000	22.51	7.49	12.51
75	4375	15000	24.12	5.88	14.12
55	4750	15000	19.21	10.79	9.21
60	4750	15000	20.95	9.05	10.95
65	4750	15000	22.70	7.30	12.70
70	4750	15000	24.44	5.56	14.44
75	4750	15000	26.19	3.81	16.19
55	5125	15000	20.72	9.28	10.72
60	5125	15000	22.61	7.39	12.61
65	5125	15000	24.49	5.51	14.49
70	5125	15000	26.37	3.63	16.37
75	5125	15000	28.26	1.74	18.26
55	5500	15000	22.24	7.76	12.24
60	5500	15000	24.26	5.74	14.26
65	5500	15000	26.28	3.72	16.28
70	5500	15000	28.30	1.70	18.30
75	5500	15000	30.32	-0.32	20.32
55	4000	20000	21.56	8.44	11.56
60	4000	20000	23.52	6.48	13.52
65	4000	20000	25.49	4.51	15.49
70	4000	20000	27.45	2.55	17.45
75	4000	20000	29.41	0.59	19.41
55	4375	20000	23.59	6.41	13.59
60	4375	20000	25.73	4.27	15.73
65	4375	20000	27.87	2.13	17.87
70	4375	20000	30.02	-0.02	20.02
75	4375	20000	32.16	-2.16	22.16
55	4750	20000	25.61	4.39	15.61
60	4750	20000	27.94	2.06	17.94
65	4750	20000	30.26	-0.26	20.26
70	4750	20000	32.59	-2.59	22.59
75	4750	20000	34.92	-4.92	24.92
55	5125	20000	27.63	2.37	17.63
60	5125	20000	30.14	-0.14	20.14
65	5125	20000	32.65	-2.65	22.65
70	5125	20000	35.16	-5.16	25.16
75	5125	20000	37.68	-7.68	27.68
55	5500	20000	29.65	0.35	19.65
60	5500	20000	32.35	-2.35	22.35
65	5500	20000	35.04	-5.04	25.04
70	5500	20000	37.74	-7.74	27.74

AMPR Weight Factor (%)	Cost($)/lb	TOGW (lbs)	Cost ($Million)	Constraint ($Million)	Success ($Million)
75	5500	20000	40.43	-10.43	30.43
55	4000	25000	26.96	3.04	16.96
60	4000	25000	29.41	0.59	19.41
65	4000	25000	31.86	-1.86	21.86
70	4000	25000	34.31	-4.31	24.31
75	4000	25000	36.76	-6.76	26.76
55	4375	25000	29.48	0.52	19.48
60	4375	25000	32.16	-2.16	22.16
65	4375	25000	34.84	-4.84	24.84
70	4375	25000	37.52	-7.52	27.52
75	4375	25000	40.20	-10.20	30.20
55	4750	25000	32.01	-2.01	22.01
60	4750	25000	34.92	-4.92	24.92
65	4750	25000	37.83	-7.83	27.83
70	4750	25000	40.74	-10.74	30.74
75	4750	25000	43.65	-13.65	33.65
55	5125	25000	34.54	-4.54	24.54
60	5125	25000	37.68	-7.68	27.68
65	5125	25000	40.82	-10.82	30.82
70	5125	25000	43.96	-13.96	33.96
75	5125	25000	47.10	-17.10	37.10
55	5500	25000	37.06	-7.06	27.06
60	5500	25000	40.43	-10.43	30.43
65	5500	25000	43.80	-13.80	33.80
70	5500	25000	47.17	-17.17	37.17
75	5500	25000	50.54	-20.54	40.54
55	4000	30000	32.35	-2.35	22.35
60	4000	30000	35.29	-5.29	25.29
65	4000	30000	38.23	-8.23	28.23
70	4000	30000	41.17	-11.17	31.17
75	4000	30000	44.11	-14.11	34.11
55	4375	30000	35.38	-5.38	25.38
60	4375	30000	38.60	-8.60	28.60
65	4375	30000	41.81	-11.81	31.81
70	4375	30000	45.03	-15.03	35.03
75	4375	30000	48.24	-18.24	38.24
55	4750	30000	38.41	-8.41	28.41
60	4750	30000	41.90	-11.90	31.90
65	4750	30000	45.40	-15.40	35.40
70	4750	30000	48.89	-18.89	38.89
75	4750	30000	52.38	-22.38	42.38
55	5125	30000	41.44	-11.44	31.44
60	5125	30000	45.21	-15.21	35.21
65	5125	30000	48.98	-18.98	38.98
70	5125	30000	52.75	-22.75	42.75

AMPR Weight Factor (%)	Cost($)/lb	TOGW (lbs)	Cost ($Million)	Constraint ($Million)	Success ($Million)
75	5125	30000	56.51	-26.51	46.51
55	5500	30000	44.48	-14.48	34.48
60	5500	30000	48.52	-18.52	38.52
65	5500	30000	52.56	-22.56	42.56
70	5500	30000	56.61	-26.61	46.61
75	5500	30000	60.65	-30.65	50.65

In Table A.2 the results from Table A.1 are sorted by increasing "constraint" values (the difference between the constraint and the calculated cost value). The potential α values are identified in columns six through eight. The values are the difference between the nominal values for each of the uncertain variables and the actual values from each DOE run. The selected value of α is identified by determining when the sign of column two changes from negative to positive. This indicates that the constraint is no longer being violated.

Recall that the objective is to maximize the Robustness Function (α), which occurs at the point where the nominal value is the furthest from the constraint. In order to be conservative in this technique, the value for α before the constraint is known to be violated is selected as the value of α.

Table A-2: Potential α Values for Info-Gap Example Problem

		Actual Values			Potential α Values		
	Constraint ($Million)	AMPR Weight Factor (%)	Cost($)/lb	TOGW (lbs)	AMPR Weight Factor (%)	Cost($)/lb	TOGW (lbs)
1	-30.65	75	5500	30000	10	800	10000
2	-26.61	70	5500	30000	5	800	10000
3	-26.51	75	5125	30000	10	425	10000
4	-22.75	70	5125	30000	5	425	10000
5	-22.56	65	5500	30000	0	800	10000
6	-22.38	75	4750	30000	10	50	10000
7	-20.54	75	5500	25000	10	800	5000
8	-18.98	65	5125	30000	0	425	10000
9	-18.89	70	4750	30000	5	50	10000
10	-18.52	60	5500	30000	5	800	10000

		Actual Values			Potential α Values		
	Constraint ($Million)	AMPR Weight Factor (%)	Cost($)/lb	TOGW (lbs)	AMPR Weight Factor (%)	Cost($)/lb	TOGW (lbs)
11	-18.24	75	4375	30000	10	325	10000
12	-17.17	70	5500	25000	5	800	5000
13	-17.10	75	5125	25000	10	425	5000
14	-15.40	65	4750	30000	0	50	10000
15	-15.21	60	5125	30000	5	425	10000
16	-15.03	70	4375	30000	5	325	10000
17	-14.48	55	5500	30000	10	800	10000
18	-14.11	75	4000	30000	10	700	10000
19	-13.96	70	5125	25000	5	425	5000
20	-13.80	65	5500	25000	0	800	5000
21	-13.65	75	4750	25000	10	50	5000
22	-11.90	60	4750	30000	5	50	10000
23	-11.81	65	4375	30000	0	325	10000
24	-11.44	55	5125	30000	10	425	10000
25	-11.17	70	4000	30000	5	700	10000
26	-10.82	65	5125	25000	0	425	5000
27	-10.74	70	4750	25000	5	50	5000
28	-10.43	75	5500	20000	10	800	0
29	-10.43	60	5500	25000	5	800	5000
30	-10.20	75	4375	25000	10	325	5000
31	-8.60	60	4375	30000	5	325	10000
32	-8.41	55	4750	30000	10	50	10000
33	-8.23	65	4000	30000	0	700	10000
34	-7.83	65	4750	25000	0	50	5000
35	-7.74	70	5500	20000	5	800	0
36	-7.68	75	5125	20000	10	425	0
37	-7.68	60	5125	25000	5	425	5000
38	-7.52	70	4375	25000	5	325	5000
39	-7.06	55	5500	25000	10	800	5000
40	-6.76	75	4000	25000	10	700	5000
41	-5.38	55	4375	30000	10	325	10000
42	-5.29	60	4000	30000	5	700	10000
43	-5.16	70	5125	20000	5	425	0
44	-5.04	65	5500	20000	0	800	0
45	-4.92	75	4750	20000	10	50	0
46	-4.92	60	4750	25000	5	50	5000
47	-4.84	65	4375	25000	0	325	5000
48	-4.54	55	5125	25000	10	425	5000
49	-4.31	70	4000	25000	5	700	5000
50	-2.65	65	5125	20000	0	425	0
51	-2.59	70	4750	20000	5	50	0
52	-2.35	60	5500	20000	5	800	0
53	-2.35	55	4000	30000	10	700	10000
54	-2.16	75	4375	20000	10	325	0
55	-2.16	60	4375	25000	5	325	5000

		Actual Values			Potential α Values		
	Constraint ($Million)	AMPR Weight Factor (%)	Cost($)/lb		Constraint ($Million)	AMPR Weight Factor (%)	Cost($)/lb
56	-2.01	55	4750	25000	10	50	5000
57	-1.86	65	4000	25000	0	700	5000
58	-0.32	75	5500	15000	10	800	5000
59	-0.26	65	4750	20000	0	50	0
60	-0.14	60	5125	20000	5	425	0
61	-0.02	70	4375	20000	5	325	0
62	0.35	55	5500	20000	10	800	0
63	0.52	55	4375	25000	10	325	5000
64	0.59	75	4000	20000	10	700	0
65	0.59	60	4000	25000	5	700	5000
66	1.70	70	5500	15000	5	800	5000
67	1.74	75	5125	15000	10	425	5000
68	2.06	60	4750	20000	5	50	0
69	2.13	65	4375	20000	0	325	0
70	2.37	55	5125	20000	10	425	0
71	2.55	70	4000	20000	5	700	0
72	3.04	55	4000	25000	10	700	5000
73	3.63	70	5125	15000	5	425	5000
74	3.72	65	5500	15000	0	800	5000
75	3.81	75	4750	15000	10	50	5000
76	4.27	60	4375	20000	5	325	0
77	4.39	55	4750	20000	10	50	0
78	4.51	65	4000	20000	0	700	0
79	5.51	65	5125	15000	0	425	5000
80	5.56	70	4750	15000	5	50	5000
81	5.74	60	5500	15000	5	800	5000
82	5.88	75	4375	15000	10	325	5000
83	6.41	55	4375	20000	10	325	0
84	6.48	60	4000	20000	5	700	0
85	7.30	65	4750	15000	0	50	5000
86	7.39	60	5125	15000	5	425	5000
87	7.49	70	4375	15000	5	325	5000
88	7.76	55	5500	15000	10	800	5000
89	7.95	75	4000	15000	10	700	5000
90	8.44	55	4000	20000	10	700	0
91	9.05	60	4750	15000	5	50	5000
92	9.09	65	4375	15000	0	325	5000
93	9.28	55	5125	15000	10	425	5000
94	9.42	70	4000	15000	5	700	5000
95	9.78	75	5500	10000	10	800	10000
96	10.70	60	4375	15000	5	325	5000
97	10.79	55	4750	15000	10	50	5000
98	10.89	65	4000	15000	0	700	5000
99	11.13	70	5500	10000	5	800	10000
100	11.16	75	5125	10000	10	425	10000

		Actual Values			Potential α Values		
	Constraint ($Million)	AMPR Weight Factor (%)	Cost($)/lb		Constraint ($Million)	AMPR Weight Factor (%)	Cost($)/lb
101	12.31	55	4375	15000	10	325	5000
102	12.36	60	4000	15000	5	700	5000
103	12.42	70	5125	10000	5	425	10000
104	12.48	65	5500	10000	0	800	10000
105	12.54	75	4750	10000	10	50	10000
106	13.67	65	5125	10000	0	425	10000
107	13.70	70	4750	10000	5	50	10000
108	13.83	60	5500	10000	5	800	10000
109	13.83	55	4000	15000	10	700	5000
110	13.92	75	4375	10000	10	325	10000
111	14.87	65	4750	10000	0	50	10000
112	14.93	60	5125	10000	5	425	10000
113	14.99	70	4375	10000	5	325	10000
114	15.17	55	5500	10000	10	800	10000
115	15.30	75	4000	10000	10	700	10000
116	16.03	60	4750	10000	5	50	10000
117	16.06	65	4375	10000	0	325	10000
118	16.19	55	5125	10000	10	425	10000
119	16.28	70	4000	10000	5	700	10000
120	17.13	60	4375	10000	5	325	10000
121	17.20	55	4750	10000	10	50	10000
122	17.26	65	4000	10000	0	700	10000
123	18.21	55	4375	10000	10	325	10000
124	18.24	60	4000	10000	5	700	10000
125	19.22	55	4000	10000	10	700	10000

Table A.3 presents the normalized α values and the final combined α values. The values are normalized based on the maximum and minimum potential α values. The final combined α is shown in the yellow row (row 62).

Table A-3: Normalized α Values for Info-Gap Example Problem

	Potential α Values			Normalized Potential α Values			
	AMPR Weight Factor (%)	Cost($)/lb	TOGW (lbs)	AMPR Weight Factor (%)	Cost($)/lb	TOGW (lbs)	Potential Combined α
1	10	800	10000	100	100.00	100	173.21
2	5	800	10000	50	100.00	100	150.00
3	10	425	10000	100	50.00	100	150.00
4	5	425	10000	50	50.00	100	122.47
5	0	800	10000	0	100.00	100	141.42
6	10	50	10000	100	0.00	100	141.42

	Potential α Values			Normalized Potential α Values			
	AMPR Weight Factor (%)	Cost($)/lb	TOGW (lbs)		AMPR Weight Factor (%)	Cost($)/lb	TOGW (lbs)
7	10	800	5000	100	100.00	50	150.00
8	0	425	10000	0	50.00	100	111.80
9	5	50	10000	50	0.00	100	111.80
10	5	800	10000	50	100.00	100	150.00
11	10	325	10000	100	36.67	100	146.10
12	5	800	5000	50	100.00	50	122.47
13	10	425	5000	100	50.00	50	122.47
14	0	50	10000	0	0.00	100	100.00
15	5	425	10000	50	50.00	100	122.47
16	5	325	10000	50	36.67	100	117.66
17	10	800	10000	100	100.00	100	173.21
18	10	700	10000	100	86.67	100	165.86
19	5	425	5000	50	50.00	50	86.60
20	0	800	5000	0	100.00	50	111.80
21	10	50	5000	100	0.00	50	111.80
22	5	50	10000	50	0.00	100	111.80
23	0	325	10000	0	36.67	100	106.51
24	10	425	10000	100	50.00	100	150.00
25	5	700	10000	50	86.67	100	141.46
26	0	425	5000	0	50.00	50	70.71
27	5	50	5000	50	0.00	50	70.71
28	10	800	0	100	100.00	0	141.42
29	5	800	5000	50	100.00	50	122.47
30	10	325	5000	100	36.67	50	117.66
31	5	325	10000	50	36.67	100	117.66
32	10	50	10000	100	0.00	100	141.42
33	0	700	10000	0	86.67	100	132.33
34	0	50	5000	0	0.00	50	50.00
35	5	800	0	50	100.00	0	111.80
36	10	425	0	100	50.00	0	111.80
37	5	425	5000	50	50.00	50	86.60
38	5	325	5000	50	36.67	50	79.65
39	10	800	5000	100	100.00	50	150.00
40	10	700	5000	100	86.67	50	141.46
41	10	325	10000	100	36.67	100	146.10
42	5	700	10000	50	86.67	100	141.46
43	5	425	0	50	50.00	0	70.71
44	0	800	0	0	100.00	0	100.00
45	10	50	0	100	0.00	0	100.00
46	5	50	5000	50	0.00	50	70.71
47	0	325	5000	0	36.67	50	62.00
48	10	425	5000	100	50.00	50	122.47
49	5	700	5000	50	86.67	50	111.85
50	0	425	0	0	50.00	0	50.00

	Potential α Values			Normalized Potential α Values			
	AMPR Weight Factor (%)	Cost($)/lb	TOGW (lbs)	AMPR Weight Factor (%)	Cost($)/lb	TOGW (lbs)	Potential Combined α
51	5	50	0	50	0.00	0	50.00
52	5	800	0	50	100.00	0	111.80
53	10	700	10000	100	86.67	100	165.86
54	10	325	0	100	36.67	0	106.51
55	5	325	5000	50	36.67	50	79.65
56	10	50	5000	100	0.00	50	111.80
57	0	700	5000	0	86.67	50	100.06
58	10	800	5000	100	100.00	50	150.00
59	0	50	0	0	0.00	0	0.00
60	5	425	0	50	50.00	0	70.71
61	5	325	0	50	36.67	0	62.00
62	10	800	0	100	100.00	0	141.42
63	10	325	5000	100	36.67	50	117.66
64	10	700	0	100	86.67	0	132.33
65	5	700	5000	50	86.67	50	111.85
66	5	800	5000	50	100.00	50	122.47
67	10	425	5000	100	50.00	50	122.47
68	5	50	0	50	0.00	0	50.00
69	0	325	0	0	36.67	0	36.67
70	10	425	0	100	50.00	0	111.80
71	5	700	0	50	86.67	0	100.06
72	10	700	5000	100	86.67	50	141.46
73	5	425	5000	50	50.00	50	86.60
74	0	800	5000	0	100.00	50	111.80
75	10	50	5000	100	0.00	50	111.80
76	5	325	0	50	36.67	0	62.00
77	10	50	0	100	0.00	0	100.00
78	0	700	0	0	86.67	0	86.67
79	0	425	5000	0	50.00	50	70.71
80	5	50	5000	50	0.00	50	70.71
81	5	800	5000	50	100.00	50	122.47
82	10	325	5000	100	36.67	50	117.66
83	10	325	0	100	36.67	0	106.51
84	5	700	0	50	86.67	0	100.06
85	0	50	5000	0	0.00	50	50.00
86	5	425	5000	50	50.00	50	86.60
87	5	325	5000	50	36.67	50	79.65
88	10	800	5000	100	100.00	50	150.00
89	10	700	5000	100	86.67	50	141.46
90	10	700	0	100	86.67	0	132.33
91	5	50	5000	50	0.00	50	70.71
92	0	325	5000	0	36.67	50	62.00
93	10	425	5000	100	50.00	50	122.47
94	5	700	5000	50	86.67	50	111.85

	Potential α Values			Normalized Potential α Values			
	AMPR Weight Factor (%)	Cost($)/lb	TOGW (lbs)		AMPR Weight Factor (%)	Cost($)/lb	TOGW (lbs)
95	10	800	10000	100	100.00	100	173.21
96	5	325	5000	50	36.67	50	79.65
97	10	50	5000	100	0.00	50	111.80
98	0	700	5000	0	86.67	50	100.06
99	5	800	10000	50	100.00	100	150.00
100	10	425	10000	100	50.00	100	150.00
101	10	325	5000	100	36.67	50	117.66
102	5	700	5000	50	86.67	50	111.85
103	5	425	10000	50	50.00	100	122.47
104	0	800	10000	0	100.00	100	141.42
105	10	50	10000	100	0.00	100	141.42
106	0	425	10000	0	50.00	100	111.80
107	5	50	10000	50	0.00	100	111.80
108	5	800	10000	50	100.00	100	150.00
109	10	700	5000	100	86.67	50	141.46
110	10	325	10000	100	36.67	100	146.10
111	0	50	10000	0	0.00	100	100.00
112	5	425	10000	50	50.00	100	122.47
113	5	325	10000	50	36.67	100	117.66
114	10	800	10000	100	100.00	100	173.21
115	10	700	10000	100	86.67	100	165.86
116	5	50	10000	50	0.00	100	111.80
117	0	325	10000	0	36.67	100	106.51
118	10	425	10000	100	50.00	100	150.00
119	5	700	10000	50	86.67	100	141.46
120	5	325	10000	50	36.67	100	117.66
121	10	50	10000	100	0.00	100	141.42
122	0	700	10000	0	86.67	100	132.33
123	10	325	10000	100	36.67	100	146.10
124	5	700	10000	50	86.67	100	141.46
125	10	700	10000	100	86.67	100	165.86

APPENDIX B

The charts in this appendix are the results from the six example problems in Chapter 8.

The three different design approaches are designated as follows:

RandO for Robust and Opportunistic Design

Ronly for Robust Design

OppOnly for Opportunistic Design

The different weight groups are designated by W1, W2, W3, and W4 as applicable.

Example A: Simple equation, 1 metric, constant constraint and desirement, complementary constraint and desirement

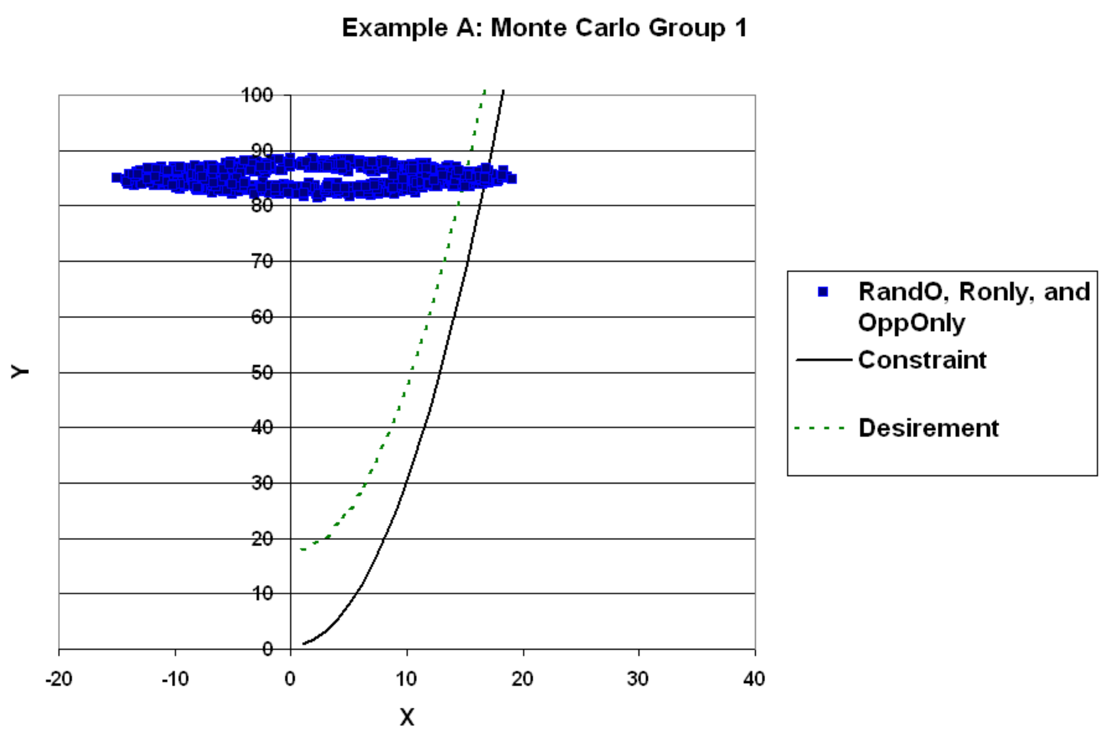

Figure B-1: Example A Monte Carlo Data - Uncertainty Group 1

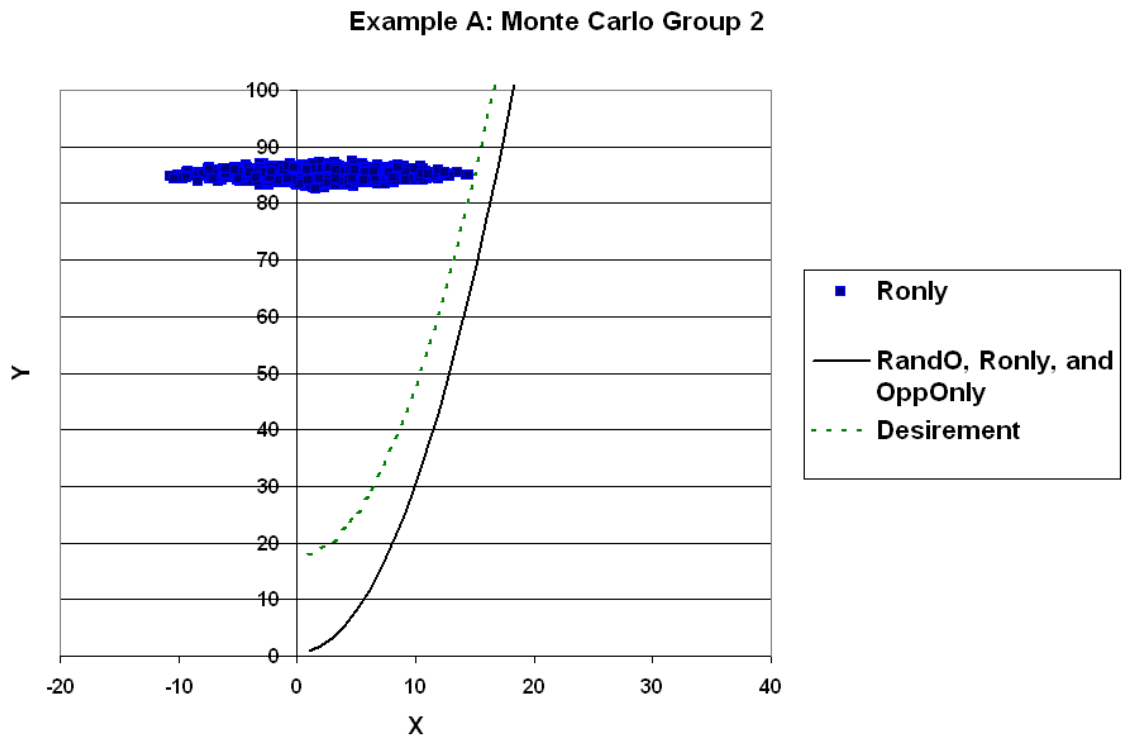

Figure B-2: Example A Monte Carlo Data - Uncertainty Group 2

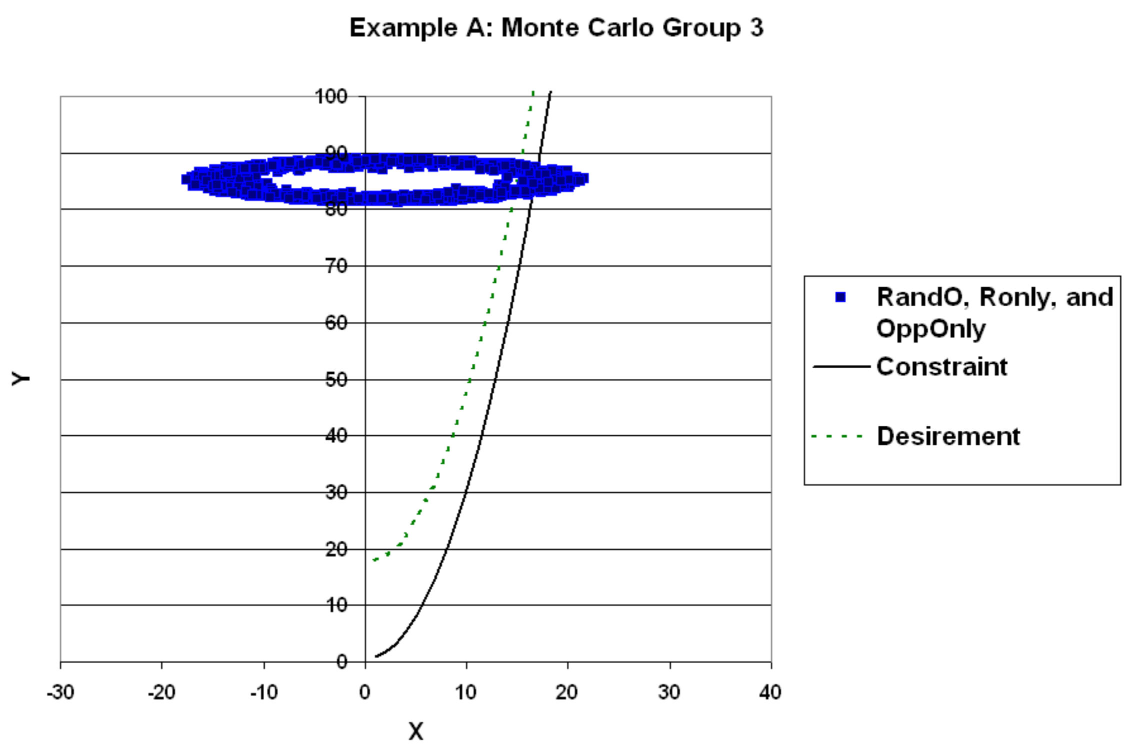

Figure B-3: Example A Monte Carlo Data - Uncertainty Group 3

Example B: Simple equation, 1 metric, constant constraint and desirement, competing constraint and desirement

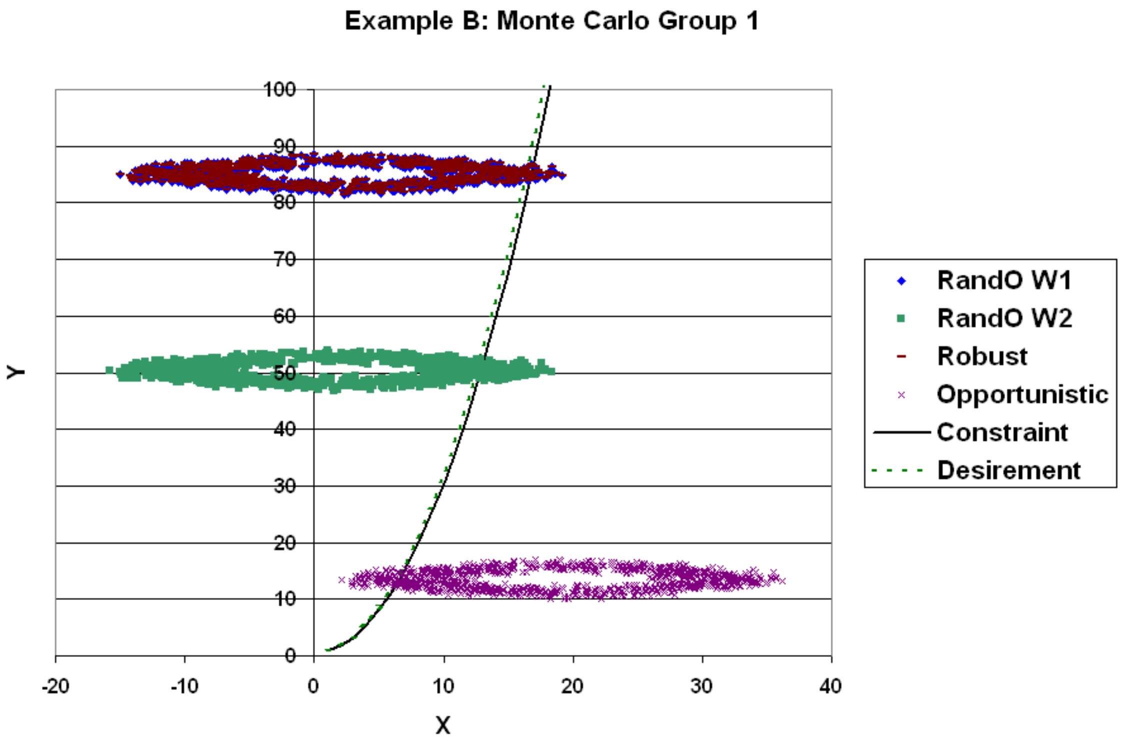

Figure B-4: Example B Monte Carlo Data - Uncertainty Group 1

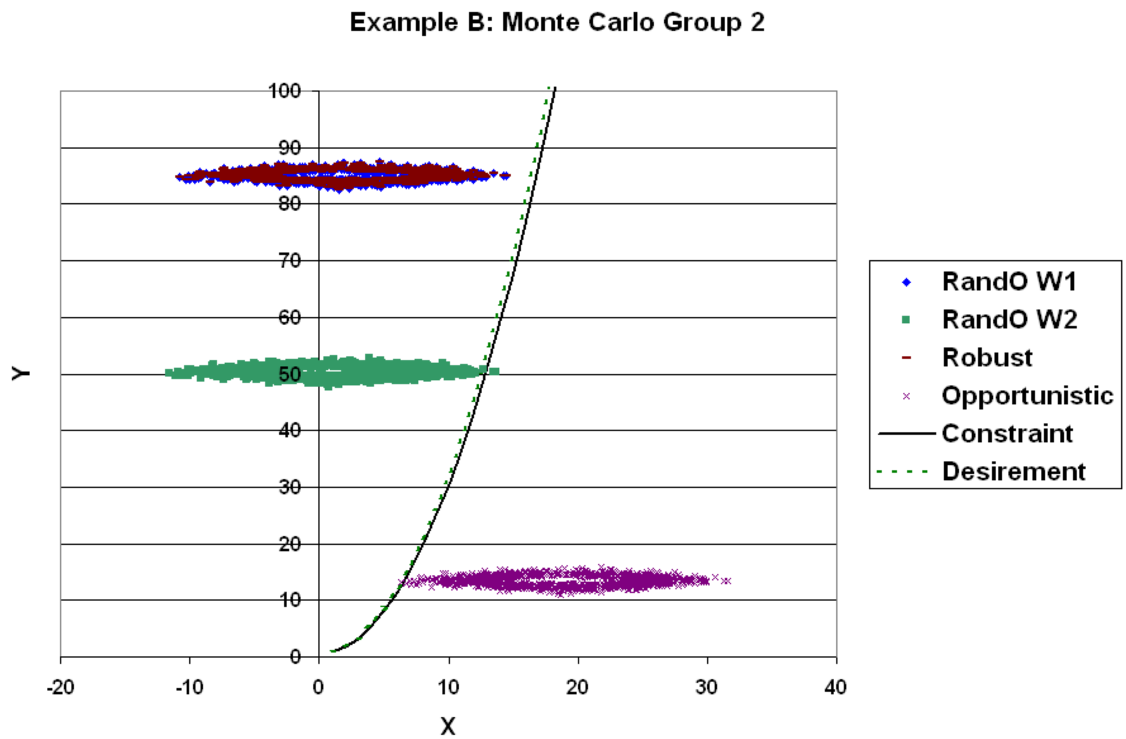

Figure B-5: Example B Monte Carlo Data - Uncertainty Group 2

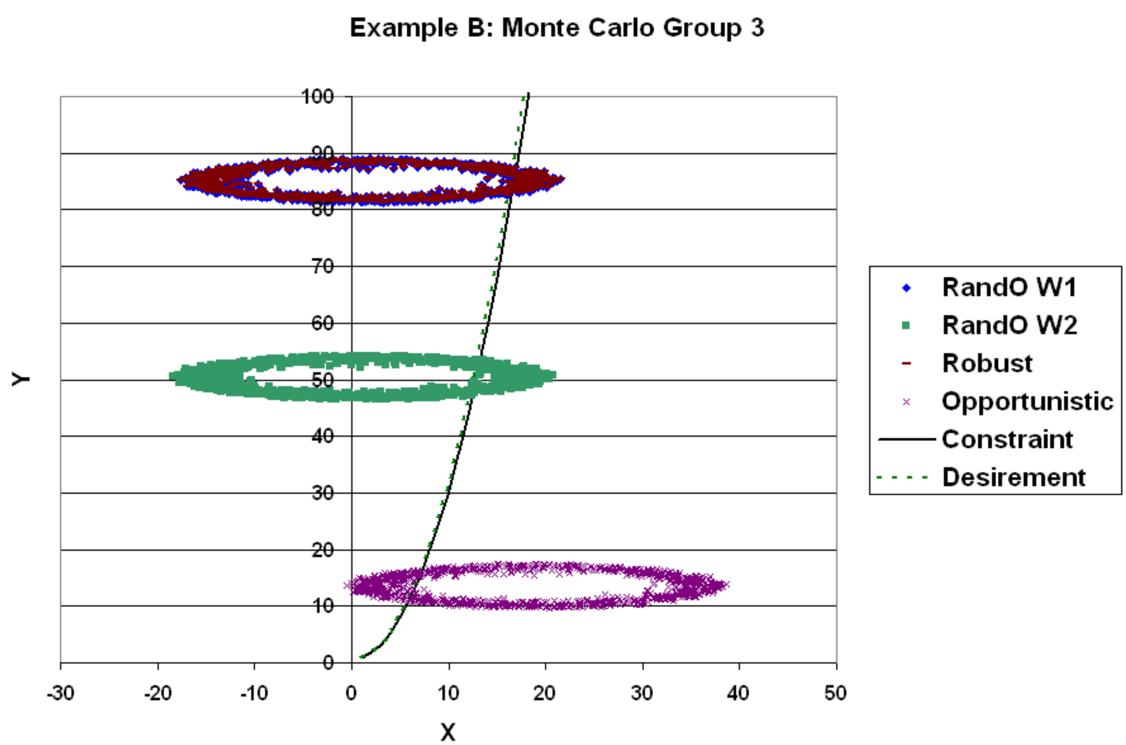

Figure B-6: Example B Monte Carlo Data - Uncertainty Group 3

<u>Example C: Simple equation, 1 metric, variable constraint and desirement, complementary constraint and desirement</u>

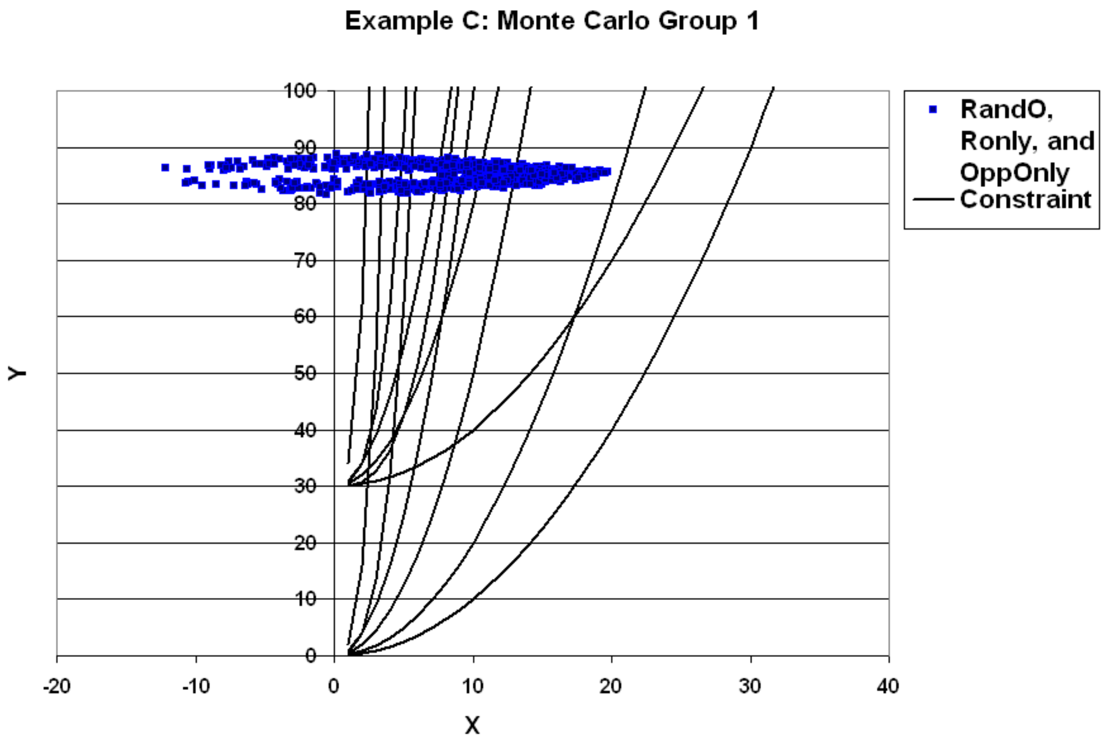

Figure B-7: Example C Monte Carlo Data - Uncertainty Group 1

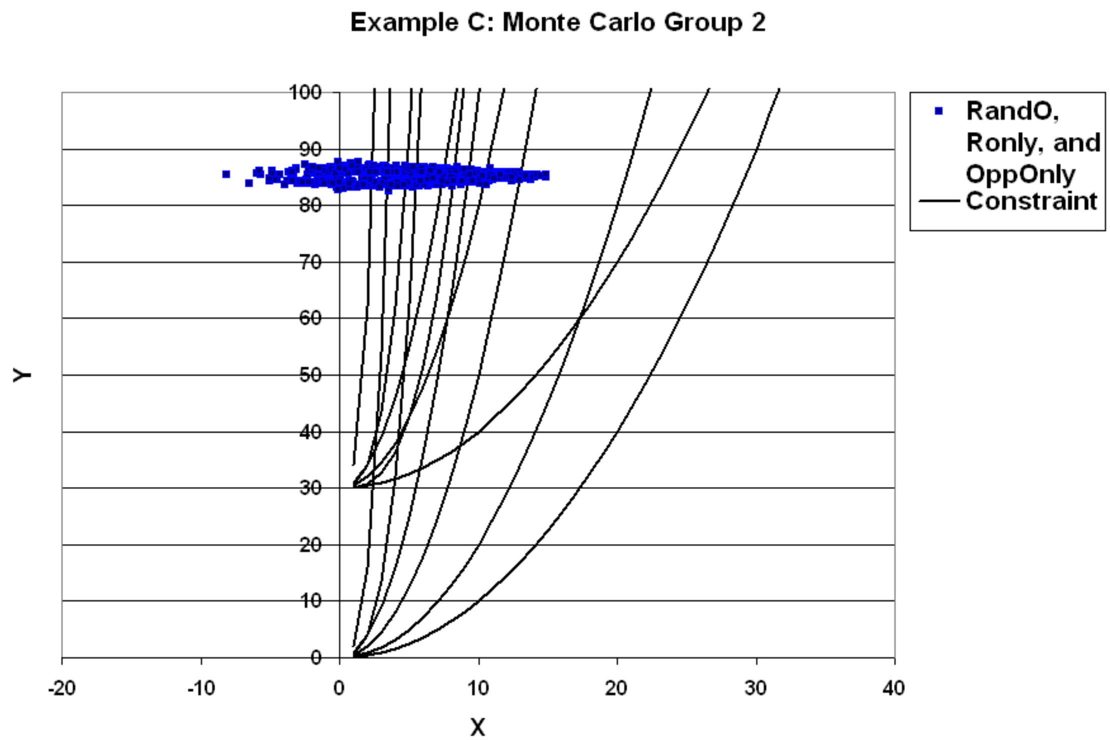

Figure B-8: Example C Monte Carlo Data - Uncertainty Group 2

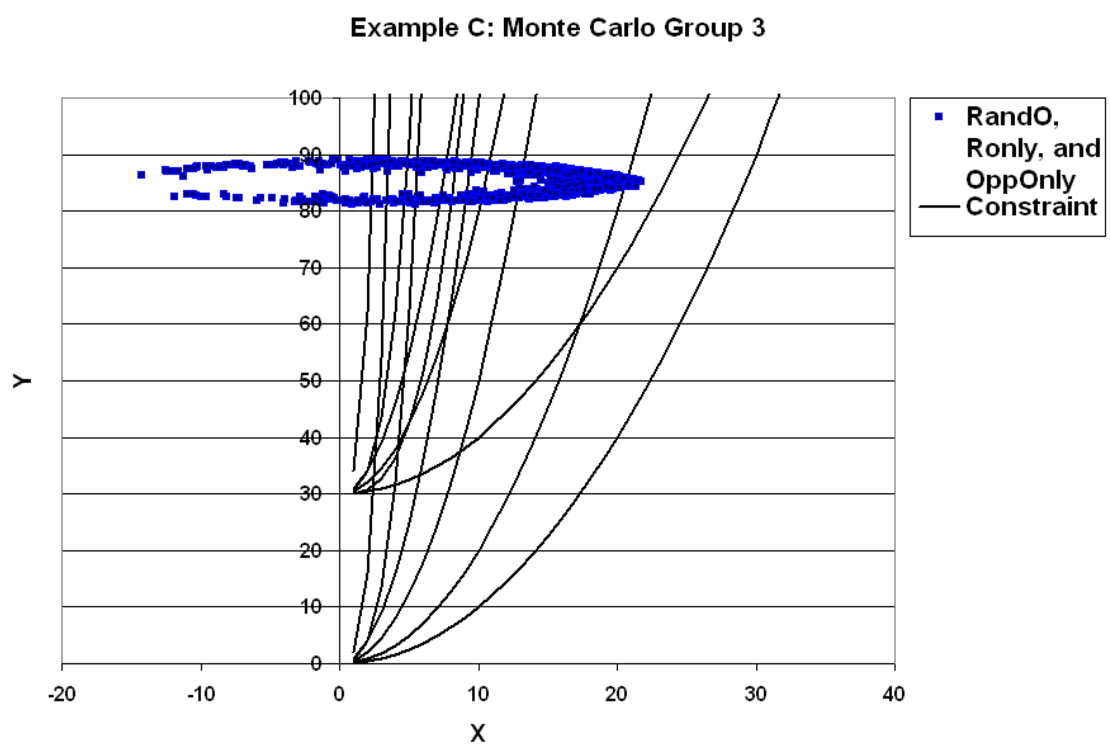

Figure B-9: Example C Monte Carlo Data - Uncertainty Group 3

Example D: Simple equation, 1 metric, variable constraint and desirement, competing constraint and desirement

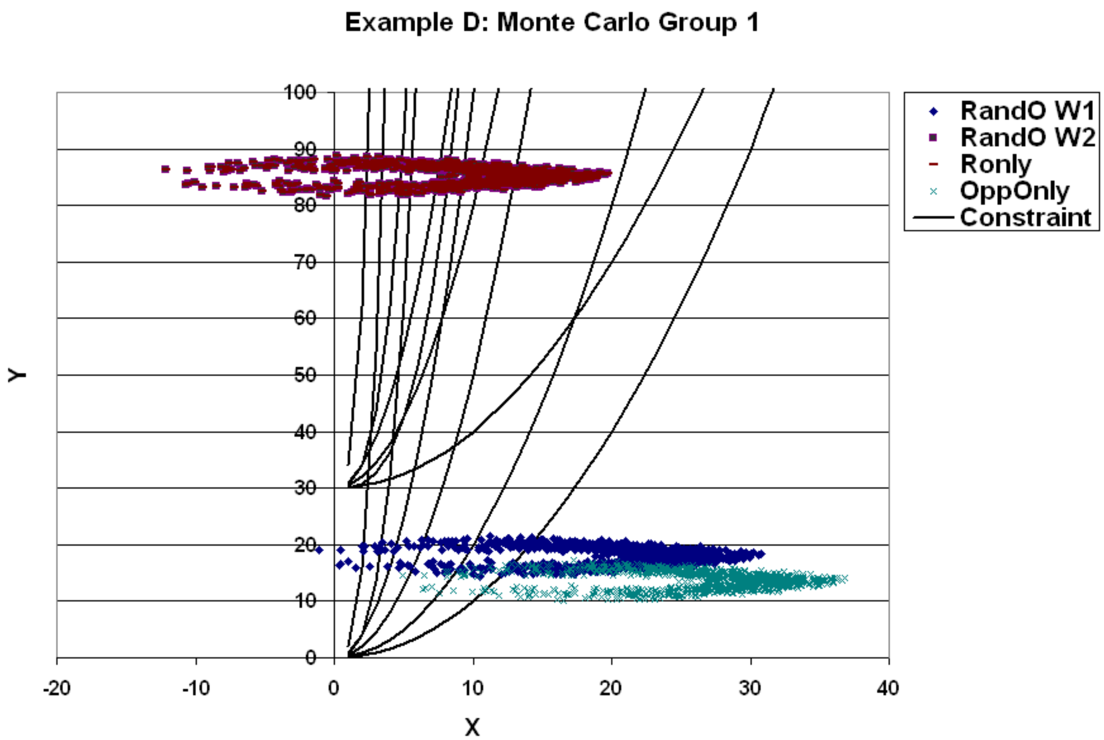

Figure B-10: Example D Monte Carlo Data - Uncertainty Group 1

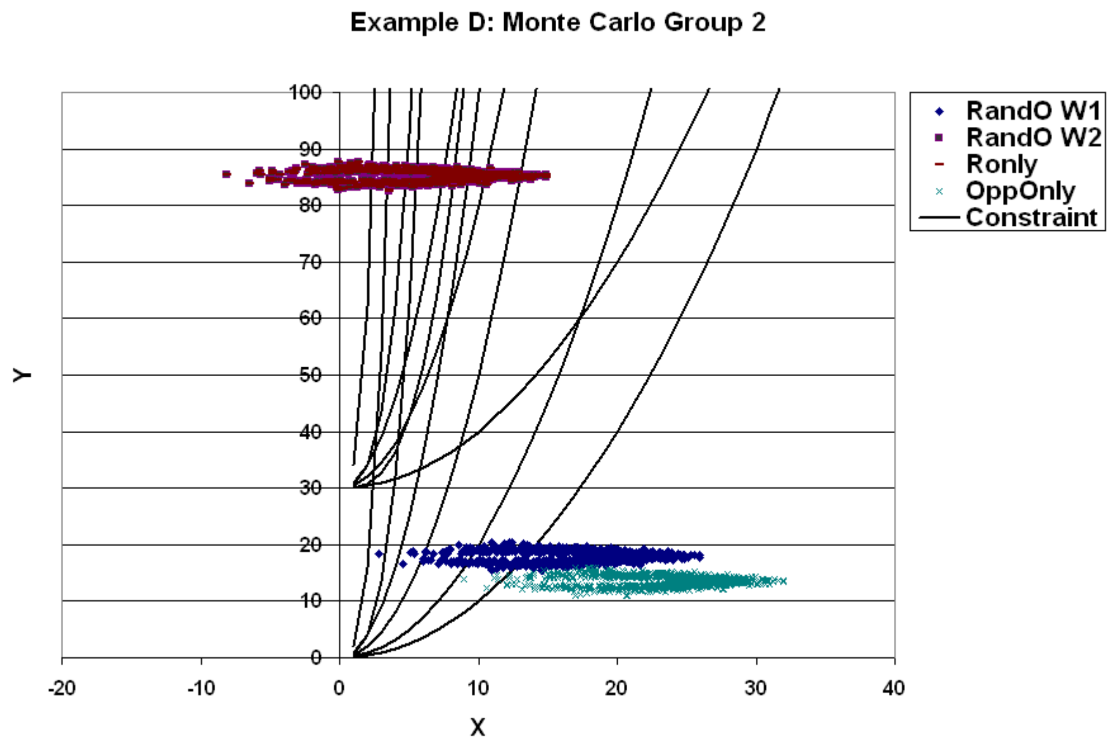

Figure B-11: Example D Monte Carlo Data - Uncertainty Group 2

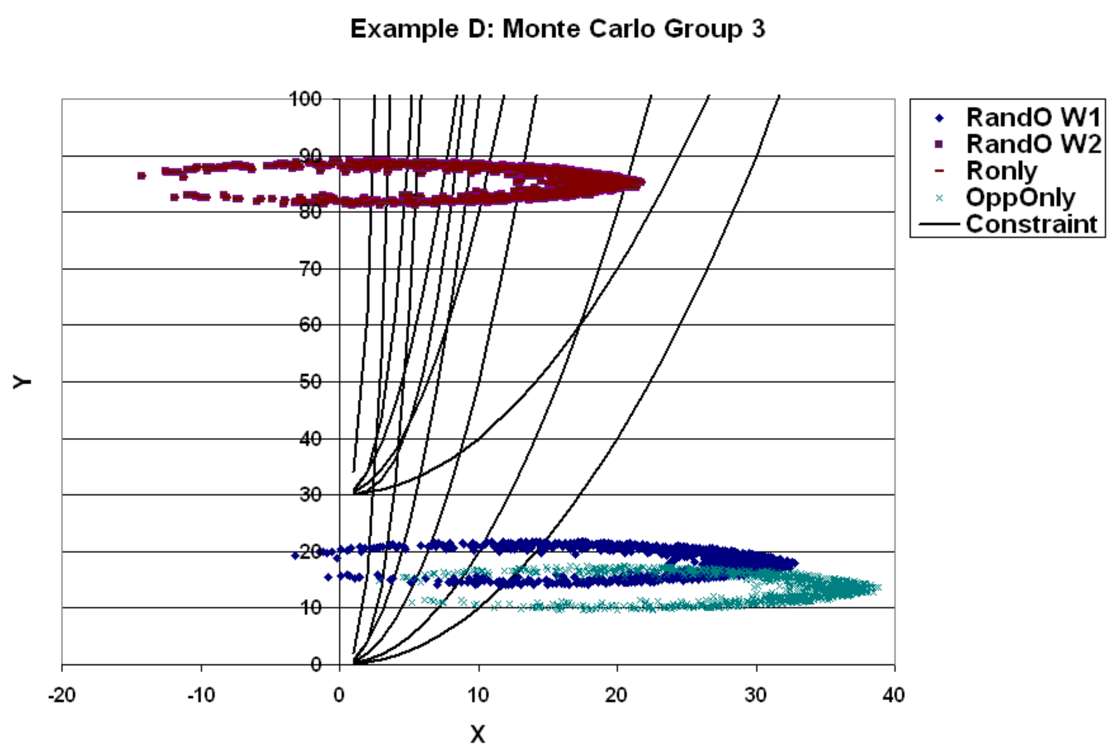

Figure B-12: Example D Monte Carlo Data - Uncertainty Group 3

Example E: Aircraft Design example

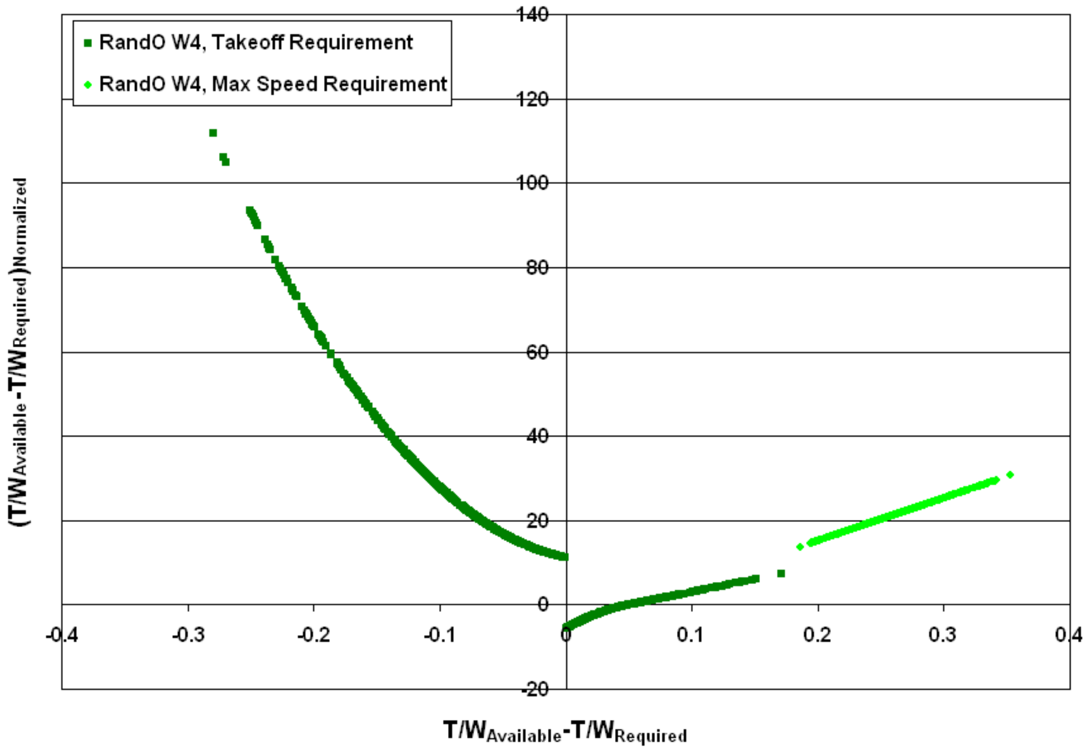

Figure B-13: Comparison of Normalized T/W Metric values versus Actual T/W values from Example E Monte Carlo Data - Uncertainty Group 1

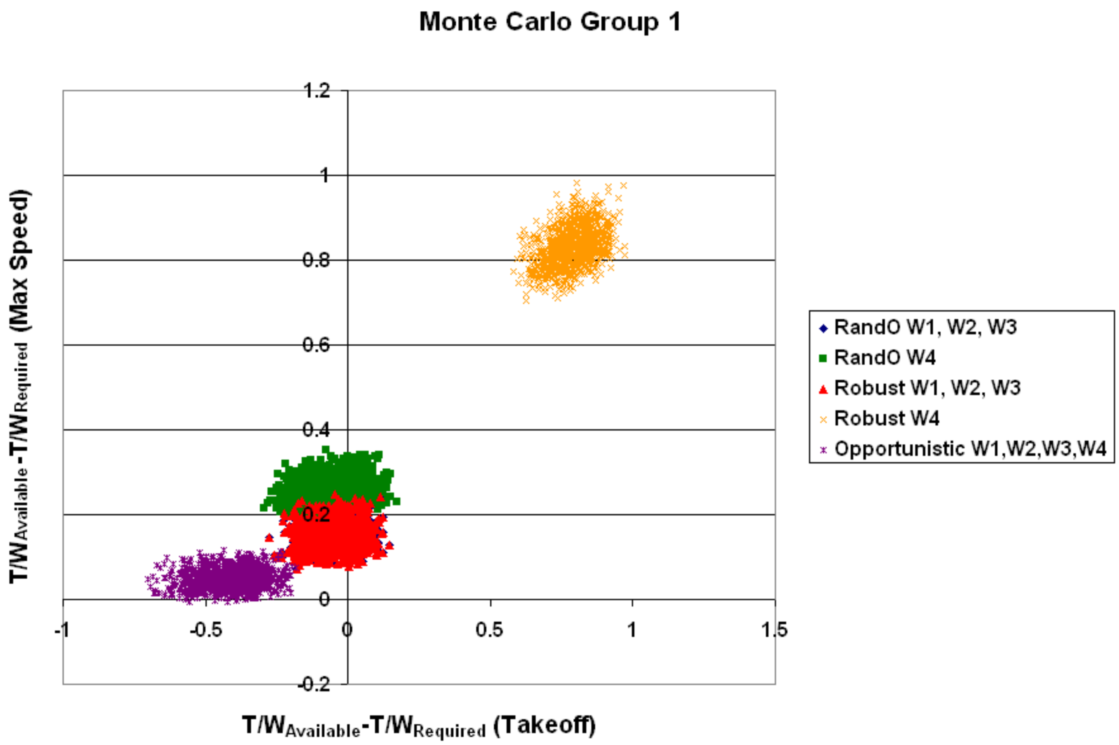

Figure B-14: Example E Monte Carlo Data for Actual T/W Metrics - Uncertainty Group 1

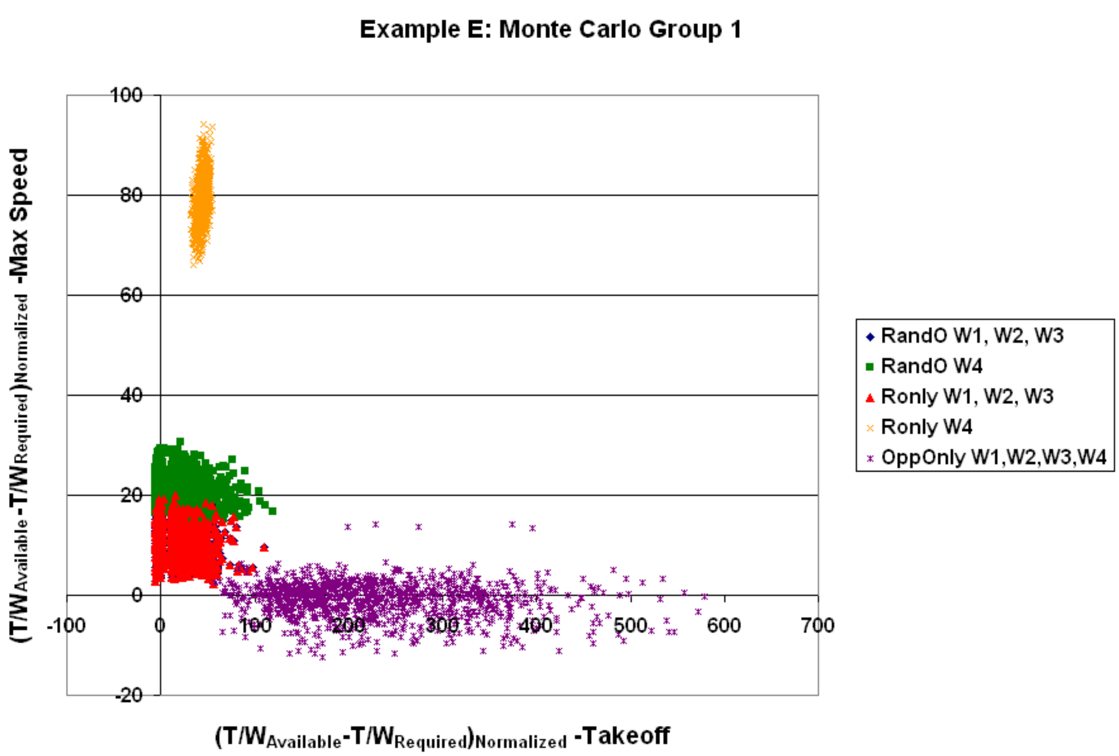

Figure B-15: Example E Monte Carlo Data for Normalized T/W Metrics - Uncertainty Group 1

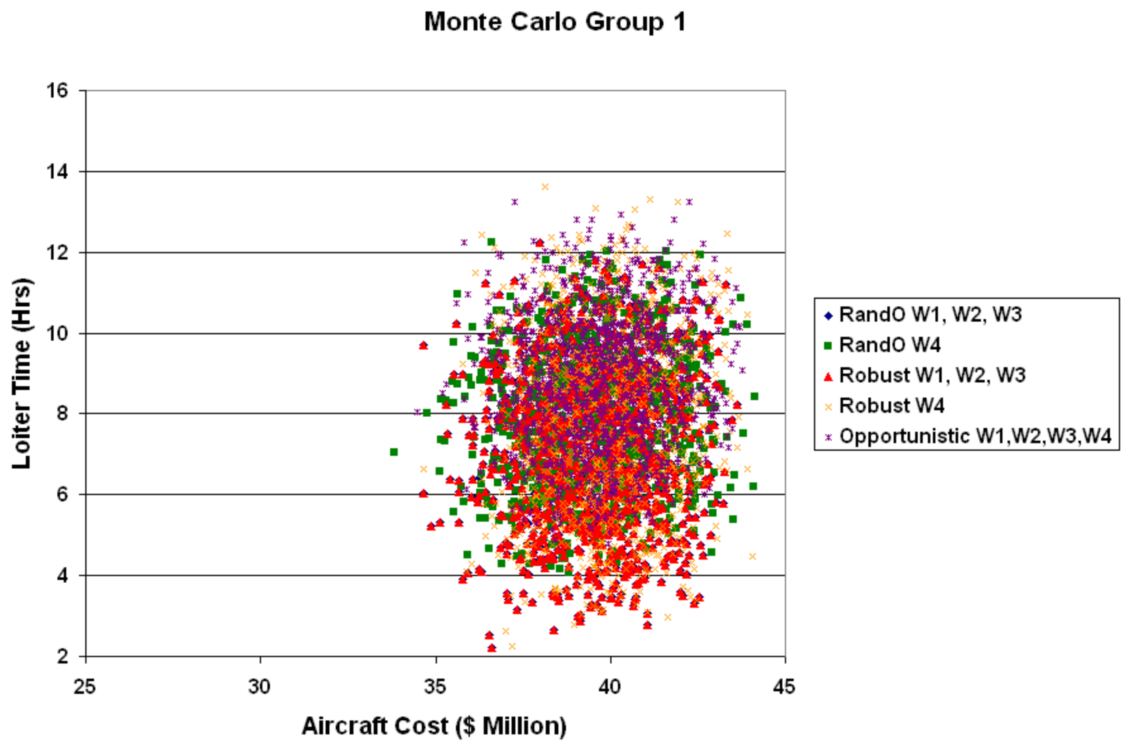

Figure B-16: Example E Monte Carlo Data for Actual Cost and LT Metrics - Uncertainty Group 1

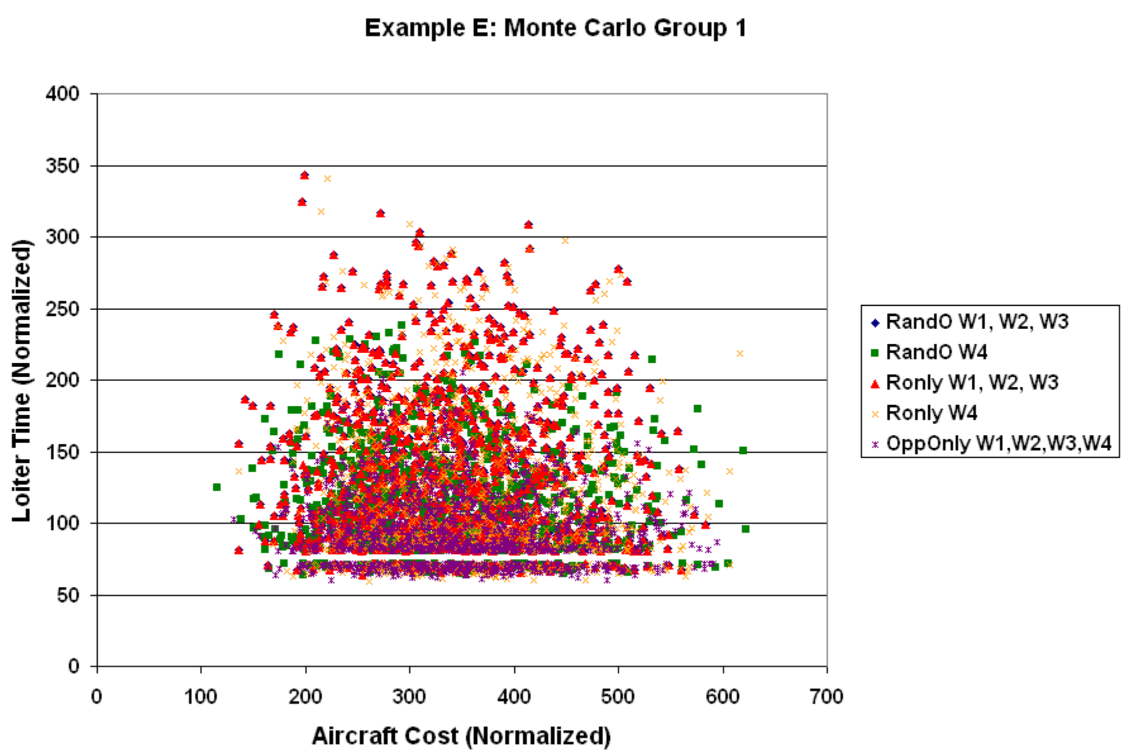

Figure B-17: Example E Monte Carlo Data for Normalized Cost and LT Metrics - Uncertainty Group 1

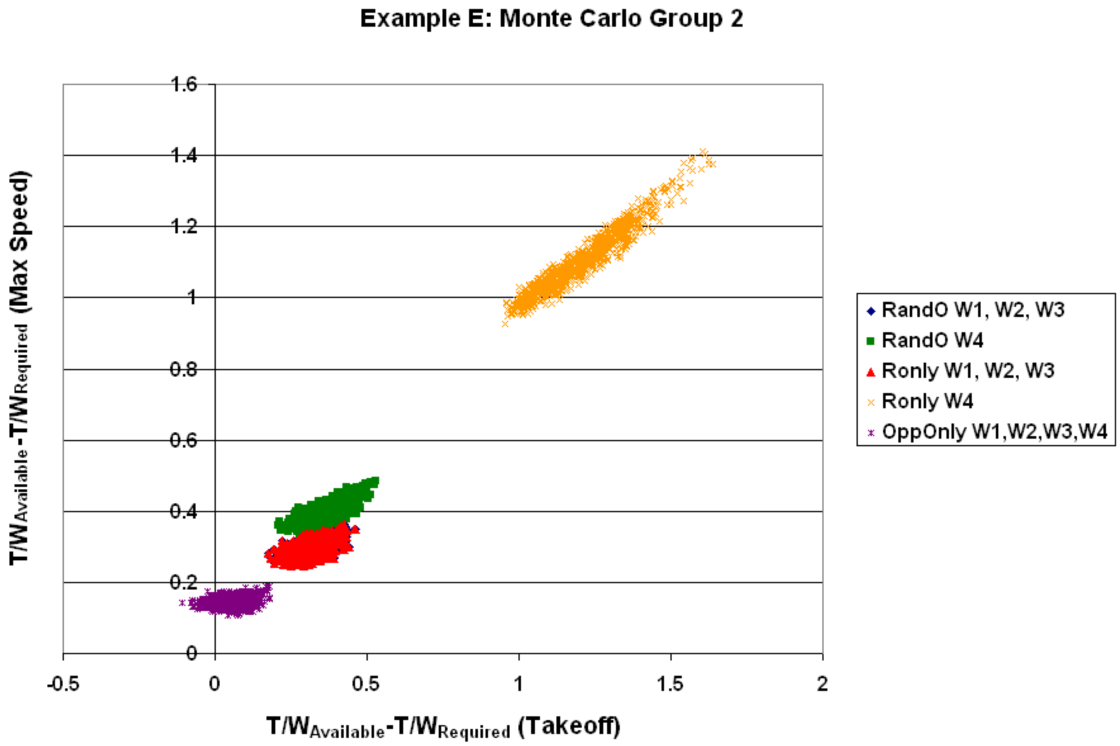

Figure B-18: Example E Monte Carlo Data for Actual T/W Metrics - Uncertainty Group 2

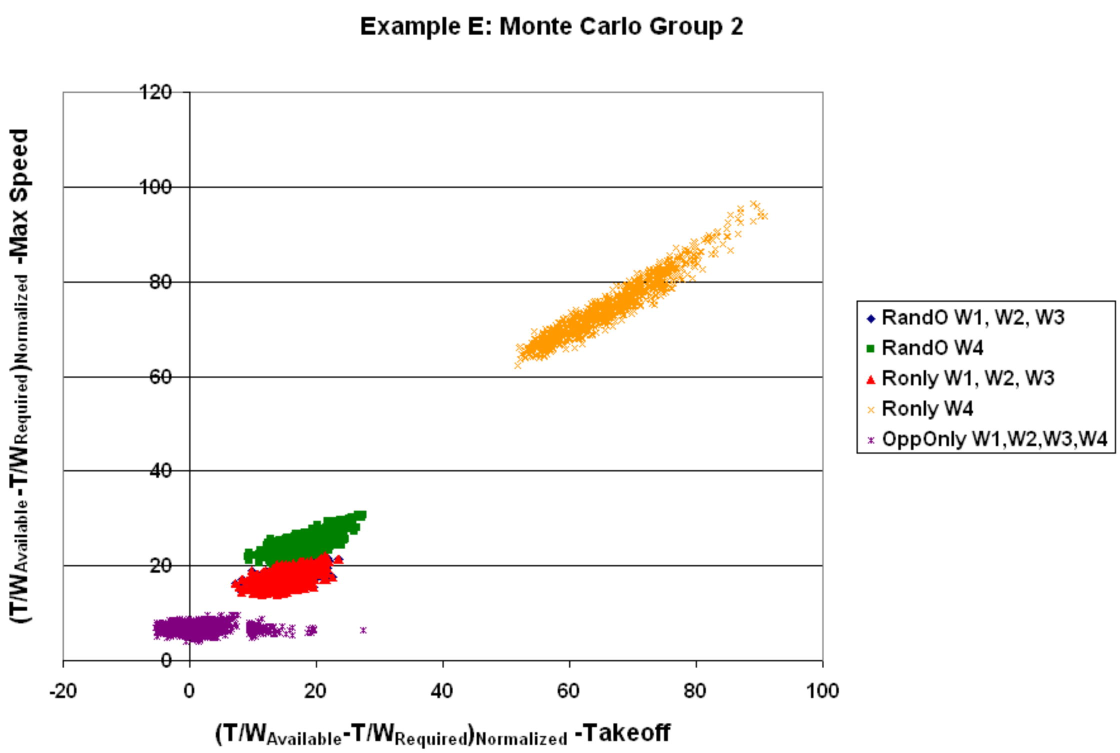

Figure B-19: Example E Monte Carlo Data for Normalized T/W Metrics - Uncertainty Group 2

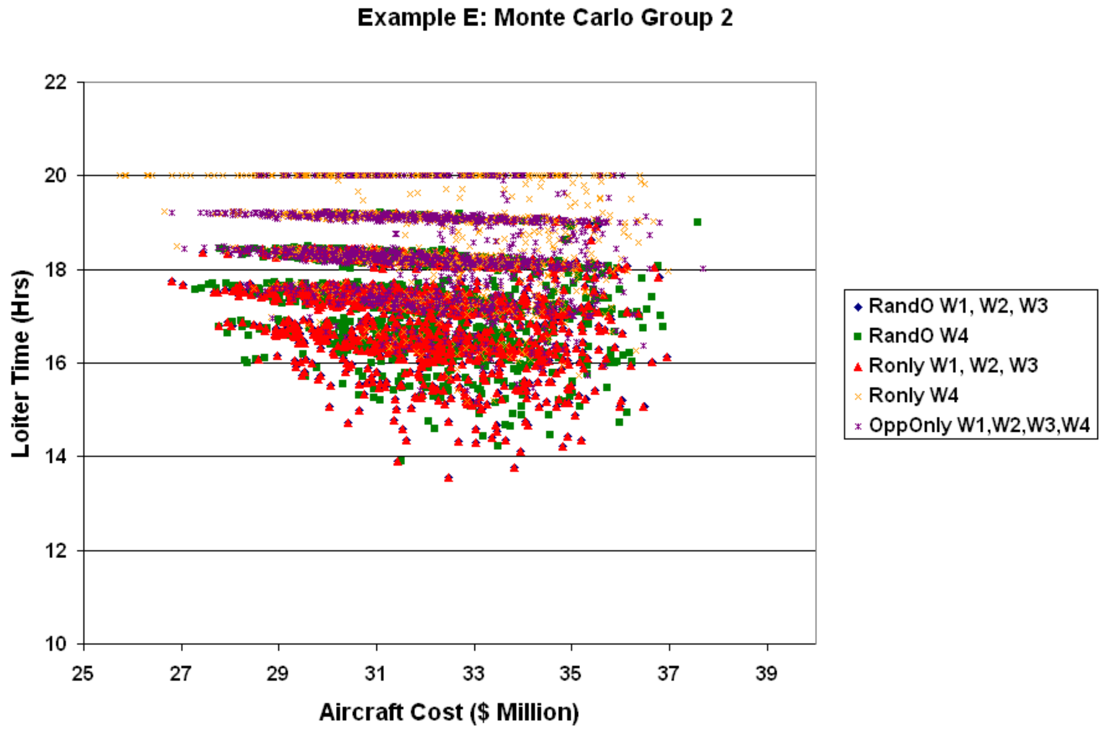

Figure B-20: Example E Monte Carlo Data for Actual Cost and LT Metrics - Uncertainty Group 2

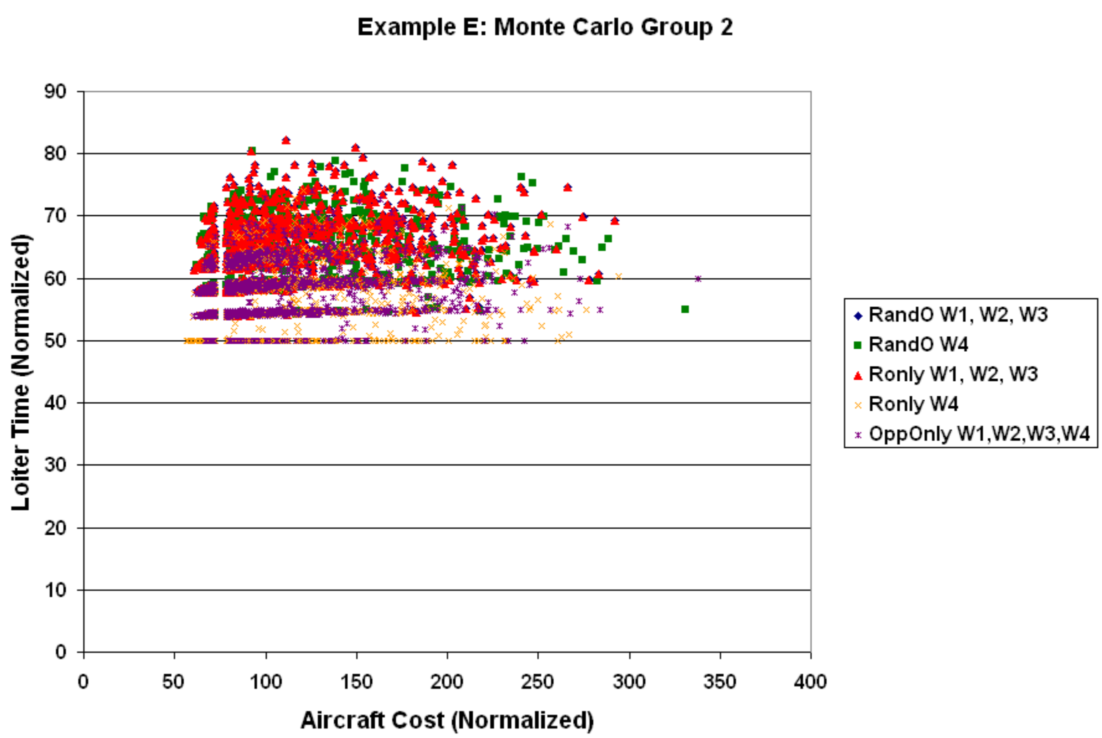

Figure B-21: Example E Monte Carlo Data for Normalized Cost and LT Metrics - Uncertainty Group 2

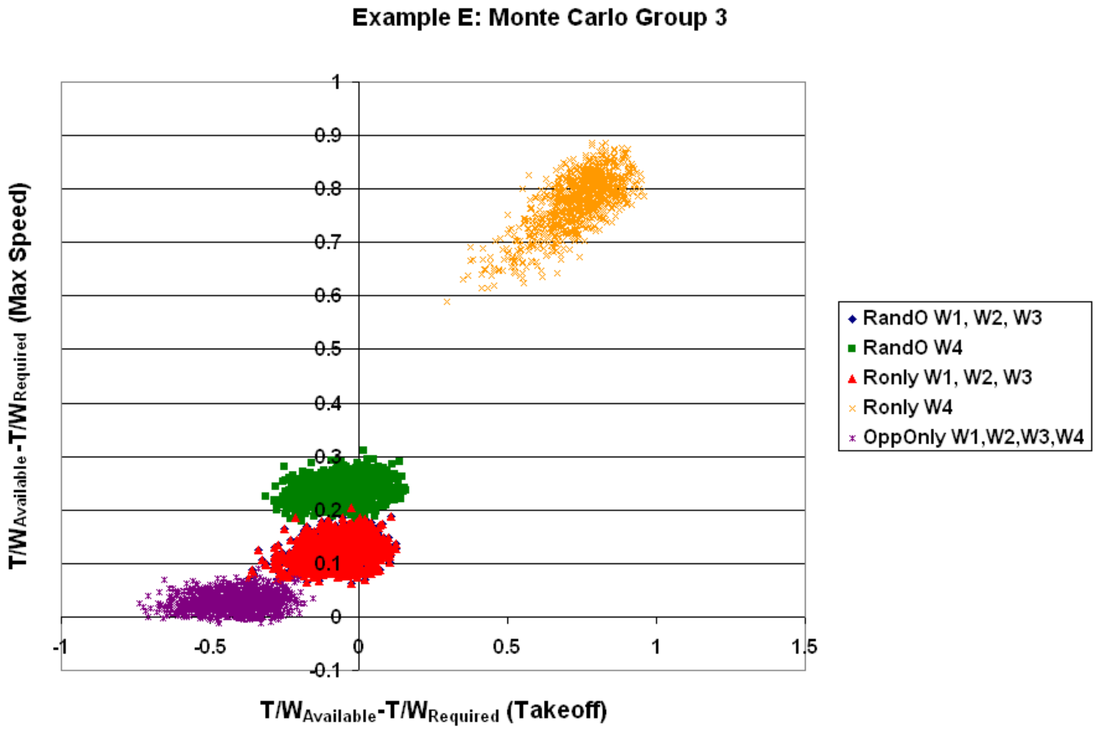

Figure B-22: Example E Monte Carlo Data for Actual T/W Metrics - Uncertainty Group 3

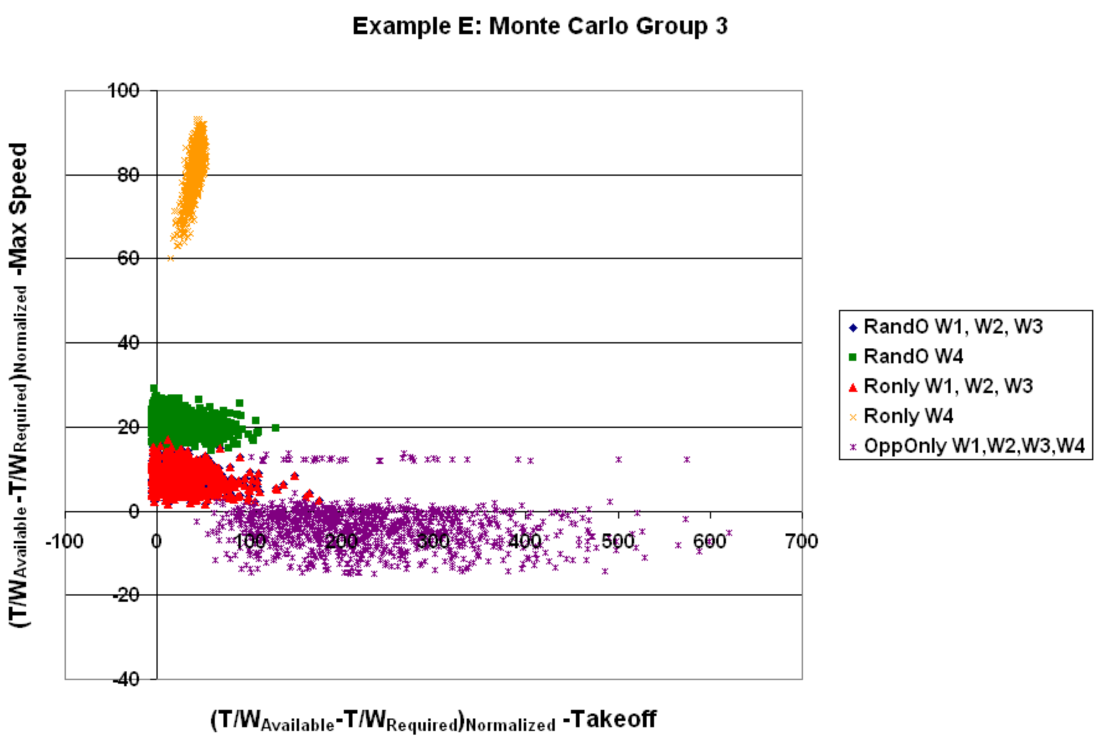

Figure B-23: Example E Monte Carlo Data for Normalized T/W Metrics - Uncertainty Group 3

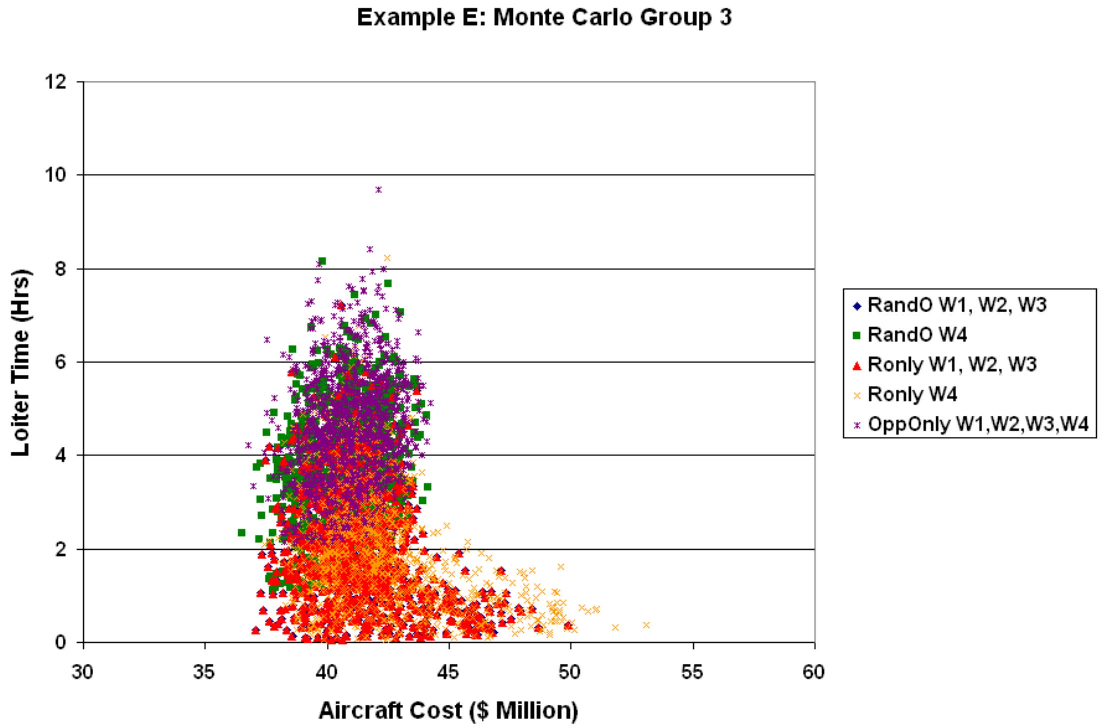

Figure B-24: Example E Monte Carlo Data for Actual Cost and LT Metrics - Uncertainty Group 3

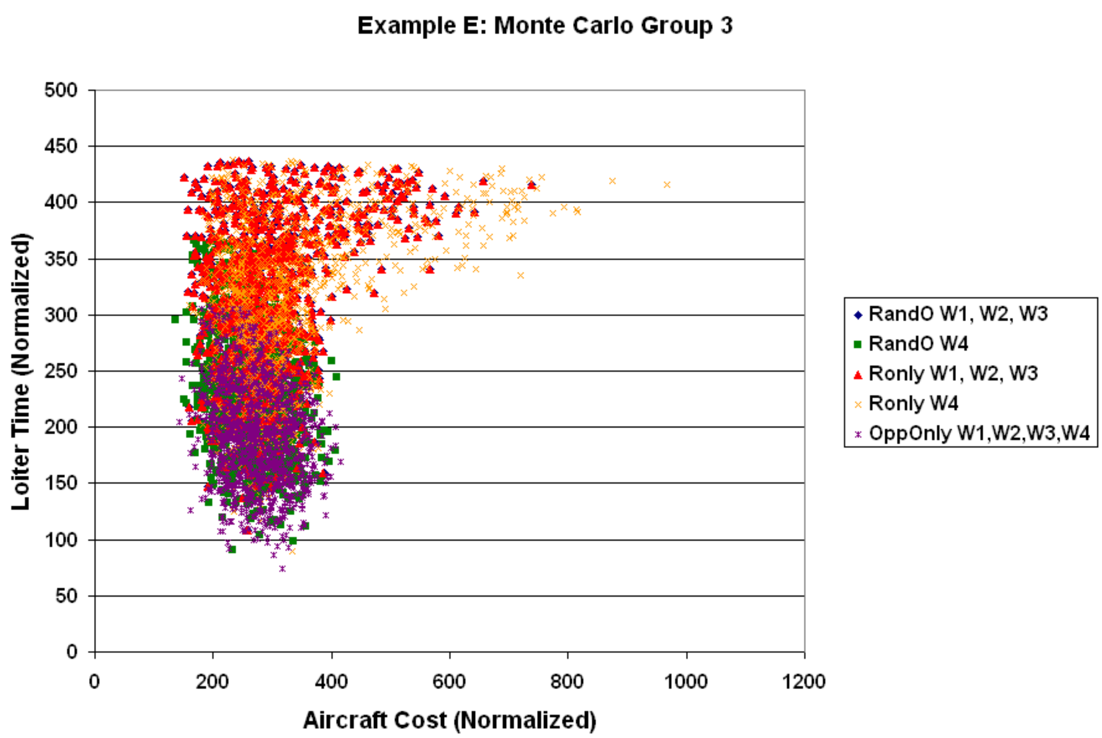

Figure B-25: Example E Monte Carlo Data for Normalized Cost and LT Metrics - Uncertainty Group 3

Example F: Fleet Design example

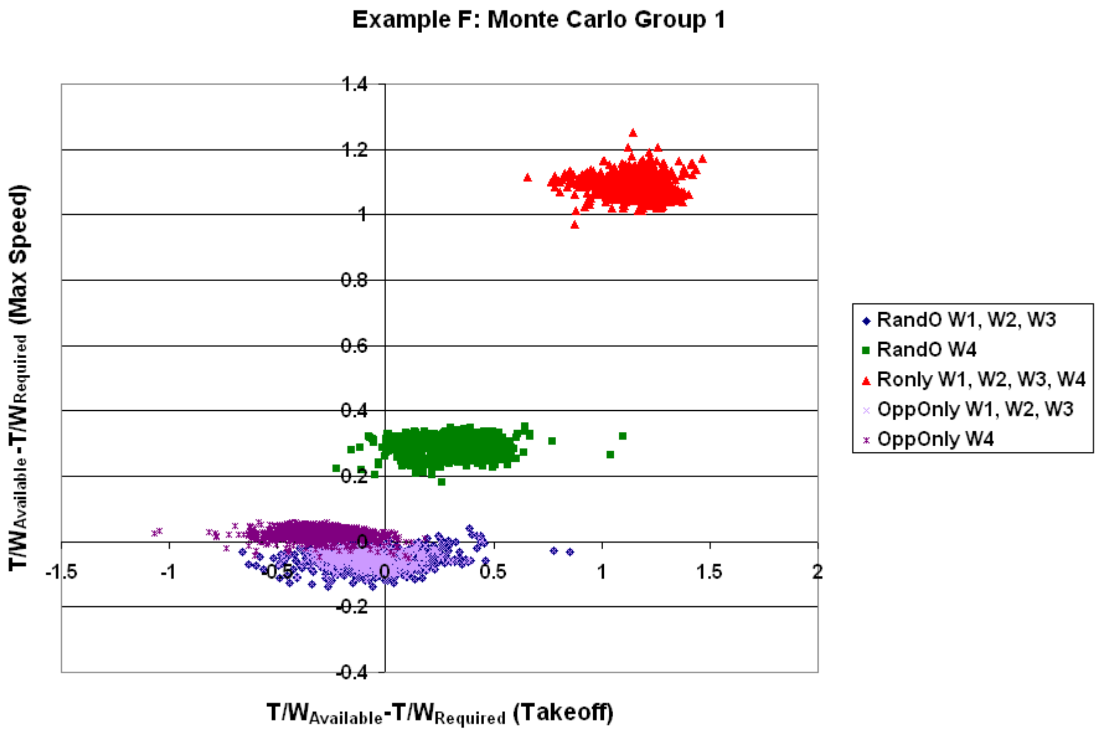

Figure B-26: Example F Monte Carlo Data for Actual T/W Metrics - Uncertainty Group 1

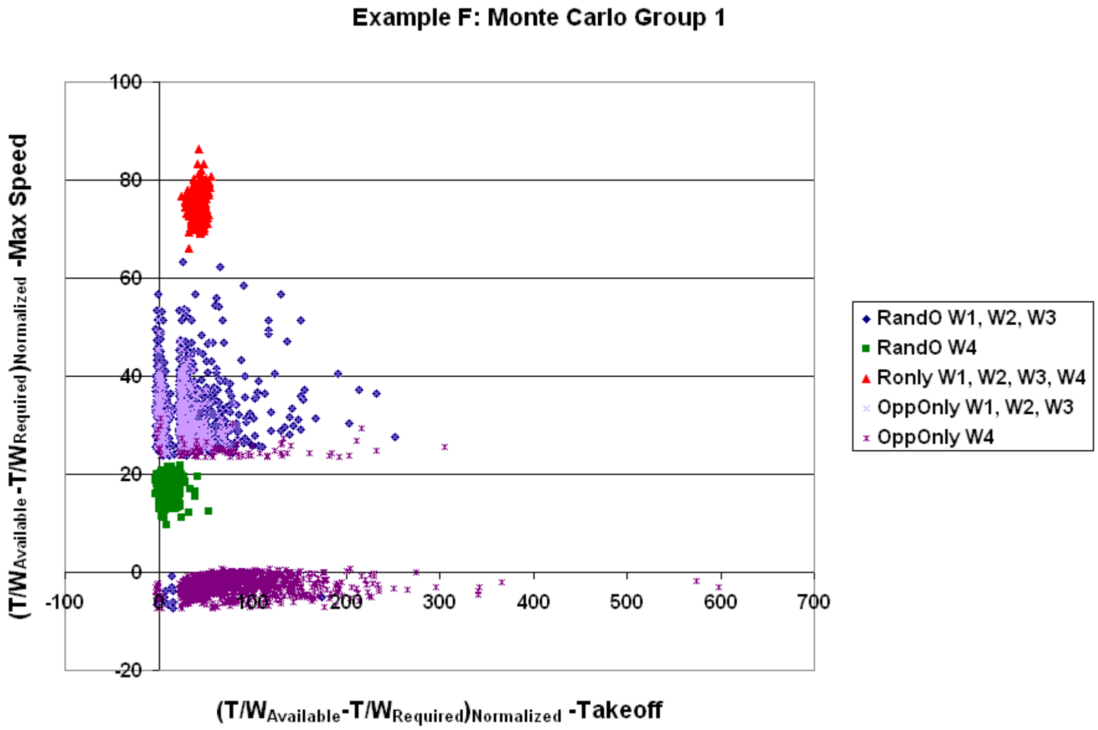

Figure B-27: Example F Monte Carlo Data for Normalized T/W Metrics - Uncertainty Group 1

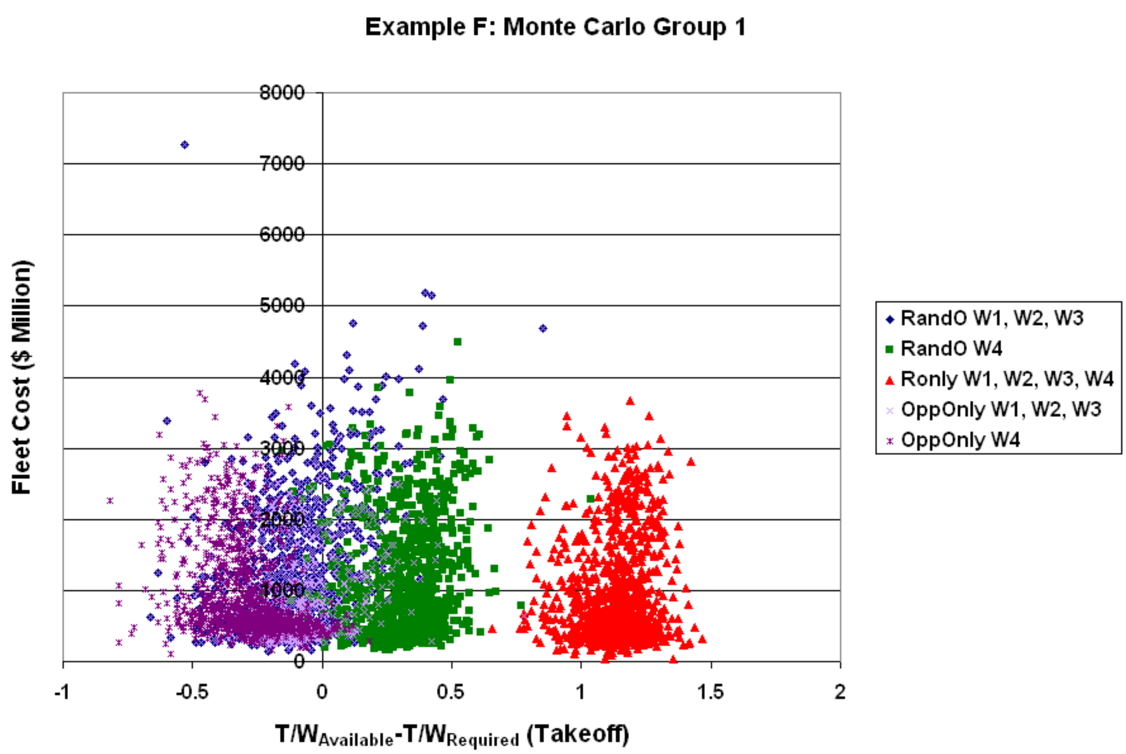

Figure B-28: Example F Monte Carlo Data for Actual Fleet Cost Metric - Uncertainty Group 1

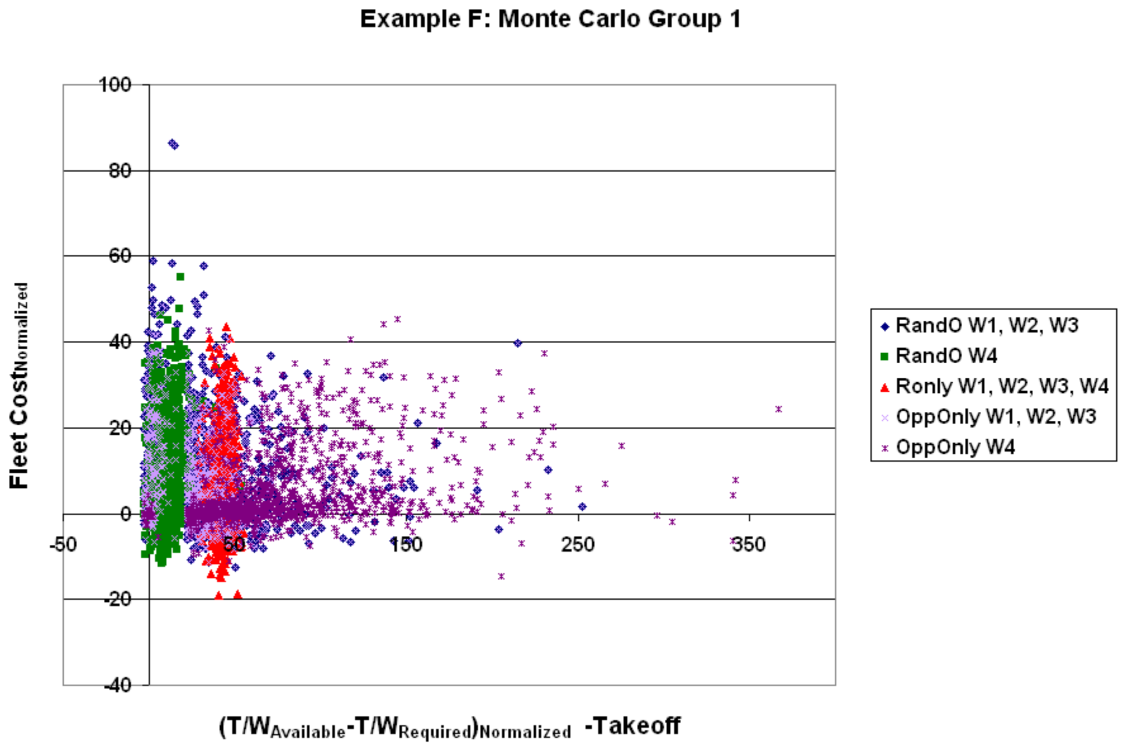

Figure B-29: Example F Monte Carlo Data for Normalized Fleet Cost Metric - Uncertainty Group 1

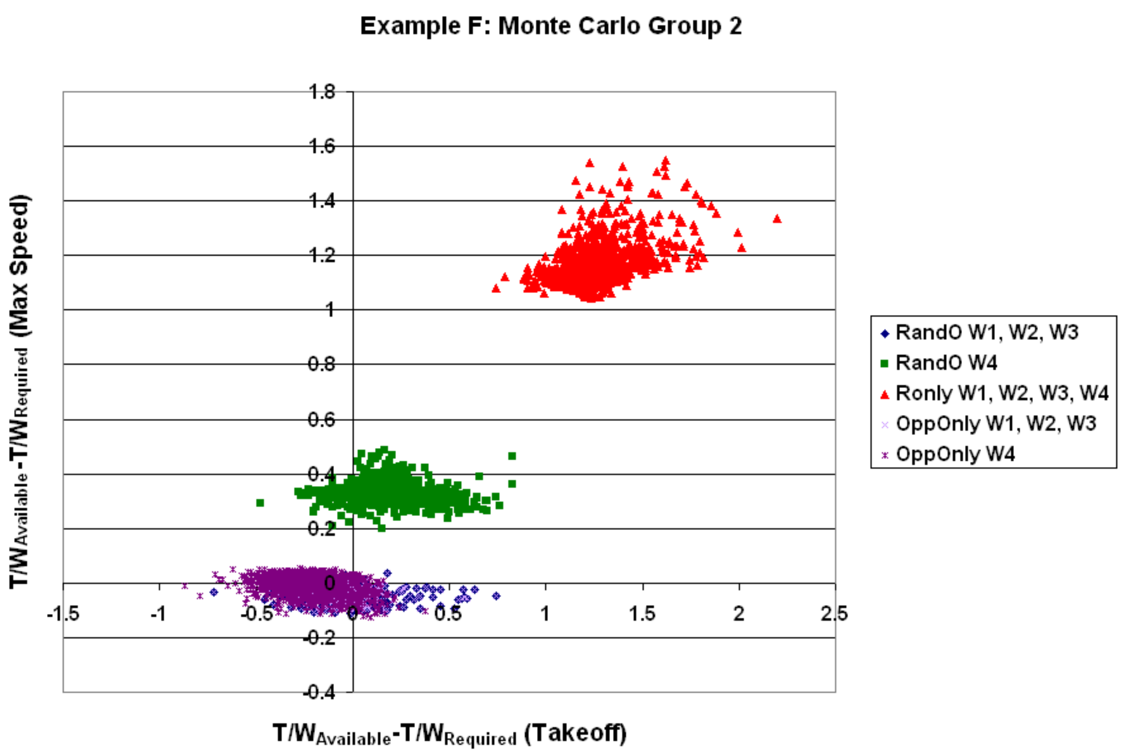

Figure B-30: Example F Monte Carlo Data for Actual T/W Metrics - Uncertainty Group 2

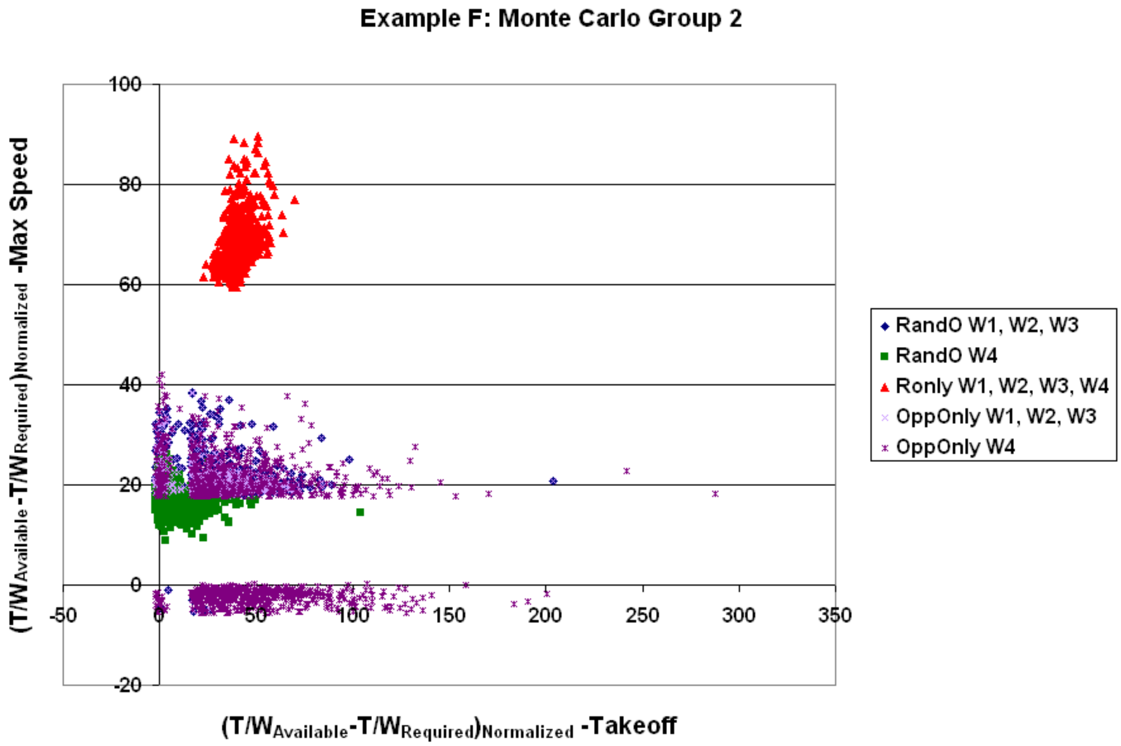

Figure B-31: Example F Monte Carlo Data for Normalized T/W Metrics - Uncertainty Group 2

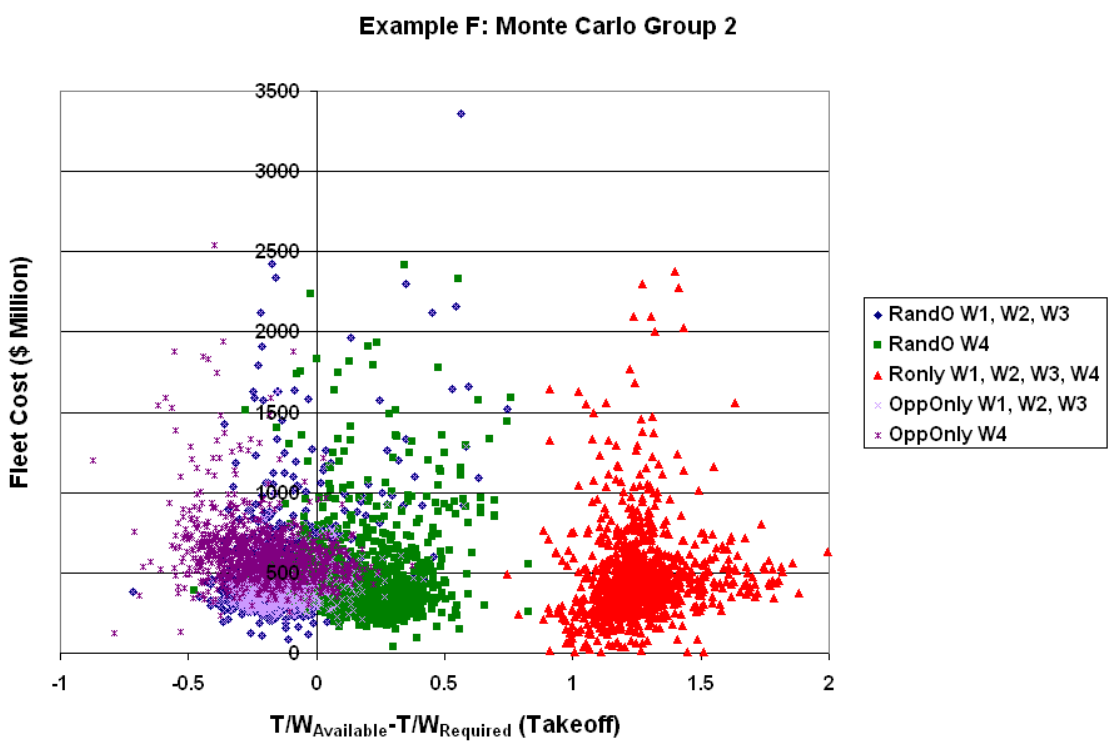

Figure B-32: Example F Monte Carlo Data for Actual Fleet Cost Metric - Uncertainty Group 2

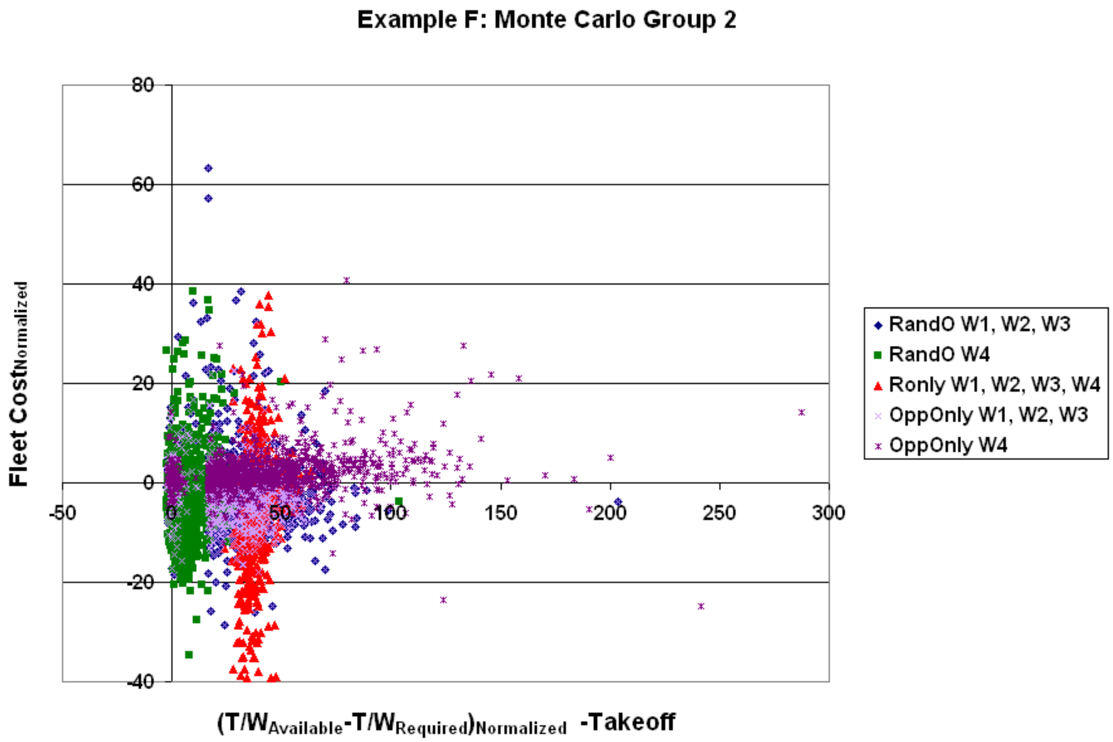

Figure B-33: Example F Monte Carlo Data for Normalized Fleet Cost Metric - Uncertainty Group 2

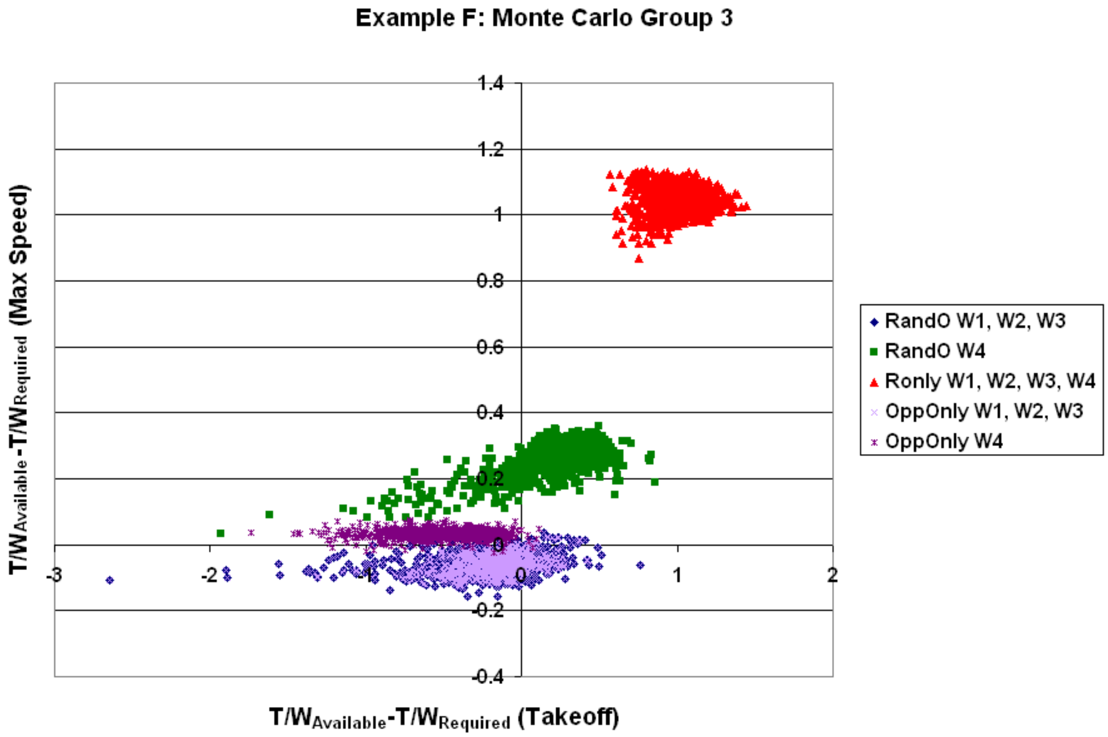

Figure B-34: Example F Monte Carlo Data for Actual T/W Metrics - Uncertainty Group 3

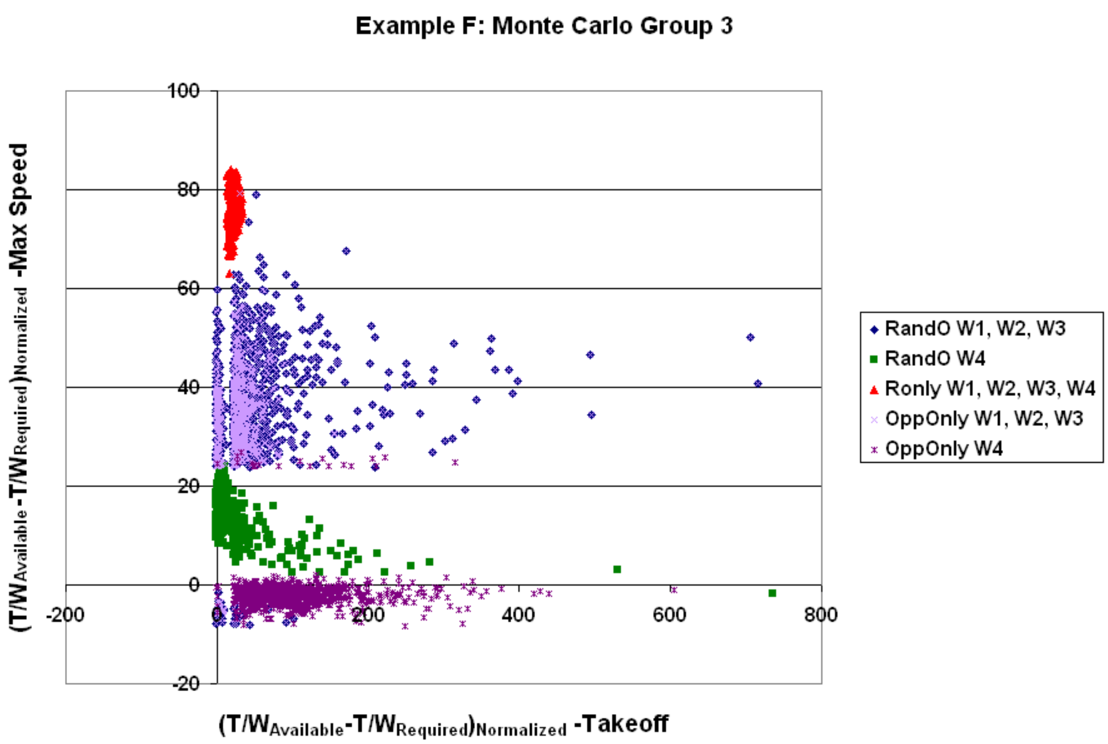

Figure B-35: Example F Monte Carlo Data for Normalized T/W Metrics - Uncertainty Group 3

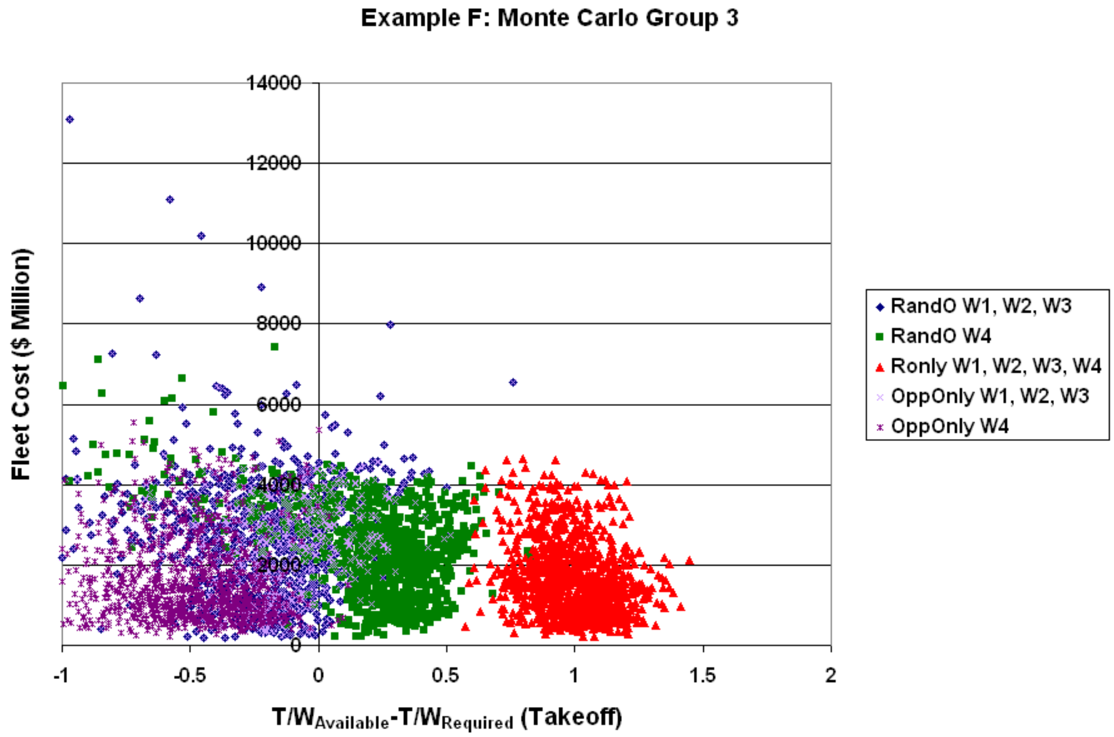

Figure B-36: Example F Monte Carlo Data for Actual Fleet Cost Metric - Uncertainty Group 3

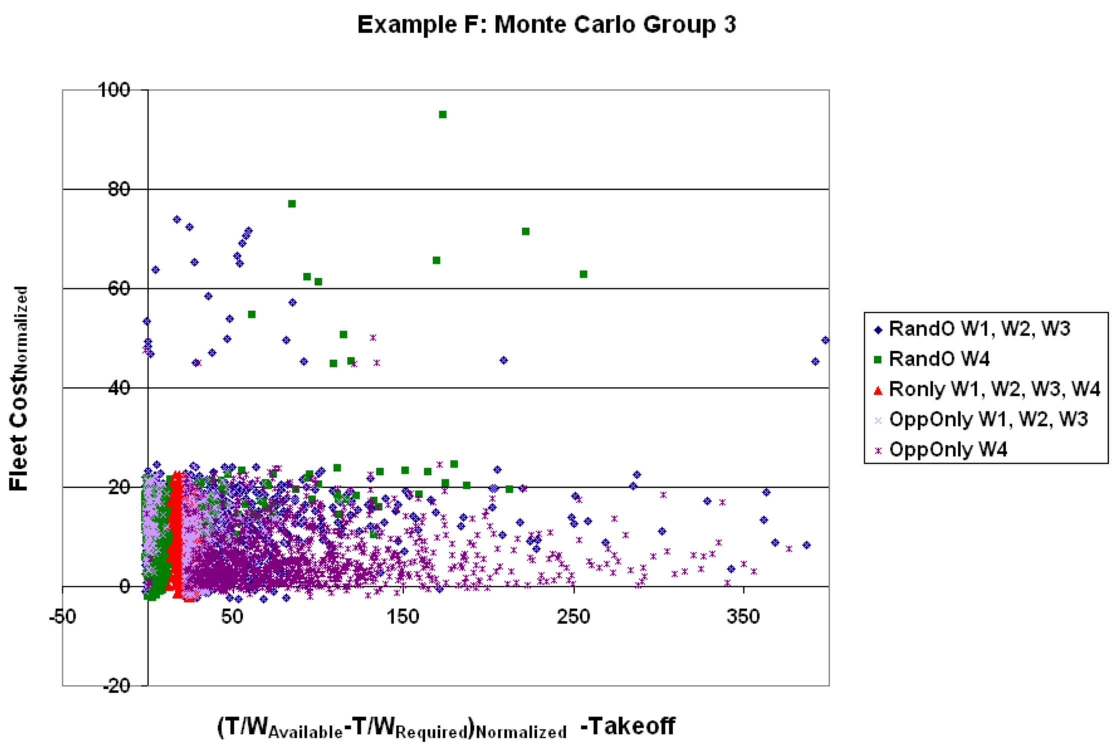

Figure B-37: Example F Monte Carlo Data for Normalized Fleet Cost Metric - Uncertainty Group 3

APPENDIX C

The following Pareto Plots for the Fleet Design Example Problem in Chapter 9 were generated in JMP Version 7.0.

Term	Orthog Estimate
CruiseDistance	20070.39
LoverD	-14317.48
CruiseVelocity	-11598.19
Mmax	8564.12
ThrustSL	-2716.22
Wpayload	1849.99
CostPerPound	1501.43
Clmax	-753.71
RevisitTime	655.71
WingArea	576.52
GroundTime	308.95
TOP	80.89

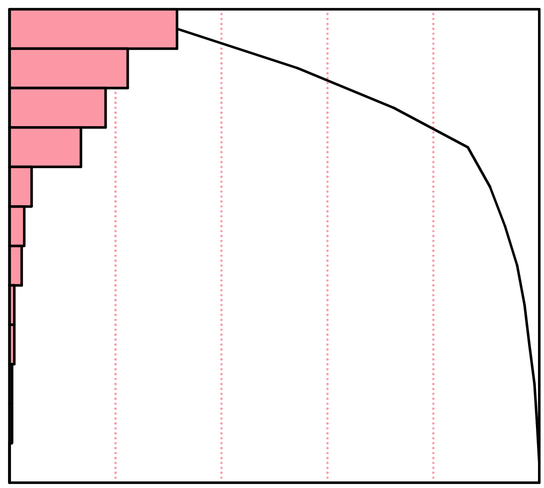

Figure C-1: Pareto Plot - Fleet Cost Metric

Term	Orthog Estimate
CruiseDistance	-1.954631
LoverD	1.295734
CruiseVelocity	1.132965
Mmax	-1.022956
ThrustSL	0.657639
WingArea	0.606348
TOP	0.474472
Clmax	0.442000
Wpayload	-0.214485
RevisitTime	-0.121785
CostPerPound	-0.068692
GroundTime	-0.007213

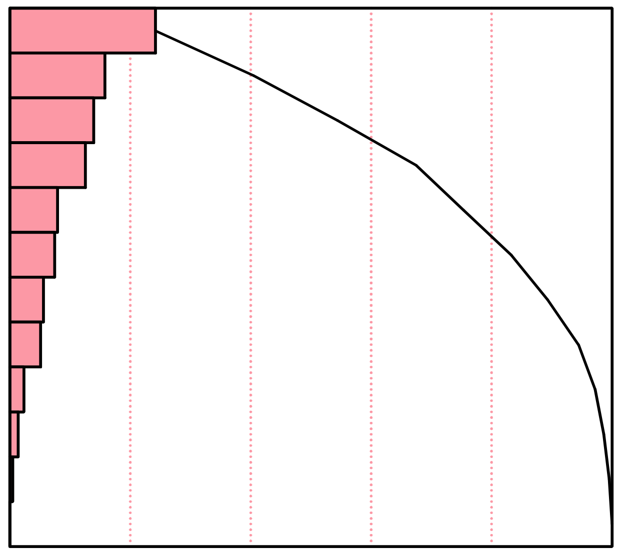

Figure C-2: Pareto Plot - (T/W Available – T/W Required) for Takeoff

Term	Orthog Estimate
ThrustSL	0.4973230
CruiseDistance	-0.2164974
LoverD	0.1193548
CruiseVelocity	0.0994230
Mmax	-0.0840595
Wpayload	-0.0517739
CostPerPound	-0.0132506
WingArea	0.0102277
RevisitTime	-0.0097635
Clmax	0.0086264
GroundTime	-0.0051552
TOP	-0.0033112

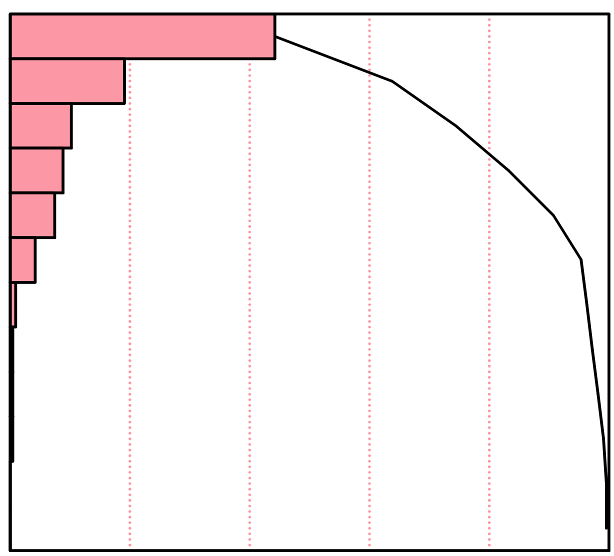

Figure C-3: Pareto Plot - (T/W Available – T/W Required) for Max Speed

APPENDIX D

The capabilities of Aircraft TCT Mission Analysis Simulation (ATMAS) were significantly increased between versions 2 and 3, in order to model all of the important uncertainty and characteristic of the Operation Desert Storm Scud Hunt. These characteristics were identified in the first three steps of the RandO Design Method.

ATMAS Version 3 was programmed in MATLAB and was run with multiple versions of the software program.

The primary systems of the code are the hunter/killer aircraft, tanker aircraft, weapons, targets, and decoys. The code is agent based where all of the primary systems are modeled as independent agents with multiple missions that operate on a time step basis in this simulation. The actions of the agent are based upon its current mission. The missions for each of the systems are discussed in the following sections.

This modeling and simulation environment is capable of modeling the following:

- Multiple bases
- Multiple refueling tracks
- Multiple tankers in single refueling orbit
- Multiple killboxes
- Multiple aircraft searching killbox at same time (wingmen)
- Cloud cover

<u>Hunter/Killer (H/K) Aircraft</u>

Weight modeling:

The weight of the aircraft is calculated at every time step, and is calculated from a form of the Brequet Range equation. [153]

Equation D-1

$$W_{new} = \exp\left(\frac{-\Delta s \cdot TSFC}{V \cdot \frac{L}{D}}\right) \cdot W_{org}$$

The lift to drag ratio is determined by assuming the following: small thrust-angle approximation and steady level flight. Considering these assumptions the equation for drag from Reference 147 is presented in Equation D-2.

Equation D-2

$$D = \left(\frac{\frac{1}{2}\rho \cdot V^2 \cdot C_{D0}}{\frac{W}{S}} + C_{D0,L} + \frac{\frac{W}{S}}{\frac{1}{2}\pi \cdot e \cdot AR \cdot \rho \cdot V^2}\right)$$

Since this is for steady level flight, the lift is equal to the weight.

A payload drop decreases the weight instantaneously and refueling increases the weight of the aircraft incrementally with each time step until the aircraft's maximum weight is reached.

Performance Assumptions:

Takeoff, climb, and landing are not modeled in the simulation. While altitude is taken into account in scenario, ascent and descent are not considered. The emphasis of this simulation tools is on the performance of the entire SoS and not on the performance of the aircraft. This simplification is adequate for SoS design at the OES Level (Chapter 2). If subsequent levels are considered for design, then either additional simulation tools will be required or the aircraft performance modeling in this tool will need to be expanded.

The missions for the H/K aircraft are as follows:

H/K Mission 0: At base and available

For this section of the code the aircraft is ready to be deployed and is at its home base.

H/K Mission 1: Deployed to Refuel with Tanker

Mission 1.1: Aircraft is deployed and on route to tanker to refuel before proceeding to designated search area (killbox).

Mission 1.2: Aircraft is assigned specific tanker once it reaches refueling orbit.

Mission 1.3: Aircraft is flying to meet up with the specified tanker.

Mission 1.4: Aircraft is flying along side the tanker, waiting to refuel once the tanker is available.

Mission 1.5: Aircraft is refueling.

H/K Mission 2: Aircraft is flying to designated killbox.

Aircraft is flying to the closest corner of the killbox to the tanker orbit.

H/K Mission 3: Aircraft is in killbox searching for aircraft.

Mission 3.1: Aircraft flies a switchback pattern to effectively cover the entire search area. The radius of the turns is based on the sensor radius of the aircraft, and the radius between the search swaths is based on the sensor radius of the aircraft. A different sensor system can be specified for day or night use. The aircraft is assumed to search at a constant specified altitude.

Mission 3.2: Pilot digresses from switchback search pattern and instead flies to a random location in the killbox. This is used to model unpredictable actions of the pilot while searching.

H/K Mission 4: Dashing to identified target and attacking if in range

Mission 4.1: Aircraft is dashing to scud missile launcher.

Mission 4.2: Aircraft is dashing to decoy.

In both potential variants of this mission, the aircraft flies to the identified target and releases a bomb when within range. The aircraft loiters around the target location in a random search pattern until it can be confirmed that the target has been destroyed. If the weapon malfunctions or misses the target, the aircraft releases another weapon.

H/K Mission 5: Scud launch detected, dashing to locate launcher

Once a scud missile is launched the nearest two available hunter/killer aircraft are routed to try and locate the launcher. Once the aircraft reach the estimated launch site they commence in a random search pattern for a specified period of time. If no targets are detected the aircraft return to their previous mission.

H/K Mission 6: Returning to Search Area

In some cases after leaving a killbox, an aircraft will return to the search area before returning to base. One case may be that the aircraft left to refuel, and another may be that it left to search for a launcher after a missile has been launched. This mission phase addresses the travel period where the aircraft is returning to the killbox. The aircraft flies to the nearest corner of the killbox in relation to their current location.

H/K Mission 7: Returning to Base

If the aircraft has expended all of its weapons, it has reached the end of its maximum mission time, or it is at bingo fuel and no tanker is available for refueling, the aircraft will fly back to its home base.

H/K Mission 8: Aircraft damaged but flying to base

Within this scenario an aircraft could be damaged from a variety of threats. Specific threats are not modeled but an aircraft attrition and malfunction factor are included. The malfunction factor includes not only malfunctions but battle damage as well.

H/K Mission 9: Aircraft refueling and reloading at base

During this mission the aircraft is refueling and reloading for the duration of its groundtime and is not available for other missions.

H/K Mission 10: Aircraft being repaired at base

This mission is when an aircraft has been damaged or malfunctioned and repairs are necessary. After the average repair time has passed the aircraft transitions into Mission 9.

H/K Mission 11: Aircraft eliminated from scenario

Due to the threats in the mission, it is possible that an aircraft may be shot down. This is modeled by the attrition factor. When this occurs the aircraft immediately becomes unavailable.

Multiple Aircraft in Search Group

Multiple hunter/killer aircraft are sent to search the same killbox at the same time. This is how wingmen are modeled. This simulation allows the number of aircraft searching one box to be a variable.

<u>Tanker Aircraft</u>

Within ATMAS Version 3, tankers are stationed at user specified bases and fly to user specified track patterns. The user can specify for multiple tankers to be flying in the same pattern, thereby creating a tanker cell. If a new tanker is needed to fill a hole in the cell a new tanker is created.

Aircraft creation:

If there is not a tanker assigned to the space in the cell, the code searches the available bases to determine if there is an available tanker. If there are multiple available tankers, the code selects the nearest tanker to fill the hole in the cell space. If no available aircraft is identified a new aircraft is created for the scenario.

When a new aircraft is created for the scenario there are several parameters that are set:

The aircraft is created at the nearest base, and the new aircraft is assigned to be in the track pattern of interest. It is assigned a spot (starting with the highest available spot) in the track pattern. Multiple tankers are flying the same track but at different altitudes. The aircraft will always return to its home base in the scenario.

There are two types of tankers that have been included in the simulation: KC-135s and KC-10s.[51] The user specifies the percentage of each type of tanker used in the simulation. The type of tanker determines all of the performance and refueling characteristics of the aircraft.

Weight modeling:

The weight of the aircraft is calculated at every time step and is calculated from a form of the Brequet Range equation (D-1). [153]

$$W_{new} = \exp\left(\frac{-\Delta s \cdot TSFC}{V \cdot \frac{L}{D}}\right) \cdot W_{org}$$

The lift to drag ratio is determined by assuming the following: small thrust-angle approximation and steady level flight. Considering these assumptions the equation for drag from Reference 147 was presented in Equation D-2.

$$D = \left(\frac{\frac{1}{2}\rho \cdot V^2 \cdot C_{D0}}{\frac{W}{S}} + C_{D0,L} + \frac{\frac{W}{S}}{\frac{1}{2}\pi \cdot e \cdot AR \cdot \rho \cdot V^2}\right)$$

Since this is for steady level flight, the lift is equal to the weight. The weight of the tanker decreased incrementally per timestep based on the refueling rate of the aircraft.

If the minimum fuel amount is reached, the tanker leaves the track pattern and returns to its home base and its mission becomes Mission 4.

Performance Assumptions:

Takeoff, climb, and landing are not modeled in the simulation. While altitude is taken into account in scenario, ascent and descent are not considered. The emphasis of the tools of this simulation is on the performance of the entire SoS and not on the performance of the aircraft. This simplification is adequate for SoS design at the OES Level (Chapter 2). If subsequent levels are considered for design, then either additional simulation tools will be required or the aircraft performance modeling in this tool will need to be expanded.

Missions

The potential mission scenarios include:

Tanker Mission 0: Tanker at base and available

For this section of the code the aircraft is ready to be deployed and is at its home base. The fuel load of the aircraft is set to the maximum fuel for refueling for the next mission.

Tanker Mission 1: Tanker deployed to Refueling Pattern

Aircraft is enroute to refueling pattern.

Tanker Mission 2: Tanker flying in Refueling Pattern

The tanker flies a simplified ellipse pattern where the long sides of the ellipse are straight lines. The basic pattern is similar to that shown for the tanker orbit in: Chapter 10 of FAA7610.

The long sides of the track pattern are called the longleg of the pattern and the shortleg of the pattern is twice the length of the turn radius of the pattern.

Tanker Mission 3: Tanker refueling Hunter/Killer Aircraft

Tanker is refueling a hunter/killer aircraft. The weight of the tanker descreases based upon the refueling rate for the tanker and the aircraft.

Tanker Mission 4: Tanker returning to base

The tanker has reached bingo fuel (meaning it has just enough fuel to return to base), has left the refueling pattern and is on its way back to its home base.

Weapons

Performance Assumptions:

The distance traveled by the weapon system to reach the target is only based upon the 2-D distance from the hunter/killer aircraft to the target. The emphasis of this simulation tools is on the performance of the entire SoS and not on the performance of the aircraft.

This simplification is adequate for SoS design at the OES Level (Chapter 2). If subsequent levels are considered for design, then either additional simulation tools will be required or the aircraft performance modeling in this tool will need to be expanded.

There are three possible ways the weapon system can malfunction:

- The weapon release system can malfunction, resulting in the aircraft remaining on the aircraft.
- The weapon could malfunction while in route and miss the target. This is modeled in conjunction with the possibility that the pilot could just miss the target, even if the weapon performs perfectly.
- The weapon could also fail to detonate once it reaches the target.

Missions

The potential mission scenarios include:

Weapon Mission 0: Weapon is on aircraft and is available.

Weapon Mission 1: Weapon is released from aircraft and is heading towards target.

Weapon Mission 2: Weapon is at target.

Weapon Mission 3: Release system of weapon malfunctions. Weapon remains on the aircraft and is unusable.

Weapon Mission 4: Malfunctioned weapon is removed from aircraft at base and is unusable.

Weapon Mission 5: Weapon is lost.

Weapon was released but never made it to target.

Weapon Mission 6: Weapon was destroyed at target.

Weapon Mission 7: Weapon was destroyed at original target location, but target unaffected. While the weapon made it to its specified destination, the target had moved.

Targets

Performance Assumptions:

The mobile missile launchers were modeled to have two different speeds. One speed was for moving on roads and the other was its average off-road speed.

Targets would not emerge from hiding if hunter/killer aircraft were around. If an aircraft was around, the target would hide for a user specified amount of time.

Missions

The potential mission scenarios include:

Target Mission 1: Mobile missile launcher is hiding before firing.

If a missile launcher is hiding, it is not detectable by the hunter/killer aircraft.

Target Mission 2: Mobile missile launcher is moving to launch site.

Target Mission 3: Mobile missile launcher is launching scud.

Target Mission 4: Mobile missile launcher is moving to hiding location after having fired missile.

Target Mission 5: Mobile missile launcher is hiding after fired missile.

Target Mission 6: Mobile missile launcher is moving to new hiding location. It is assumed that in order to move to a new hiding location the launcher will travel on a nearby road. This is factored into the speed that the mobile missile launcher could travel.

Target Mission 7: Mobile missile launcher is destroyed.

Target Mission 8: Mobile missile launcher is damaged.

Target Mission 9: Mobile missile launcher is out of scud missiles.

Decoys

There were two types of decoys modeled. The first type was randomly distributed throughout the search area. The purpose of these decoys was to mislead the pilots into believing that launchers were located in these areas.

The second type was located near the actual model scud launchers. The purpose of these decoys was to distract the pilots from the actual mobile launchers.

The simulation was setup so that the number of decoys was evenly distributed between these groups.

Missions

The potential mission scenarios include:

Decoy Mission 1: Decoy is hiding.

If a decoy is hiding, it is undetectable to the hunter/killer aircraft.

Decoy Mission 2: Decoy is moving to decoy location.

Decoy Mission 3: Decoy is in decoy position.

Decoy Mission 4: Decoy is destroyed.

If a decoy is destroyed it is apparent that this "target" is no longer of interest. If a decoy is hit, but not destroyed the hunter/killer aircraft will continue to strike the decoy until it is determined to be eliminated in the scenario.

Weather

Additionally cloud cover was modeled such that it impedes the visibility of the ground for the searching aircraft.

The area of interest is divided into boxes. The size of the box is specified by the user. For the example problem a box of size 10x10nm was used. Based upon the likelihood of the visibility being restricted by cloud cover, the weather box is either given a flag with a value of 1 or 0. If this weather flag is 0, the box is covered by clouds, otherwise the box is clear. The weather was updated based on a user specified amount of time. For the Operation Desert Storm Scud Hunt example problem, the weather was updated every 15 minutes.

APPENDIX E

The following Pareto Plots for the Operation Desert Storm Scud Hunt Example Problem in Chapter 11 were generated in JMP Version 7.0.

Term	Orthog Estimate
weaponrange	3.245161
HKRangeType(1)	-3.105082
numberofHKflyingtogether	3.079355
HKPropulsionType(2)	2.887181
AvgRepairTimeforMalfHK(1)	-2.871617
numtargets	2.870969
LengthofEngagement	2.747103
TargetAverageHidingTimeHours_hour	2.669281
MaxtimetoSearchforFiredScuds	-2.669281
HKDASType(1)	-2.529202
HKCASType(2)	-2.249044
HKAttrition(1)	-2.249044
ProbWeaponMalfunction1	-2.171223
WeatherMaxLikelyCloudiness	-2.140094
WeaponTracking	2.078853
HKMaxMissionTimeType(1)	1.891065
MINtargetsetuptime_min	-1.859936
HKSenradType(2)	-1.828808
HKMalfunction(2)	1.828808
ProbWeaponMalfunction3	1.719857
HKLASType(1)	-1.688729
HKLASType(2)	1.688729
HKTOGWType(2)	1.470828
HKRangeType(2)	-1.330749
numberofTankersinPattern	1.002030
HKPropulsionType(1)	0.988334
WeatherMinLikelyCloudiness	-0.926077
HKTOGWType(1)	-0.910513
HKAttrition(2)	0.894948
HKDASType(2)	-0.630355
ProbWeaponMalfunction2	0.536969
LengthSearchArea	-0.505840
HKCASType(1)	-0.381326
NumberofDecoy	-0.303504
RandomPilotfactor	0.287907
MINtargetwindowoppafterfiring_min	-0.256811
HKGroundTimeType	-0.194554
HKMalfunction(1)	0.116732
AvgRepairTimeforMalfHK(2)	-0.116732
HKSenradType(1)	-0.085604
WeightofWeapon	-0.054475
HKMaxMissionTimeType(2)	0.023346

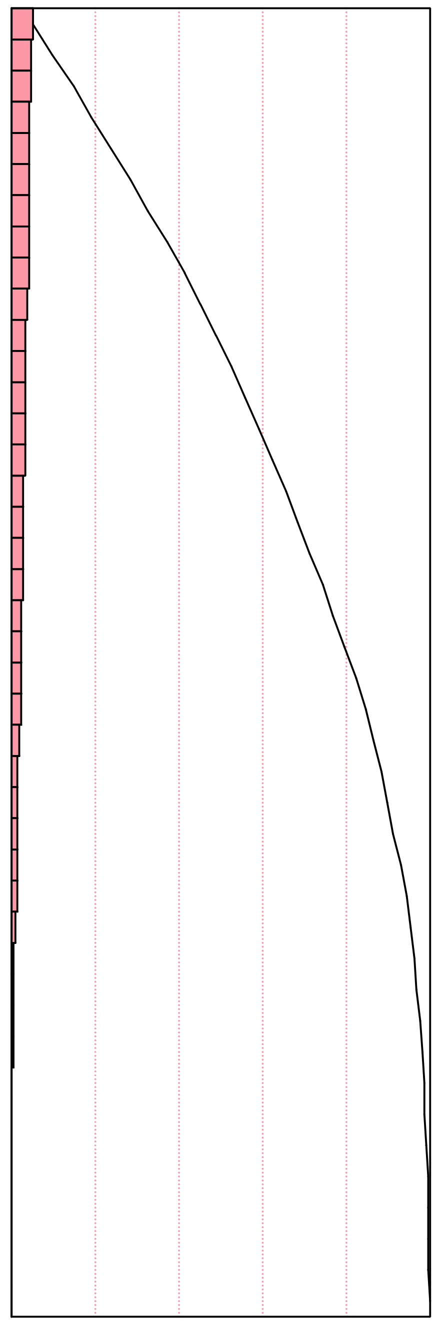

Figure E-1: Pareto Plot – Number of Scuds Launched

Term	Orthog Estimate
HKSenradType(2)	1.439700
numberofHKflyingtogether	1.348430
HKRangeType(1)	-1.315185
numtargets	1.299365
AvgRepairTimeforMalfHK(1)	-1.237363
LengthSearchArea	-0.848255
HKSenradType(1)	0.785998
HKRangeType(2)	-0.723741
LengthofEngagement	0.708177
HKDASType(1)	-0.677048
HKCASType(2)	0.645919
TargetAverageHidingTimeHours_hour	0.630355
HKMalfunction(1)	0.583662
ProbWeaponMalfunction1	-0.568098
HKTOGWType(2)	-0.552533
WeaponTracking	-0.540907
WeightofWeapon	0.490276
HKTOGWType(1)	0.459147
HKPropulsionType(1)	-0.412454
HKAttrition(2)	-0.412454
HKMaxMissionTimeType(2)	0.381326
ProbWeaponMalfunction2	-0.365762
NumberofDecoy	0.350197
WeatherMinLikelyCloudiness	0.319069
HKGroundTimeType	0.303504
HKAttrition(1)	0.287940
WeatherMaxLikelyCloudiness	0.272376
ProbWeaponMalfunction3	-0.256811
HKPropulsionType(2)	0.241247
HKMaxMissionTimeType(1)	-0.210118
MINtargetsetuptime_min	0.210118
MaxtimetoSearchforFiredScuds	-0.178990
HKMalfunction(2)	-0.178990
HKLASType(1)	0.163425
weaponrange	0.101168
HKLASType(2)	-0.101168
HKDASType(2)	-0.085604
RandomPilotfactor	-0.054488
AvgRepairTimeforMalfHK(2)	-0.054475
MINtargetwindowoppafterfiring_min	-0.038911
numberofTankersinPattern	0.020601
HKCASType(1)	-0.007782

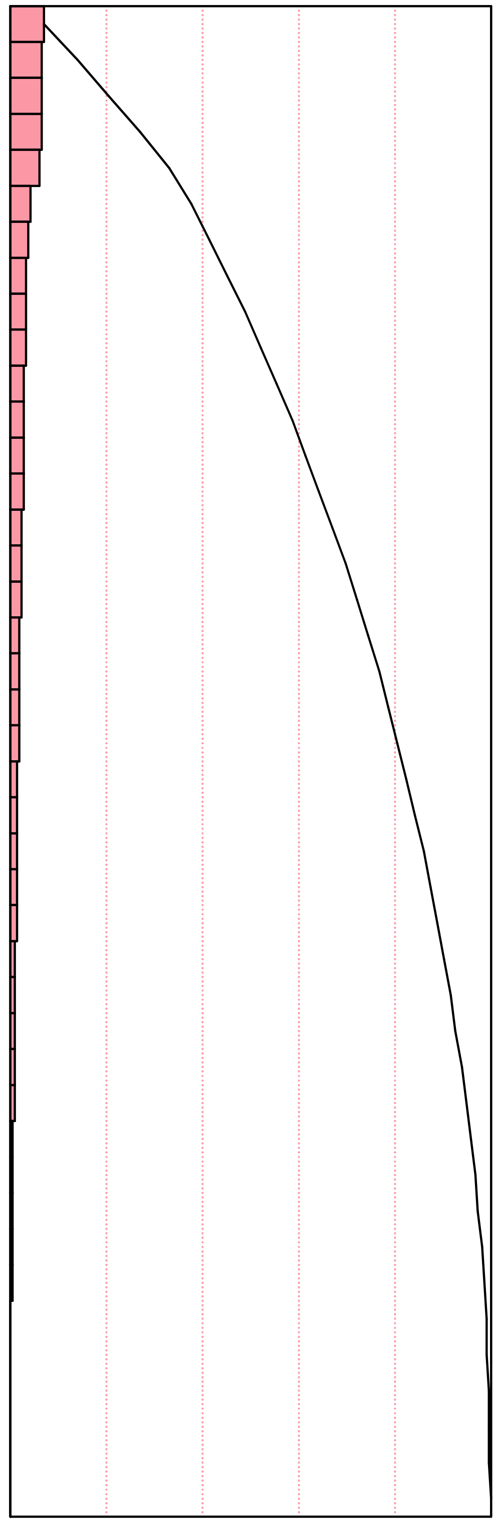

Figure E-2: Pareto Plot – Number of Mobile Scud Launchers Destroyed

Term	Orthog Estimate
NumberofDecoy	21.97682
HKSenradType(2)	8.93392
HKSenradType(1)	8.74715
numtargets	-5.08868
HKTOGWType(2)	-4.99615
MINtargetsetuptime_min	4.29575
numberofHKflyingtogether	3.68037
HKLASType(1)	3.51754
HKTOGWType(1)	-3.39302
HKMalfunction(1)	-3.26851
HKAttrition(2)	-2.91053
LengthSearchArea	-2.72376
AvgRepairTimeforMalfHK(1)	2.21013
HKMaxMissionTimeType(2)	2.17900
HKMalfunction(2)	2.07005
HKMaxMissionTimeType(1)	-1.68095
TargetAverageHidingTimeHours_hour	1.64982
HKRangeType(1)	1.46305
weaponrange	1.29184
HKPropulsionType(1)	-1.15176
LengthofEngagement	0.96499
HKCASType(1)	0.91829
WeatherMaxLikelyCloudiness	0.88717
HKPropulsionType(2)	0.80934
WeightofWeapon	-0.79378
HKGroundTimeType	-0.79378
MINtargetwindowoppafterfiring_min	0.74709
ProbWeaponMalfunction3	-0.65370
HKCASType(2)	-0.48249
ProbWeaponMalfunction1	-0.42024
MaxtimetoSearchforFiredScuds	0.42024
WeaponTracking	0.38959
HKAttrition(1)	-0.38911
RandomPilotfactor	0.37359
HKRangeType(2)	0.37354
numberofTankersinPattern	-0.30170
HKLASType(2)	0.29572
AvgRepairTimeforMalfHK(2)	0.29572
HKDASType(2)	-0.23346
ProbWeaponMalfunction2	0.18677
WeatherMinLikelyCloudiness	0.17121
HKDASType(1)	-0.10895

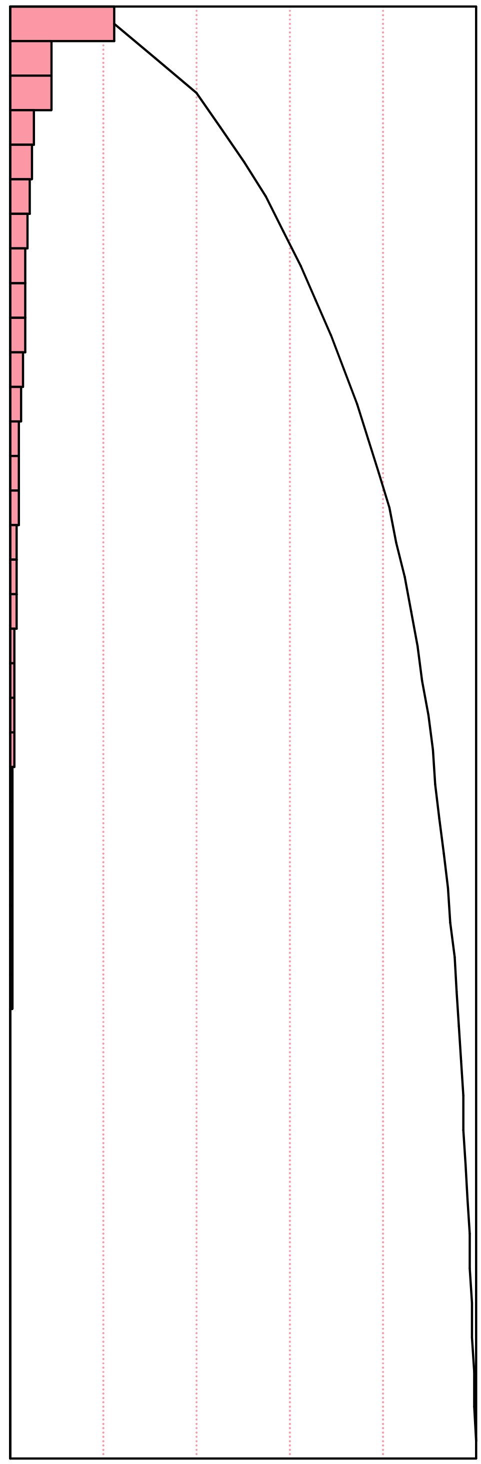

Figure E-3: Pareto Plot – Number of Decoys Destroyed

Term	Orthog Estimate
numberofHKflyingtogether	146.4973
LengthSearchArea	-146.3280
HKGroundTimeType	40.8486
HKAttrition(1)	-21.6889
HKMaxMissionTimeType(2)	-20.6149
HKMaxMissionTimeType(1)	-17.4398
HKRangeType(1)	-14.9184
HKTOGWType(1)	-14.3892
RandomPilotfactor	13.5320
LengthofEngagement	-12.9262
HKPropulsionType(2)	11.4787
HKRangeType(2)	-11.4631
HKCASType(1)	9.7510
HKLASType(1)	7.9611
HKPropulsionType(1)	7.9611
HKSenradType(1)	6.9339
HKCASType(2)	6.4514
HKDASType(1)	-6.2179
MINtargetsetuptime_min	5.7510
NumberofDecoy	-4.8950
HKSenradType(2)	4.2568
ProbWeaponMalfunction2	-3.9300
WeatherMaxLikelyCloudiness	3.7743
WeatherMinLikelyCloudiness	-3.7277
ProbWeaponMalfunction3	3.6031
HKMalfunction(2)	-3.4164
weaponrange	3.2140
HKTOGWType(2)	-3.1673
TargetAverageHidingTimeHours_hour	-3.0895
MINtargetwindowoppafterfiring_min	2.8872
HKDASType(2)	-2.7004
MaxtimetoSearchforFiredScuds	2.6848
WeightofWeapon	-2.2646
AvgRepairTimeforMalfHK(1)	2.0467
HKLASType(2)	-1.8911
HKMalfunction(1)	-1.7510
WeaponTracking	1.5442
AvgRepairTimeforMalfHK(2)	1.1595
numtargets	-0.9180
HKAttrition(2)	0.9105
numberofTankersinPattern	0.2371
ProbWeaponMalfunction1	-0.0545

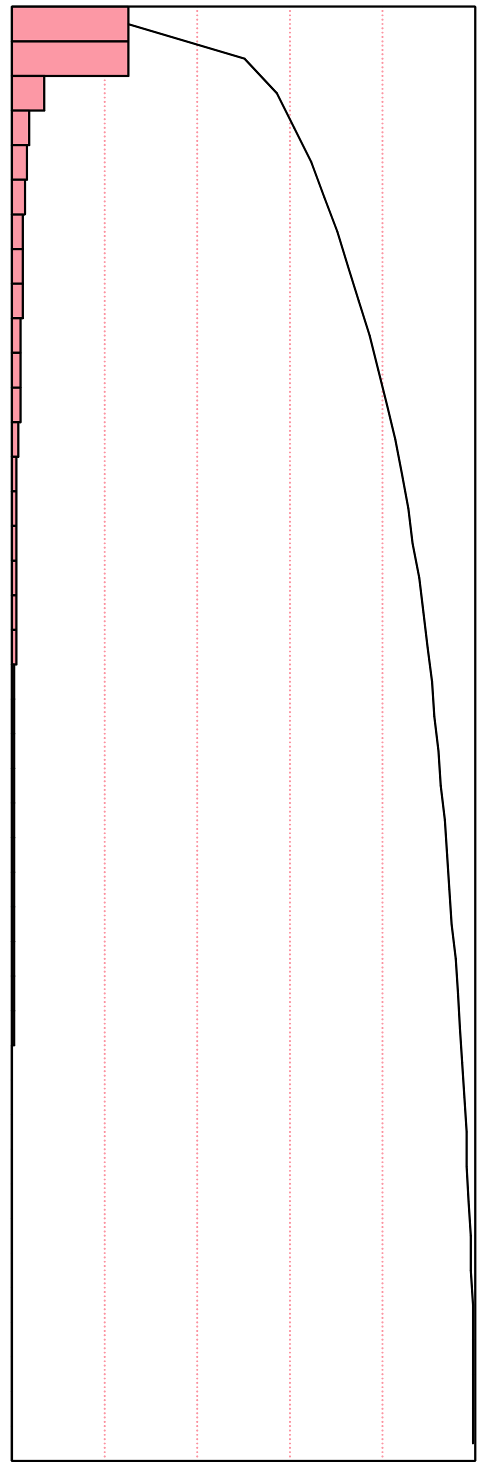

Figure E-4: Pareto Plot – Number of Hunter/Killer Aircraft

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